PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. Consider an anti-symmetric tensor P_0 with the indices i and j running from 1 to 5. The number of independent components of the tensor is

(a) 3 (b) 10 (c) 9 (d) 6

2. The value of the integral $\oint_C \frac{e^z \sin(z)}{z^2} dz$, where the contour C is the unit circle $|z-2|=1$, is

(a) $2\pi i$ (b) $4\pi i$ (c) πi (d) 0

3. The eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

(a) 5, 2, –2 (b) –5, –1, –1 (c) 5, 1, –1 (d) –5, 1, 1

4. If $f(x) = \begin{cases} 0 & \text{for} \quad x < 3 \\ x-3 & \text{for} \quad x \geq 3 \end{cases}$, then the Laplace transform of $f(x)$ is

(a) $s^{-2} e^{3s}$ (b) $s^{2} e^{-3s}$ (c) s^{-2} (d) $s^{-2} e^{-3s}$

5. The valence electrons do not directly determine the following property of a metal.

(a) Electrical conductivity (b) Thermal conductivity
(c) Shear modulus (d) Metallic lustre

6. Consider X-ray diffraction from a crystal with a face-centered-cubic (fcc) lattice. The lattice plane for which there is NO diffraction peak is

(a) (2, 1, 2) (b) (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

7. The Hall coefficient, R_H of sodium depends on

(b) The effective charge carrier mass and carrier density
(b) The charge carrier density and relaxation time
(c) The charge carrier density only
(d) The effective charge carrier mass

8. The Bloch theorem states that within a crystal, the wave function $\psi(\vec{r})$, of an electron has the form

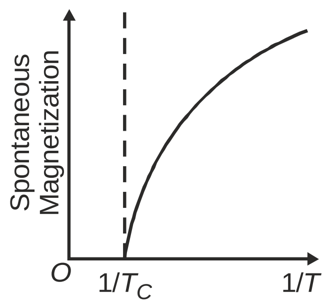
(a) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}\cdot\vec{r}}$, where $u(\vec{r})$ is an arbitrary function and $\vec{k}$ is an arbitrary vector.

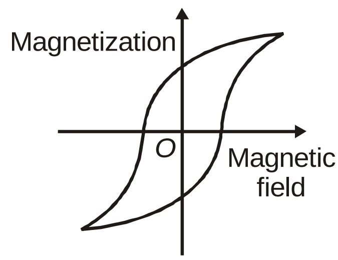
(b) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}\cdot\vec{r}}$, where $u(\vec{r})$ is an arbitrary function and $\vec{G}$ is a reciprocal lattice vector.

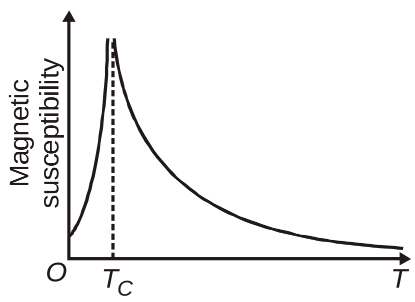
(c) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}\cdot\vec{r}}$, where $u(\vec{r}) = u(\vec{r}+\vec{A})$, $\vec{A}$ is a lattice vector and $\vec{G}$ is a reciprocal lattice vector.

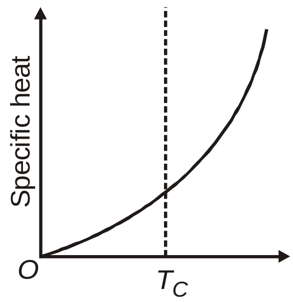
(d) $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}\cdot\vec{r}}$, where $u(\vec{r}) = u(\vec{r}+\vec{A})$, $\vec{A}$ is a lattice vector and $\vec{k}$ is an arbitrary vector.

9. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represent the property of the medium ?
(T_C is the Curie temperature)

(a) 

(b) 

(c) 

(d) 

10. The thermal conductivity of a given material reduces, when it undergoes a transition from its normal state to the superconducting state. The reason is
(a) The cooper pairs cannot transfer energy to the lattice.
(b) Upon the formation of Cooper pairs, the lattice becomes less efficient in heat transfer.
(c) The electrons in the normal state loose their ability to transfer heat because of their coupling to the Cooper pairs.
(d) The heat capacity increases on transition to the superconducting state leading to a reduction in thermal conductivity.

11. The basic process underlying the neutron β-decay is
(a) $d \to u + e^- + \bar{\nu}_e$ (b) $d \to u + e^-$ (c) $s \to u + e^- + \bar{\nu}_e$ (d) $u \to d + e^- + \bar{\nu}_e$

12. In the nuclear shell model the spin parity of ^{15}N is given by
(a) $\frac{1}{2}^-$ (b) $\frac{1}{2}^+$ (c) $\frac{3}{2}^-$ (d) $\frac{3}{2}^+$

13. Match the reactions on the left with the associated interactions on the right
(1) $\pi^+ \to \mu^+ + \nu_\mu$ (i) Strong
(2) $\pi^0 \to \gamma + \gamma$ (ii) Electromagnetic
(3) $\pi^0 + n \to \pi^- + p$ (iii) Weak
(a) 1-iii, 2-ii, 3-i (b) 1-i, 2-ii, 3-iii (c) 1-ii, 2-i, 3-iii (d) 1-iii, 2-i, 3-ii

14. To detect trace amounts of a gaseous species in a mixture of gases, the preferred probing tools is
(a) Ionization spectroscopy with X-rays (b) NMR spectroscopy
(c) ESR spectroscopy (d) Laser spectroscopy

15. A collection of N atoms is exposed to a strong resonant electromagnetic radiation with N_g atoms in the ground state and N_e atoms in the excited state, such that $N_g + N_e = N$. This collection of two-level atoms will have the following population distribution:
(a) $N_g \ll N_e$ (b) $N_g \gg N_e$ (c) $N_g \approx N_e \approx \frac{N}{2}$ (d) $N_g - N_e \approx \frac{N}{2}$

16. Two states of an atom have definite parities. An electric dipole transition between these states is
(a) Allowed if both the states have even parity
(b) Allowed if both the states have odd parity
(c) Allowed if both the states have opposite parities
(d) Not allowed unless a static electric field is applied

17. The spectrum of radiation emitted by a black body at a temperature 1000 K peaks in the
(a) Visible range of frequencies (b) Infrared range of frequencies
(c) Ultraviolet range of frequencies (d) Microwave range of frequencies

18. An insulating sphere of radius a carries a charge density $\rho(\vec{r}) = \rho_0(a^2 - r^2)\cos\theta$; $r < a$. The leading order term for the electric field at a distance d, far away from the charge distribution, is proportional to

(a) d^{-1} (b) d^{-2} (c) d^{-3} (d) d^{-4}

19. The voltage resolution of a 12-bit digital to analog converter (DAC), whose output varies from –10 V to +10 V is, approximately

(a) 1 mV (b) 5 mV (c) 20 mV (d) 100 mV

20. In the of the following circuits, negative feedback does not operate for a negative input. Which one is it ? The opamps are running from ± 15 V supplies.

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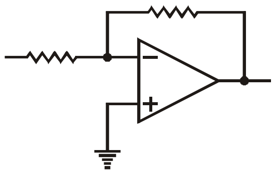

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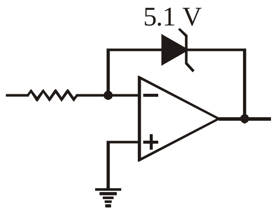

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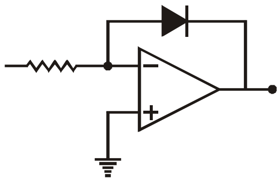

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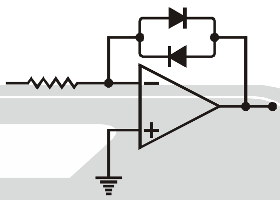


21. A system of N non-interacting classical point particles is constrained to move on the two-dimensional surface of a sphere. The internal energy of the system is

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{1}{2}Nk_BT$ (c) Nk_BT (d) $\frac{5}{2}Nk_BT$

22. Which of the following atoms cannot exhibit Bose-Einstein condensation, even in principle ?

(a) ${}^{1}H_1$ (b) ${}^{4}He_2$ (c) ${}^{23}Na_{11}$ (d) ${}^{40}K_{19}$

23. For the set of all Lorentz transformation with velocities along the x-axis, consider the two statements given below:

P : If L is a Lorentz transformation then, L^{-1} is also a Lorentz transformation

Q : If L_1 and L_2 are Lorentz transformation then, L_1L_2 is necessarily a Lorentz transformation

Choose the correct option.

(a) P is true and Q is false (b) Both P and Q are true
(c) Both P and Q are false (d) P is false and Q is true

24. Which of the following is an allowed wavefunction for a particle in a bound state ? N is a constant and $\alpha, \beta > 0$.

(a) $\psi = N\frac{e^{-\alpha r}}{r^3}$ (b) $\psi = N(1 - e^{-\alpha r})$

(c) $\psi = Ne^{-\alpha x}e^{-\beta(x^2+y^2+z^2)}$ (d) $\psi = \begin{cases} \text{non-zero constant} & \text{if } r < R \\ 0 & \text{if } r > R \end{cases}$

25. A particle is confined within a spherical region of radius one femtometer (10^{-15} m). Its momentum can be expected to be about

(a) $20\frac{keV}{c}$ (b) $200\frac{keV}{c}$ (c) $200\frac{MeV}{c}$ (d) $2\frac{GeV}{c}$

Q.26 – Q.55 : Carry TWO marks each.

26. For the complex function, $f(z)=\dfrac{e^{\sqrt{z}}-e^{-\sqrt{z}}}{\sin\left(\sqrt{z}\right)}$, which of the following statements is correct ?

(a) $z=0$ is a branch point
(b) $z=0$ is a pole of order one
(c) $z=0$ is a removable singularity
(d) $z=0$ is an essential singularity

27. The solution of the differential equation for $y(t):\dfrac{d^2y}{dt^2}-y=2\cosh(t)$, subject to the initial conditions $y(0)=0$ and $\left.\dfrac{dy}{dt}\right|_{t=0}=0$, is

(a) $\frac{1}{2}\cosh(t)+t\sinh(t)$
(b) $-\sinh(t)+t\cosh(t)$
(c) $t\cosh(t)$
(d) $t\sinh(t)$

28. Given the recurrence relation for the Legendre polynomials
$(2n+1)\,x\,P_n(x)=(n+1)\,P_{n+1}(x)+n\,P_{n-1}(x),$
which of the following integrals has a non-zero value ?

(a) $\int_{-1}^{+1} x^2 P_n(x)P_{n+1}(x)\,dx$
(b) $\int_{-1}^{+1} x\,P_n(x)P_{n+2}(x)\,dx$
(c) $\int_{-1}^{+1} x\left[P_n(x)\right]^2 dx$
(d) $\int_{-1}^{+1} x^2\,P_n(x)P_{n+2}(x)\,dx$

29. For a two-dimensional free electron gas, the electronic density n, and the Fermi energy E_F, are related by

(a) $n=\dfrac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$
(b) $n=\dfrac{mE_F}{\pi\hbar^2}$
(c) $n=\dfrac{mE_F}{2\pi\hbar^2}$
(d) $n=\dfrac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

30. Far away from any of the resonance frequencies of a medium the real part of the dielectric permittivity is
(a) Always independent of frequency
(b) Monotonically decreasing with frequency
(c) Monotonically increasing with frequency
(d) A non-monotonic function of frequency

31. The ground state wavefunction of deuteron is in a superposition of *s* and *d* states. Which of the following is NOT true as a consequence ?
(a) It has a non-zero quadrupole moment
(b) The neutron-proton potential is non-central
(c) The orbital wavefunction is not spherically symmetric
(d) The Hamiltonian does not conserve the total angular momentum

32. The first three energy levels of $^{228}\mathrm{Th}_{90}$ are shown below:

4^+ ———— 187 keV
2^+ ———— 57.5 keV
0^+ ———— 0 keV

The expected spin-parity and energy of the next level are given by

(a) (6^+ ; 400 keV)
(b) (6^+ ; 300 keV)
(c) (2^+ ; 400 keV)
(d) (4^+ ; 300 keV)

33. The quark content of Σ^+, K^-, π^- and p is indicated:

$$\left|\Sigma^+\right\rangle = |uus\rangle;\ \left|K^-\right\rangle = |s\bar{u}\rangle;\ \left|\pi^-\right\rangle = |\bar{u}d\rangle;\ |p\rangle = |uud\rangle.$$

In the process, $\pi^- + p \to K^- + \Sigma^+$, considering strong interactions only, which of the following statements is true ?

(a) The process is allowed because $\Delta S = 0$

(b) The process is allowed because $\Delta l_3 = 0$

(c) The process is not allowed because $\Delta S \neq 0$ and $\Delta l_3 \neq 0$

(d) The process is not allowed because the baryon number is violated

34. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigenvalues are

(a) $\dfrac{\hbar^2}{2I} l(l+1) + \dfrac{\hbar^2 m_l^2}{2}\left(\dfrac{1}{I_z} - \dfrac{1}{I}\right)$

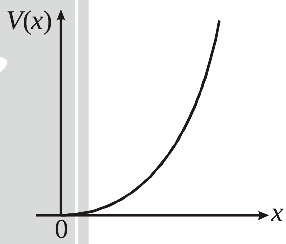
(b) $\dfrac{\hbar^2}{2I} l(l+1)$

(c) $\dfrac{\hbar^2 m_\ell^2}{2}\left(\dfrac{1}{I_z} - \dfrac{1}{I}\right)$

(d) $\dfrac{\hbar^2}{2I} l(l+1) + \dfrac{\hbar^2 m_\ell^2}{2}\left(\dfrac{1}{I_z} + \dfrac{1}{I}\right)$

35. A particle of mass m is confined in the potential

$$V(x) = \begin{cases} \dfrac{1}{2} m\omega^2 x^2 & \text{for} \quad x > 0 \\ \infty & \text{for} \quad x \leq 0 \end{cases}$$



Let the wavefunction of the particle be given by $\psi(x) = -\dfrac{1}{\sqrt{5}}\psi_0 + \dfrac{2}{\sqrt{5}}\psi_1$, where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state respectively. The expectation value of the energy is

(a) $\dfrac{31}{10}\hbar\omega$ (b) $\dfrac{25}{10}\hbar\omega$ (c) $\dfrac{13}{10}\hbar\omega$ (d) $\dfrac{11}{10}\hbar\omega$

36. Match the typical spectra of stable molecules with the corresponding wave-number range

1. Electronic spectra — i. 10^6 cm^{-1} and above
2. Rotational spectra — ii. $10^5 - 10^6$ cm^{-1}
3. Molecular dissociation — iii. $10^0 - 10^2$ cm^{-1}

(a) 1-ii, 2-i, 3-iii (b) 1-ii, 2-iii, 3-i (c) 1-iii, 2-ii, 3-i (d) 1-i, 2-ii, 3-iii

37. Consider the operations $P: \vec{r} \to -\vec{r}$ (parity) and $T: t \to -t$ (time-reversal). For the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which of the following set of transformations is correct ?

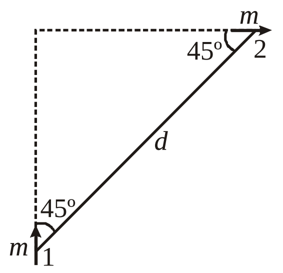
(a) $P: \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}\ ;\ T: \vec{E} \to \vec{E}, \vec{B} \to -\vec{B}$

(b) $P: \vec{E} \to \vec{E}, \vec{B} \to \vec{B}\ ;\ T: \vec{E} \to \vec{E}, \vec{B} \to \vec{B}$

(c) $P: \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}\ ;\ T: \vec{E} \to -\vec{E}, \vec{B} \to -\vec{B}$

(d) $P: \vec{E} \to \vec{E}, \vec{B} \to -\vec{B}\ ;\ T: \vec{E} \to -\vec{E}, \vec{B} \to \vec{B}$

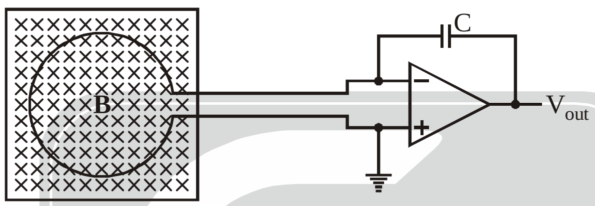
38. Two magnetic dipoles of magnitude m each are placed in a plane as shown.



The energy of interaction is given by

(a) zero (b) $\frac{\mu_0}{4\pi}\frac{m^2}{d^3}$ (c) $\frac{3\mu_0}{2\pi}\frac{m^2}{d^3}$ (d) $-\frac{3\mu_0}{8\pi}\frac{m^2}{d^3}$

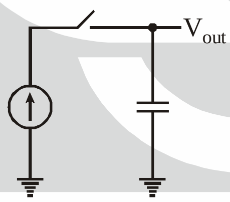
39. Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B thereby enclosing a flux ϕ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged.



If the loop is pulled out of the region of the magnetic field at a constant speed u, the final output voltage V_{out} is independent of

(a) ϕ_0 (b) u (c) R (d) C

40. The figure shows a constant current source charging a capacitor that is initially uncharged.



If the switch is closed at t = 0, which of the following plots depicts correctly the output voltage of the circuit as a function of time ?

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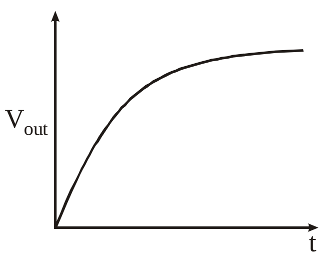

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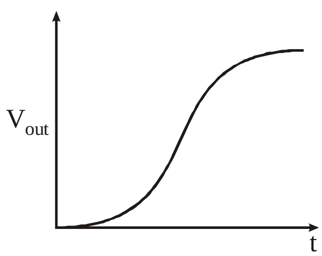

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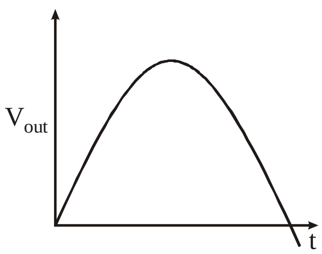

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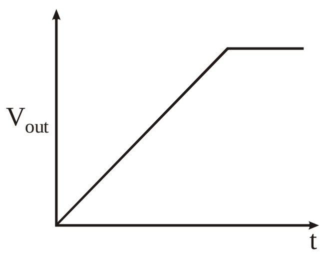
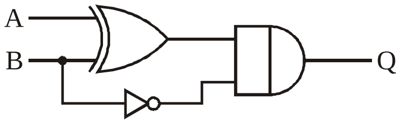
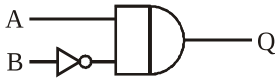
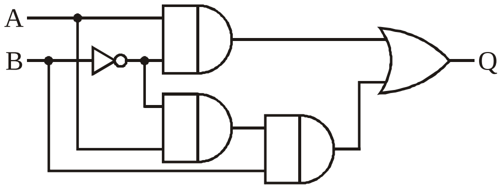
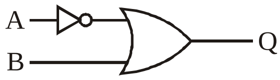


41. For any set of inputs A and B, the following circuits give the same output Q, except one. Which one is it ?

(a) 

(b) 

(c) 

(d) 

42. CO_2 molecule has the first few energy levels uniformly separated by approximately 2.5 meV. At a temperature of 300 K, the ratio of the number of molecules in the 4th excited state to the number in the 2nd excited state is about

(a) 0.5 (b) 0.6 (c) 0.8 (d) 0.9

43. Which among the following sets of Maxwell relations is correct ? (U-internal energy, H-enthalpy, A-Helmholtz free energy and G-Gibb's free energy)

(a) $T = \left(\frac{\partial U}{\partial V}\right)_S$ and $P = \left(\frac{\partial U}{\partial S}\right)_V$

(b) $V = \left(\frac{\partial H}{\partial P}\right)_S$ and $T = \left(\frac{\partial H}{\partial S}\right)_P$

(c) $P = -\left(\frac{\partial G}{\partial V}\right)_T$ and $V = \left(\frac{\partial G}{\partial P}\right)_S$

(d) $P = -\left(\frac{\partial A}{\partial S}\right)_T$ and $S = -\left(\frac{\partial A}{\partial P}\right)_V$

44. For a spin-s particle, in the eigen basis of $\vec{S}^2, S_z$, the expectation value $\left\langle s\,m\left|S_x^2\right|s\,m\right\rangle$ is

(a) $\frac{\hbar^2\left\{s(s+1)-m^2\right\}}{2}$

(b) $\hbar^2\left\{s(s+1)-2m^2\right\}$

(c) $\hbar^2\left\{s(s+1)-m^2\right\}$

(d) $\hbar^2 m^2$

45. A particle is placed in a region with the potential $V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3$, where $k, \lambda > 0$. Then,

(a) $x = 0$ and $x = \frac{k}{\lambda}$ are points of stable equilibrium

(b) $x = 0$ is a point of stable equilibrium and $x = \frac{k}{\lambda}$ is a point of unstable equilibrium

(c) $x = 0$ and $x = \frac{k}{\lambda}$ are points of unstable equilibrium

(d) There are no points of stable or unstable equilibrium

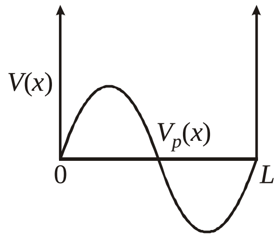
46. A π^0 meson at rest decays into two photons, which move along the x-axis. They are both detected simultaneously after a time, $t = 10\,s$. In an inertial frame moving with a velocity $\mathrm{v} = 0.6c$ in the direction of one of the photons, the time interval between the two detections is

(a) 15 s (b) 0 s (c) 10 s (d) 20 s

47. A particle of mass m is confined in an infinite potential well :

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L \\ \infty & \text{otherwise} \end{cases}$$

It is subjected to a perturbing potential $V_p(x) = V_0 \sin\left(\frac{2\pi x}{L}\right)$ within the well. Let $E^{(1)}$ and $E^{(2)}$ be the corrections to the ground state energy in the first and second order in V_0, respectively. Which of the following are true ?



(a) $E^{(1)} = 0$; $E^{(2)} < 0$ (b) $E^{(1)} > 0$; $E^{(2)} = 0$

(c) $E^{(1)} = 0$; $E^{(2)}$ depends on the sign of V_0 (d) $E^{(1)} < 0$; $E^{(2)} < 0$

Common Data for Questions 48 and 49 :

In the presence of a weak magnetic field, atomic hydrogen undergoes the transition:

${}^2\mathrm{P}_{1/2} \rightarrow {}^1\mathrm{S}_{1/2}$ by emission of radiation.

48. The number of distinct spectral lines that are observed in the resultant Zeeman spectrum is

(a) 2 (b) 3 (c) 4 (d) 6

49. The spectral line corresponding to the transition ${}^2\mathrm{P}_{1/2}\left(m_j = +\frac{1}{2}\right) \rightarrow {}^1\mathrm{S}_{1/2}\left(m_j = -\frac{1}{2}\right)$ is observed along the direction of the applied magnetic field. The emitted electromagnetic field is

(a) Circularly polarized (b) Linearly polarized

(c) Unpolarized (d) Not emitted along the magnetic field direction

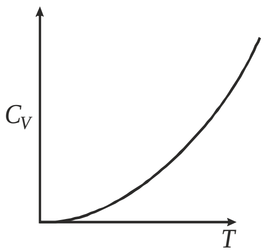
Common Data for Questions 50 and 51 :

The partition function for a gas of photons is given by

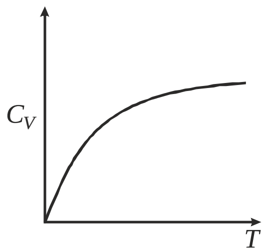
$$\ln Z = \frac{\pi^2}{45} \frac{V(k_B T)^3}{\hbar^3 C^3}$$

50. The specific heat of the photon gas varies with temperature as

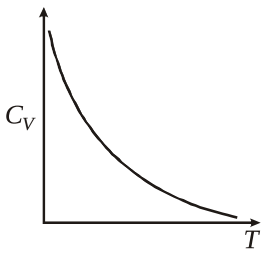
(a)



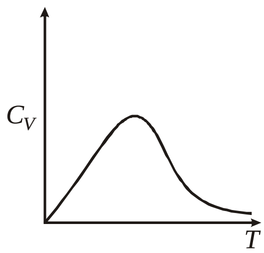
(b)



(c)



(d)



51. The pressure of the photon gas is

(a) $\frac{\pi^2}{15}\frac{(k_BT)^3}{\hbar^3C^3}$ (b) $\frac{\pi^2}{8}\frac{(k_BT)^4}{\hbar^3C^3}$ (c) $\frac{\pi^2}{45}\frac{(k_BT)^4}{\hbar^3C^3}$ (d) $\frac{\pi}{45}\frac{(k_BT)^{3/2}}{\hbar^3C^3}$

Statement for Linked Answer Questions 52 and 53:

Consider the propagation of electromagnetic waves in a linear, homogeneous and isotropic material medium with electric permittivity ε, and magnetic permittivity μ.

52. For a plane wave of angular frequency ω and propagation vector $\hat{k}$ propagating in the medium Maxwell's equations reduce to

(a) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=\omega\varepsilon\vec{H}$; $\vec{k}\times\vec{H}=-\omega\mu\vec{E}$

(b) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=-\omega\varepsilon\vec{H}$; $\vec{k}\times\vec{H}=\omega\mu\vec{E}$

(c) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=-\omega\mu\vec{H}$; $\vec{k}\times\vec{H}=\omega\varepsilon\vec{E}$

(d) $\vec{k}\cdot\vec{E}=0$; $\vec{k}\cdot\vec{H}=0$; $\vec{k}\times\vec{E}=\omega\mu\vec{H}$; $\vec{k}\times\vec{H}=-\omega\varepsilon\vec{E}$

53. If ε and μ assume negative values in a certain frequency range, then the directions of the propagation vector $\hat{k}$ and the Poynting vector $\vec{S}$ in that frequency range are related as

(a) $\vec{k}$ and $\vec{S}$ are parallel

(b) $\vec{k}$ and $\vec{S}$ are anti-parallel

(c) $\vec{k}$ and $\vec{S}$ are perpendicular to each other

(d) $\vec{k}$ and $\vec{S}$ make an angle that depends on the magnitude of $|\varepsilon|$ and $|\mu|$

Statement for Linked Answer Questions 54 and 55 :

The Lagrangian for a simple pendulum is given by :

$$L=\frac{1}{2}ml^2\dot{\theta}^2-mgl(1-\cos\theta)$$

54. Hamilton's equations are then given by

(a) $\dot{p}_\theta=-mgl\sin\theta$; $\dot{\theta}=\frac{p_\theta}{ml^2}$ (b) $\dot{p}_\theta=mgl\sin\theta$; $\dot{\theta}=\frac{p_\theta}{ml^2}$

(c) $\dot{p}_\theta=-m\ddot{\theta}$; $\dot{\theta}=\frac{p_\theta}{m}$ (d) $\dot{p}_\theta=-\left(\frac{g}{l}\right)\theta$; $\dot{\theta}=\frac{p_\theta}{ml}$

55. The Poisson bracket between θ and $\dot{\theta}$ is

(a) $\{\theta,\dot{\theta}\}=1$ (b) $\{\theta,\dot{\theta}\}=\frac{1}{ml^2}$ (c) $\{\theta,\dot{\theta}\}=\frac{1}{m}$ (d) $\{\theta,\dot{\theta}\}=\frac{g}{l}$