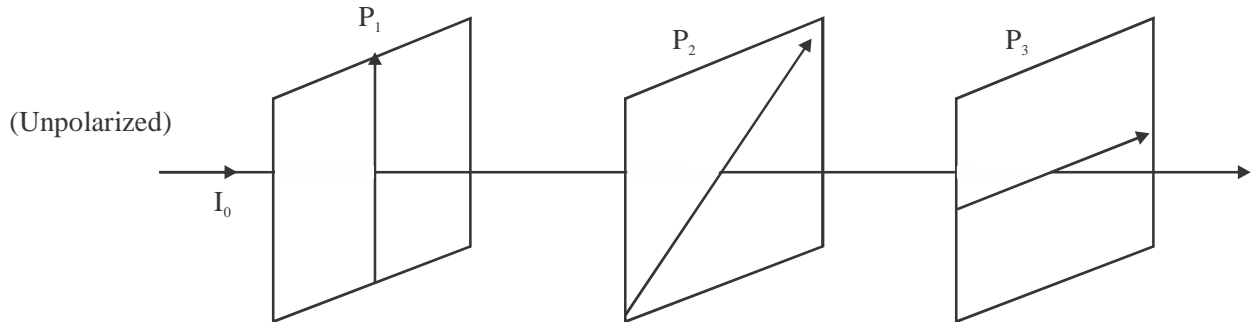


**CSIR-UGC-NET/JRF- DEC. - 2011**  
**PHYSICAL SCIENCES BOOKLET - [B]**

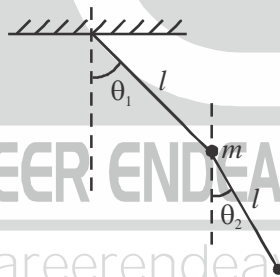
**Part B**

21. Consider three polarizers  $P_1, P_2$  and  $P_3$  placed along an axis as shown in the Figure.



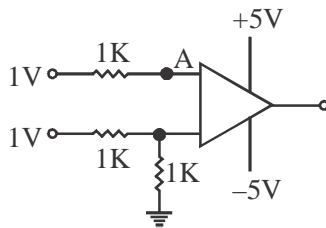
The pass axis of  $P_1$  and  $P_3$  are at right angles to each other while the pass axis of  $P_2$  makes an angle  $\theta$  with that of  $P_1$ . A beam of unpolarized light of intensity  $I_0$  is incident on  $P_1$  as shown. The intensity of light emerging from  $P_3$  is

- (a) 0                      (b)  $\frac{I_0}{2}$                       (c)  $\frac{I_0}{8} \sin^2 2\theta$                       (d)  $\frac{I_0}{4} \sin^2 2\theta$
22. A double pendulum consists of two point masses  $m$  attached by massless strings of length  $l$  as shown in the figure:



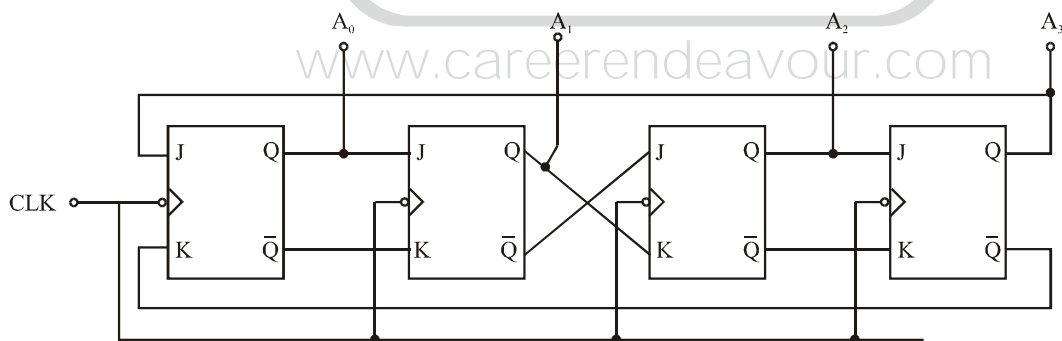
The kinetic energy of the pendulum is :

- (a)  $\frac{1}{2} m \ell^2 [\dot{\theta}_1^2 + \dot{\theta}_2^2]$                       (b)  $\frac{1}{2} m \ell^2 [2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)]$
- (c)  $\frac{1}{2} m \ell^2 [\dot{\theta}_1^2 + 2\dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)]$                       (d)  $\frac{1}{2} m \ell^2 [2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 + \theta_2)]$
23. In the operational amplifier circuit below, the voltage at point A is



- (a) 1.0 V                      (b) 0.5 V                      (c) 0 V                      (d) -5.0V

24. A point particle of mass  $m$  carrying an electric charge  $q$  is attached to a spring of stiffness constant  $k$ . A constant electric field  $E$  along the direction of the spring is switched on for a time interval  $T$  (where  $T \ll \sqrt{m/k}$ ). Neglecting radiation loss, the amplitude of oscillation after the field is switched off is :
- (a)  $qE/k$                       (b)  $qET^2/m$                       (c)  $qE\sqrt{m}/Tk^{3/2}$                       (d)  $qET/\sqrt{mk}$
25. A constant force  $F$  is applied to a relativistic particle of rest mass  $m$ . If the particle starts from rest at  $t = 0$ , its speed after a time  $t$  is
- (a)  $Ft/m$                       (b)  $c \tanh\left(\frac{Ft}{mc}\right)$                       (c)  $c(1 - e^{-Ft/mc})$                       (d)  $\frac{Fct}{\sqrt{F^2t^2 + m^2c^2}}$
26. The potential of a diatomic molecule as a function of the distance  $r$  between the atoms is given by  $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$ . The value of the potential at equilibrium separation between the atoms is :
- (a)  $-4a^2/b$                       (b)  $-2a^2/b$                       (c)  $-a^2/2b$                       (d)  $-a^2/4b$
27. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e., points where the electric field vanishes) will be found inside the square?
- (a) 3                      (b) 4                      (c) 5                      (d) 7
28. A static charge distribution gives rise to an electric field of the form  $\vec{E} = \alpha(1 - e^{-r/R})\frac{\hat{r}}{r^2}$ , where  $\alpha$  and  $R$  are positive constants. The charge contained within a sphere of radius  $R$ , centred at the origin is:
- (a)  $\pi\alpha\epsilon_0 \frac{e}{R^2}$                       (b)  $\pi\alpha\epsilon_0 \frac{e^2}{R^2}$                       (c)  $4\pi\alpha\epsilon_0 \frac{R}{e}$                       (d)  $\pi\alpha\epsilon_0 \frac{R^2}{e}$
29. A counter consists of four flip-flops connected as shown in the figure.

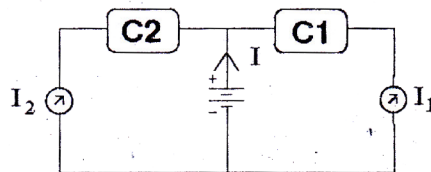


If the counter is initialized as  $A_0 A_1 A_2 A_3 = 0110$ , the state after the next clock pulse is

- (a) 1000                      (b) 0001                      (c) 0011                      (d) 1100
30. In a Young's double slit interference experiment, the slits are at a distance  $2L$  from each other and the screen is at a distance  $D$  from the slits. If a glass slab of refractive index  $\mu$  and thickness  $d$  is placed in the path of one of the beams, the minimum value of  $d$  for the central fringe to be dark is
- (a)  $\frac{\lambda D}{(\mu-1)\sqrt{D^2 + L^2}}$                       (b)  $\frac{\lambda D}{(\mu-1)L}$                       (c)  $\frac{\lambda}{(\mu-1)}$                       (d)  $\frac{\lambda}{2(\mu-1)}$



37. The energy of the first excited quantum state of a particle in the two-dimensional potential  $V(x, y) = \frac{1}{2}m\omega^2(x^2 + 4y^2)$  is:
- (a)  $2\hbar\omega$                       (b)  $3\hbar\omega$                       (c)  $\frac{3}{2}\hbar\omega$                       (d)  $\frac{5}{2}\hbar\omega$
38. The internal energy  $E$  of a system is given by  $E = \frac{bS^3}{VN}$ , where  $b$  is a constant and other symbols have their usual meaning. The temperature of this system is equal to
- (a)  $\frac{bS^2}{VN}$                       (b)  $\frac{3bS^2}{VN}$                       (c)  $\frac{bS^3}{V^3N}$                       (d)  $\left(\frac{S}{N}\right)^2$
39. Consider a particle in a one dimensional potential that satisfies  $V(x) = V(-x)$ . Let  $|\psi_0\rangle$  and  $|\psi_1\rangle$  denote the ground and the first excited states, respectively, and let  $|\psi\rangle = \alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$  be a normalized state with  $\alpha_0$  and  $\alpha_1$  being real constants. The expectation value  $\langle x \rangle$  of the position operator  $x$  in the state  $|\psi\rangle$  is given by
- (a)  $\alpha_0^2\langle\psi_0|x|\psi_0\rangle + \alpha_1^2\langle\psi_1|x|\psi_1\rangle$                       (b)  $\alpha_0\alpha_1[\langle\psi_0|x|\psi_1\rangle + \langle\psi_1|x|\psi_0\rangle]$   
(c)  $\alpha_0^2 + \alpha_1^2$                       (d)  $2\alpha_0\alpha_1$
40. A  $3 \times 3$  matrix  $M$  has  $Tr[M] = 6, Tr[M^2] = 26$  and  $Tr[M^3] = 90$ . Which of the following can be a possible set of eigenvalues of  $M$ ?
- (a)  $\{1, 1, 4\}$                       (b)  $\{-1, 0, 7\}$                       (c)  $\{-1, 3, 4\}$                       (d)  $\{2, 2, 2\}$
41. The perturbation  $H' = bx^4$ , where  $b$  is a constant, is added to the one dimensional harmonic oscillator potential  $V(x) = \frac{1}{2}m\omega^2x^2$ . Which of the following denotes the correction to the ground state energy to first order in  $b$ ?
- [Hint : The normalized ground state wave function of the one dimensional harmonic oscillator potential is  $\psi_0 = \left(\frac{m\omega}{h\pi}\right)^{1/4} e^{-\frac{m\omega x^2}{2h}}$ . You may use the following integral  $\int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx = a^{-n-\frac{1}{2}} \Gamma\left(n + \frac{1}{2}\right)$
- (a)  $\frac{3b\hbar^2}{4m^2\omega^2}$                       (b)  $\frac{3b\hbar^2}{2m^2\omega^2}$                       (c)  $\frac{3b\hbar^2}{2\pi m^2\omega^2}$                       (d)  $\frac{15b\hbar^2}{4m^2\omega^2}$
42. A battery powers two circuits  $C_1$  and  $C_2$  as shown in the figure



- The total current  $I$  drawn from the battery is estimated by measuring the currents  $I_1$  and  $I_2$  through the individual circuits. If  $I_1$  and  $I_2$  are both 200 mA and if the errors in their measurement are 3 mA and 4 mA respectively, the error in the estimate of  $I$  is:
- (a) 7.0 mA                      (b) 7.5 mA                      (c) 5.0 mA                      (d) 10.5 mA



50. A planet of mass  $m$  moves in the inverse square central force field of the Sun of mass  $M$ . If the semi-major and semi-minor axes of the orbit are  $a$  and  $b$ , respectively, the total energy of the planet is :

(a)  $-\frac{GMm}{a+b}$       (b)  $-GMm\left(\frac{1}{a}+\frac{1}{b}\right)$       (c)  $-\frac{GMm}{a}\left(\frac{1}{b}-\frac{1}{a}\right)$       (d)  $-GMm\frac{(a-b)}{(a+b)^2}$

51. Let  $|0\rangle$  and  $|1\rangle$  denote the normalized eigenstates corresponding to the ground and first excited state of a one dimensional harmonic oscillator. The uncertainty  $\Delta p$  in the state  $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ , is:

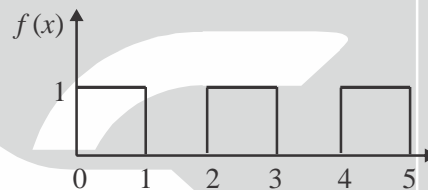
(a)  $\Delta p = \sqrt{\hbar m \omega} / 2$       (b)  $\Delta p = \sqrt{\hbar m \omega} / 2$       (c)  $\Delta p = \sqrt{\hbar m \omega}$       (d)  $\Delta p = \sqrt{2\hbar m \omega}$

52. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at  $770 \text{ cm}^{-1}$ , the approximate positions of the Stokes and the anti-Stokes lines are

(a) 481.5 nm and 520 nm      (b) 481.5 nm and 500 nm  
(c) 500 nm and 520 nm      (d) 500 nm and 600 nm

53. The graph of the function

$$f(x) = \begin{cases} 1 & \text{for } 2n \leq x \leq 2n+1 \\ 0 & \text{for } 2n+1 \leq x \leq 2n+2 \end{cases}$$



(Where  $n = 0, 1, 2, \dots$ ) is shown below.

Its Laplace transform  $\tilde{f}(s)$  is

(a)  $\frac{1+e^{-s}}{s}$       (b)  $\frac{1-e^{-s}}{s}$       (c)  $\frac{1}{s(1+e^{-s})}$       (d)  $\frac{1}{s(1-e^{-s})}$

54. The energy levels of electrons of mass ' $m$ ' and charge ' $e$ ' confined in an area  $A$  in the  $xy$ -plane with a uniform magnetic field  $B$  applied in the  $z$ -direction are given by  $E_n = \left(n + \frac{1}{2}\right) \frac{eB\hbar}{mc}$ ,  $n = 0, 1, 2, \dots$

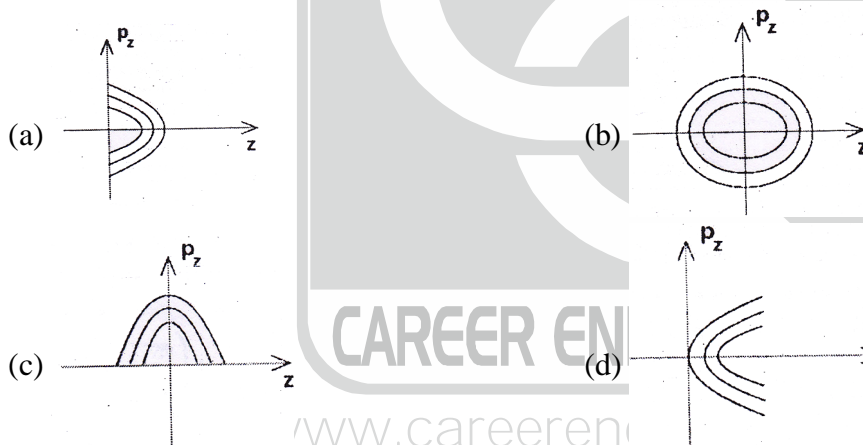
The degeneracy of each level is  $\frac{eBA}{\hbar c}$ . The lowest level is completely filled the other are empty. The

fermi energy  $\frac{\hbar^2 N}{2\pi mA}$ , where  $N$  is the total number of electrons, is:

- (a) coincident with the  $n = 0$  level      (b) coincident with the  $n = 1$  level  
(c) midway between the  $n = 0$  and the  $n = 1$  levels  
(d) midway between the  $n = 1$  and the  $n = 2$  levels.
55. An annulus of mass  $M$  made of a material of uniform density has inner and outer radii ' $a$ ' and ' $b$ ' respectively. Its principal moment of inertia along the axis of symmetry perpendicular to the plane of the annulus is :

(a)  $\frac{1}{2}M\frac{(b^4+a^4)}{(b^2-a^2)}$       (b)  $\frac{1}{2}M\pi(b^2-a^2)$       (c)  $\frac{1}{2}M(b^2-a^2)$       (d)  $\frac{1}{2}M(b^2+a^2)$

56. The principal value of the real integral  $I = \int_{-3}^{+3} \frac{dx}{x^2 + 3x + 2}$  is:
- (a)  $\frac{3\pi}{2}$                       (b)  $\ln\left(\frac{2}{5}\right)$                       (c)  $\infty$                       (d) 0
57. The minimum energy of an electron (the rest mass of which is 0.5 MeV) that can emit Cherenkov radiation while passing through water (of refractive index 1.5) is approximately
- (a) 1.0 MeV                      (b) 3.0 MeV                      (c) 0.6 MeV                      (d) 0.5 MeV
58. A heater and a thermocouple are used to measure and control temperature  $T$  of a sample at  $T_0 = 250^\circ\text{C}$ . A feedback circuit supplies power 'P' to the heater according to the equation  $P = P_0 + G(T_0 - T) - D \frac{dT}{dt}$  with appropriately tuned values of the coefficients G and D. In order to maintain temperature stability in the presence of an external heat perturbation which causes small but rapid fluctuations of temperature, it is necessary to
- (a) decrease  $D$                       (b) increase  $D$                       (c) decrease  $G$                       (d) increase  $G$
59. The trajectory on the  $zp_z$ -plane (phase-space trajectory) of a ball bouncing perfectly elastically off a hard surface at  $z = 0$  is given by approximately by (neglect friction) :

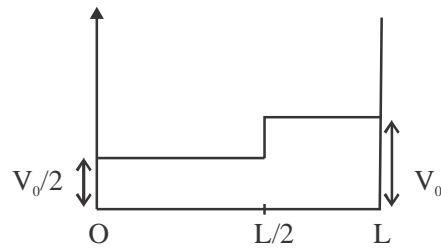


60. According to the shell model the spin and parity of the two nuclei  ${}_{51}^{125}\text{Sb}$  and  ${}_{38}^{89}\text{Sr}$  are, respectively.
- (a)  $\left(\frac{5}{2}\right)^+$  and  $\left(\frac{5}{2}\right)^+$                       (b)  $\left(\frac{5}{2}\right)^+$  and  $\left(\frac{7}{2}\right)^+$                       (c)  $\left(\frac{7}{2}\right)^+$  and  $\left(\frac{5}{2}\right)^+$                       (d)  $\left(\frac{7}{2}\right)^+$  and  $\left(\frac{7}{2}\right)^+$
61. The wave function of a particle at time  $t = 0$  is given by  $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle)$ , where  $|u_1\rangle$  and  $|u_2\rangle$  are the normalized eigenstates with eigenvalues  $E_1$  and  $E_2$  respectively, ( $E_2 > E_1$ ). The shortest time after which  $|\psi(t)\rangle$  will become orthogonal to  $|\psi(0)\rangle$  is
- (a)  $\frac{-\hbar\pi}{2(E_2 - E_1)}$                       (b)  $\frac{\hbar\pi}{E_2 - E_1}$                       (c)  $\frac{\sqrt{2}\hbar\pi}{E_2 - E_1}$                       (d)  $\frac{2\hbar\pi}{E_2 - E_1}$



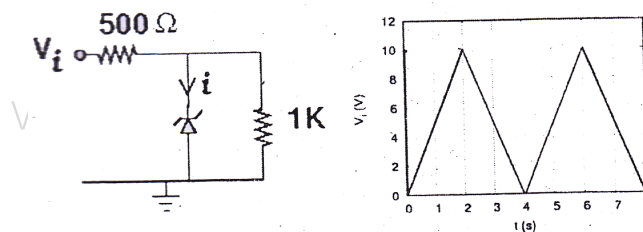


69. A constant perturbation as shown in the figure below acts on a particle of mass  $m$  confined in a infinite potential well between 0 and  $L$ .

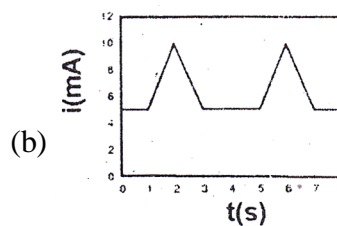
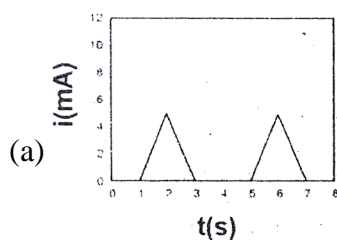


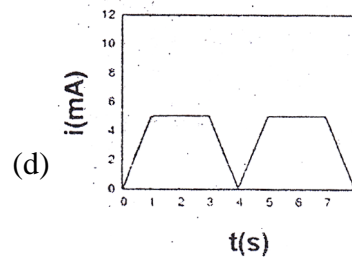
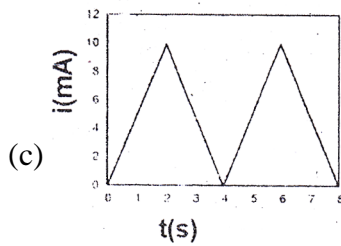
The first-order correction to the ground state energy of the particle is

- (a)  $\frac{V_0}{2}$                       (b)  $\frac{3V_0}{4}$                       (c)  $\frac{V_0}{4}$                       (d)  $\frac{3V_0}{2}$
70. An atom of mass  $M$  can be excited to a state of mass  $M + \Delta$  by photon capture. The frequency of a photon which can cause this transition is:
- (a)  $\Delta c^2 / 2h$                       (b)  $\Delta c^2 / h$                       (c)  $\Delta^2 c^2 / 2Mh$                       (d)  $\Delta(\Delta + 2M)c^2 / 2Mh$
71. The excitation of a three-dimensional solid are bosonic in nature with their frequency  $\omega$  and wave-number  $k$  are related by  $\omega \propto k^2$  in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to
- (a)  $T^{1/2}$                       (b)  $T$                       (c)  $T^{3/2}$                       (d)  $T^3$
72. Gas molecules of mass 'm' are confined in a cylinder of radius  $R$  and height  $L$  (with  $R \gg L$ ) kept vertically in the Earth's gravitational field. The average energy of the gas at low temperatures (such that  $mgL \gg k_B T$ ) is given by
- (a)  $Nk_B T / 2$                       (b)  $3Nk_B T / 2$                       (c)  $2Nk_B T$                       (d)  $5Nk_B T / 2$
73. The figure below shows a voltage regulator utilizing a Zener diode of breakdown voltage 5V and a positive triangular wave input of amplitude 10V.



For  $V_i > 5V$ , the Zener regulates the output voltage by channeling the excess current through it self. Which of the following waveforms shows the current 'i' passing through the Zener diode?





74. A constant electric current  $I$  in an infinitely long straight wire is suddenly switched on at  $t = 0$ . The vector potential at a perpendicular distance  $r$  from the wire is given by

$$\vec{A} = \frac{\hat{k}\mu_0 I}{2\pi} \ln \left[ \frac{1}{r} \left( ct + \sqrt{c^2 t^2 - r^2} \right) \right].$$
 The electric field at a distance  $r (< ct)$  is:

- (a) 0      (b)  $\frac{\mu_0 I}{2\pi t} \frac{1}{\sqrt{2}} (\hat{i} - \hat{j})$       (c)  $\frac{c\mu_0 I}{2\pi\sqrt{c^2 t^2 - r^2}} \frac{1}{\sqrt{2}} (\hat{i} + \hat{j})$       (d)  $-\frac{c\mu_0 I}{2\pi\sqrt{c^2 t^2 - r^2}} \hat{k}$

75. Monochromatic light of wavelength 660 nm and intensity  $100 \text{ mW/cm}^2$  falls on a solar cell of area  $30 \text{ cm}^2$ . The conversion efficiency of the solar cell is 10%. If each converted photon results in an electron-hole pair, what is the maximum circuit current supplied by the solar cell? (Take  $h = 6.6 \times 10^{-34} \text{ J-s}$ ,  $c = 3 \times 10^8 \text{ m/s}$  and  $e = 1.6 \times 10^{-19} \text{ C}$ ).

- (a) 160 mA      (b) 320 mA      (c) 1600 mA      (d) 3200 mA