PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$

(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

(b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

2. Two uniform thin rods of equal length L, and masses M_1 and M_2 are joined together along the length. The moment of inertia of the combined rod of length 2L about an axis passing through the mid-point perpendicular to the length of the rod is,

(a) $(M_1 + M_2)\frac{L^2}{12}$ (b) $(M_1 + M_2)\frac{L^2}{6}$ (c) $(M_1 + M_2)\frac{L^2}{3}$ (d) $(M_1 + M_2)\frac{L^2}{2}$

3. The space-time dependence of the electric field of a linearly polarized light in free space is given by $\hat{x}E_0 \cos(\omega t - kz)$, where E_0, ω and k are the amplitude, the angular frequency and the wavevector, respectively. The time averaged energy density associated with the electric field is

(a) $\frac{1}{4}\varepsilon_0 E_0^2$ (b) $\frac{1}{2}\varepsilon_0 E_0^2$ (c) $\varepsilon_0 E_0^2$ (d) $2\varepsilon_0 E_0^2$

4. If the peak output voltage of a full wave rectifier is 10 V, its d.c. voltage is

(a) 10.0V (b) 7.07V (c) 6.36 V (d) 3.18V

5. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x, y)$ is given by

$$V(x, y) = 0 \text{ for } -a < x < a \text{ and } -a < y < a$$
$$= \infty \text{ else where}$$

The energy of the first excited state for this particle is given by,

(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

6. The isothermal compressibility, κ of an ideal gas at temperature T_0 and volume V_0, is given by

(a) $-\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (b) $\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (c) $-V_0\frac{\partial P}{\partial V}\Big|_{T_0}$ (d) $V_0\frac{\partial P}{\partial P}\Big|_{T_0}$

7. The ground state of sodium atom (^{11}Na) is a $^2S_{1/2}$ state. The difference in energy levels arising in the presence of a weak external magnetic field B, given in terms of Bohr magneton, μ_B is

(a) $\mu_B B$ (b) $2\mu_B B$ (c) $4\mu_B B$ (d) $6\mu_B B$

8. For an ideal Fermi gas in three dimensions, the electron velocity V_F at the Fermi surface is related to electron concentration n as,

(a) $V_F \propto n^{2/3}$ (b) $V_F \propto n$ (c) $V_F \propto n^{1/2}$ (d) $V_F \propto n^{1/3}$

9. Which one of the following sets corresponds to fundamental particles ?

(a) proton, electron and neutron (b) proton, electron and photon

(c) electron, photon and neutrino (d) quark, electron and meson

CAREER ENDEAVOUR

10. In case of a Geiger-Muller (GM) counter, which one of the following statements is CORRECT?

(a) Multiplication factor of the detector is of the order of 10^{10}.
(b) Type of the particles detected can be identified.
(c) Energy of the particles detected can be distinguished.
(d) Operating voltage of the detector is few tens of Volts.

11. A plane electromagnetic wave traveling in free space is incident normally on a glass plate of refractive index 3/2. If there is no absorption by the glass, Its reflectivity is

(a) 4% (b) 16 % (c) 20% (d) 50%

12. A Ge semiconductor is doped with acceptor impurity concentration of 10^{15} atoms /cm^3. For the given hole mobility of $1800\ \text{cm}^2/\text{V}-\text{s}$, the resistivity of this material is:

(a) $0.288\ \Omega$ cm (b) $0.694\ \Omega$ cm (c) $3.472\ \Omega$ cm (d) $6.944\ \Omega$ cm

13. A classical gas of molecules each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are v_x, v_y and v_z. The mean value of $(v_x + v_y)^2$ is:

(a) $\dfrac{k_BT}{m}$ (b) $\dfrac{3}{2}\dfrac{k_BT}{m}$ (c) $\dfrac{1}{2}\dfrac{k_BT}{m}$ (d) $\dfrac{2k_BT}{m}$

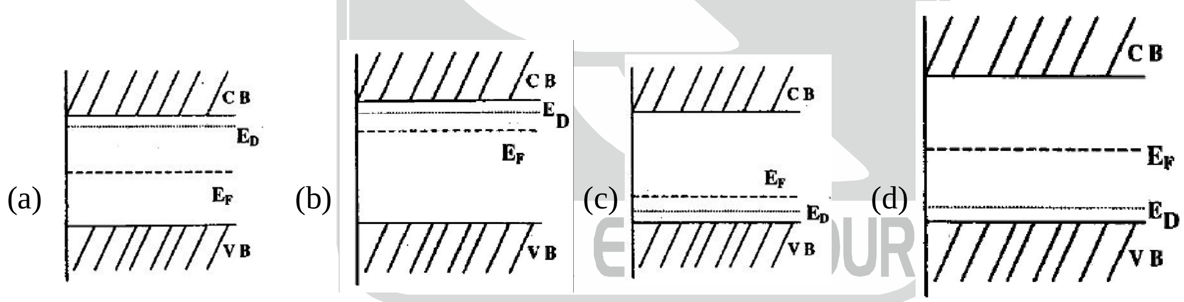
14. In a central force field the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by

$$\frac{1}{r}=\frac{m}{L^2}\left(1+\varepsilon\cos\theta\right)$$

Where, ε is the eccentricity of the particle's motion. Which one of the following choices for ε gives rise to a parabolic trajectory ?

(a) $\varepsilon=0$ (b) $\varepsilon=1$ (c) $0<\varepsilon<1$ (d) $\varepsilon>1$

15. Identify the CORRECT energy band diagram for silicon doped with Arsenic. Here CB, VB, E_D and E_F conduction band, valence band, impurity level and Fermi level, respectively.



16. The first stokes line of a rotational Raman spectrum is observed at 12.96 cm^{-1}. Considering the rigid rotor approximation, the rotational constant is given by

(a) 6.48 cm^{-1} (b) 3.24 cm^{-1} (c) 2.16 cm^{-1} (d) 1.62 cm^{-1}

17. The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E\propto\dfrac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas. Identify the CORRECT equation of state (P being the pressure),

(a) $PV=\dfrac{1}{3}E$ (b) $PV=\dfrac{2}{3}E$ (c) $PV=E$ (d) $PV=\dfrac{5}{3}E$

18. Consider the wavefunction $\psi=\psi\left(\vec{r}_1,\vec{r}_2\right)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by

$$\psi\left(\vec{r}_1,\vec{r}_2\right)=\frac{1}{\sqrt{2}}\left[\phi_1\left(\vec{r}_1\right)\phi_2\left(\vec{r}_2\right)+\phi_2\left(\vec{r}_1\right)\phi_1\left(\vec{r}_2\right)\right]$$

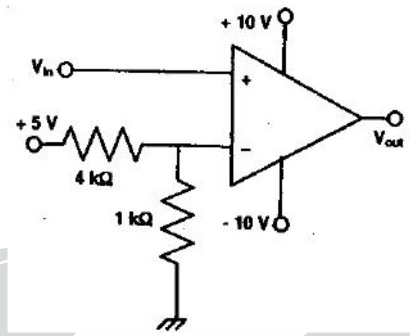
Where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wavefunction with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta+\beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta-\beta\alpha)$ (c) $\alpha\alpha$ (d) $\beta\beta$

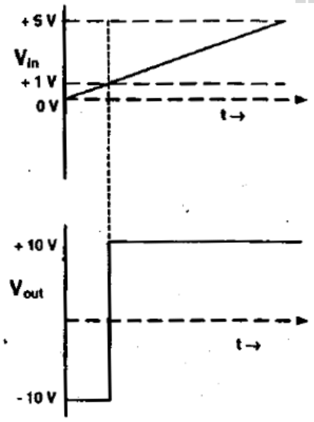
19. The electric and the magnetic fields $\vec{E}(z,t)$ and $\vec{B}(z,t)$, respectively corresponding to the scalar potential $\phi(z,t)=0$ and vector potential $\vec{A}(z,t)=\hat{i}tz$ are

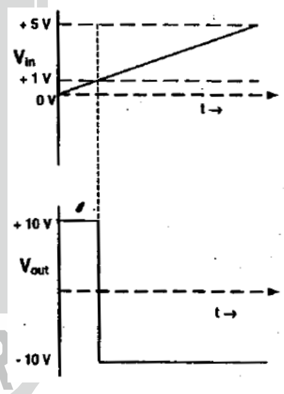
(a) $\vec{E}=\hat{i}z$ and $\vec{B}=-\hat{j}t$ (b) $\vec{E}=\hat{i}z$ and $\vec{B}=\hat{j}t$ (c) $\vec{E}=-\hat{i}z$ and $\vec{B}=-\hat{j}t$ (d) $\vec{E}=-\hat{i}z$ and $\vec{B}=\hat{j}t$

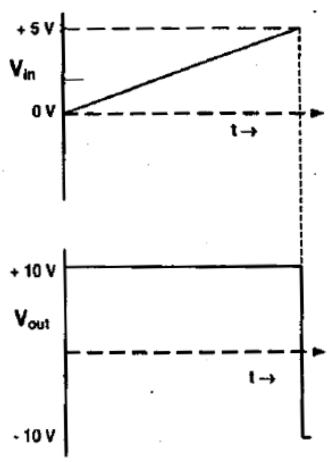
20. Consider the following OP-AMP circuit.

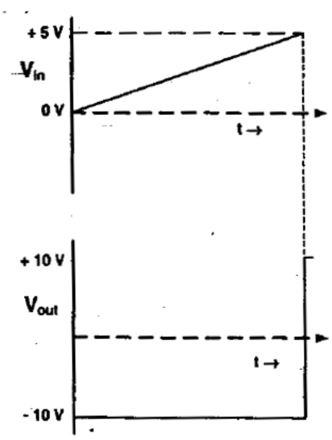


Which one of the following correctly represents the output V_{out} corresponding to the input V_{in} ?

(a)


(b)


(c)


(d)


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21. Deuteron has only one bound state with spin parity 1^+, isospin 0 and electric quadrupole moment 0.286 efm^2. These data suggest that the nuclear forces are having

(a) only spin and isospin dependence
(b) no spin dependence and no tensor components
(c) spin dependence but no tensor components
(d) spin dependence along with tensor components

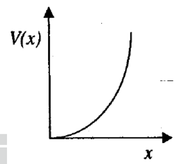
22. A particle of unit mass moves along the x-axis under the influence of a potential, $V(x) = x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

23. Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids ?

(a) Debye's T^3 law
(b) Dulong Petit's law
(c) Optical branches in lattices
(d) Thermal expansion

24. A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Which one of the following statements is CORRECT ?



(a) The parity of the first excited state is even
(b) The parity of the ground state is even
(c) The ground state energy is $\frac{1}{2}\hbar\omega$
(d) The first excited state energy is $\frac{7}{2}\hbar\omega$

25. The number of independent components of the symmetric tensor A_{ij} with indices i, j = 1, 2, 3 is

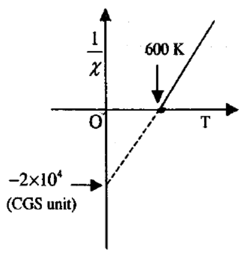
(a) 1 (b) 3 (c) 6 (d) 9

Q.26 – Q.55 : Carry TWO marks each.

26. Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The system is subjected to a perturbation of the form $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$, where $\delta << 1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are

(a) 1 and $(1+2\delta)$ (b) $(1+\delta)$ and $(1-\delta)$ (c) $(1+2\delta)$ and $(1-2\delta)$ (d) $(1+\delta)$ and $(1-2\delta)$

27. Inverse susceptibility $(1/\chi)$ as a function of temperature, T for a material undergoing paramagnetic to ferro-magnetic transition is given in the figure, where O is the origin. The values of the Curie constant, C and the Weiss molecular field constant, λ, in CGS units, are



(a) $C = 5\times10^{-5}, \lambda = 3\times10^{-2}$
(b) $C = 3\times10^{-2}, \lambda = 5\times10^{-5}$
(c) $C = 3\times10^{-2}, \lambda = 2\times10^{-4}$
(d) $C = 2\times10^{4}, \lambda = 3\times10^{-2}$

CAREER ENDEAVOUR

28. A plane polarized electromagnetic wave in free space at time t = 0 is given by $\vec{E}(x,z) = 10\hat{j}\exp[i(6x+8z)]$. The magnetic field $\vec{B}(x,z,t)$ is given by

(a) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k}-8\hat{i})\exp[i(6x+8z-10ct)]$.

(b) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k}+8\hat{i})\exp[i(6x+8z-10ct)]$.

(c) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k}-8\hat{i})\exp[i(6x+8z-ct)]$.

(d) $\vec{B}(x,z,t) = \frac{1}{c}(6\hat{k}+8\hat{i})\exp[i(6x+8z+ct)]$.

29. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(a) 0,1,1 (b) $0, -\sqrt{2}, \sqrt{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

30. Match the typical spectroscopic regions specified in List-I with the corresponding type of transitions in List-II and find the correct answer using the codes given below the list :

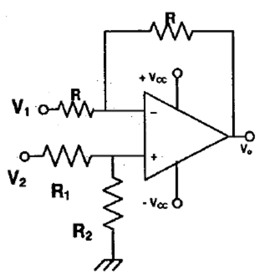
List-I	List-II
P. Infrared region	1. Electronic transitions involving valence electrons
Q. Ultraviolet visible region	2. Nuclear transitions
R. X-ray region	3. Vibrational transitions of molecules
S. γ - rays region	4. Transitions involving inner shell electrons

Codes :

	P	Q	R	S
(a)	1	3	2	4
(b)	2	4	1	3
(c)	3	1	4	2
(d)	4	1	2	3

CAREER ENDEAVOUR

31. In the following circuit, for the output voltage to $V_0 = (-V_1 + V_2/2)$ the ratio R_1/R_2 is



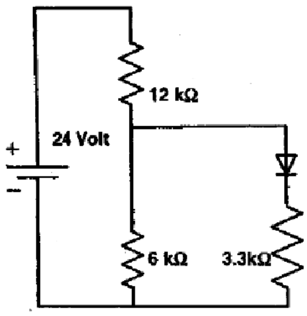
(a) 1/2 (b) 1 (c) 2 (d) 3

32. The terms $\left\{\hat{J}_1, \hat{J}_2\right\}_j$ arising from $2s^1\,3d^1$ electronic configuration in J – J scheme are

(a) $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$ (b) $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$

(c) $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$ (d) $\left\{\frac{3}{2}, \frac{1}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

33. In the following circuit, the voltage drop across the ideal diode in forward bias condition is 0.7 V.



The current passing through the diode is

(a) 0.5 mA (b) 1.0 mA (c) 1.5 mA (d) 2.0 mA

34. Choose the CORRECT statement from the following

(a) Neutron interacts through electromagnetic interaction

(b) Electron does not interact through weak interaction

(c) Neutrino interacts through weak and electromagnetic interaction

(d) Quark interacts through strong interaction but not through weak interaction

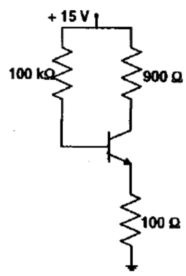
35. A rod of proper length ℓ_0 oriented parallel to the x-axis moves with speed 2c/3 along the x-axis in the S-frame, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed c/2 with respect to the S-frame. The length of the rod as measured by the observer is

(a) $0.35\,\ell_0$ (b) $0.48\,\ell_0$ (c) $0.87\,\ell_0$ (d) $0.97\,\ell_0$

36. A simple cubic crystal with lattice parameter a_c undergoes transition into a tetragonal structure with lattice parameter $a_t = b_t = \sqrt{2}\,a_c$ and $c_t = 2a_c$, below a certain temperature. The ratio of the interplanar spacings of (101) planes for the cubic and the tetragonal structure is

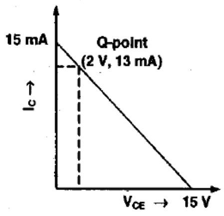
(a) $\sqrt{\frac{1}{6}}$ (b) $\frac{1}{6}$ (c) $\sqrt{\frac{3}{8}}$ (d) $\frac{3}{8}$

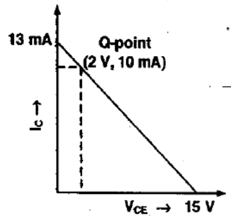
37. Consider the following circuit in which the current gain β_{dc} of the transistor is 100.

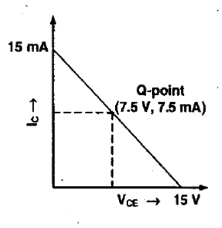


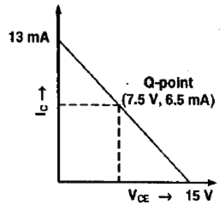
Which one of the following correctly represents the load line (collector current I_c with respect to collectr-emitter voltage V_{CE}) and Q-point of this circuit?

CAREER ENDEAVOUR

(a) 

(b) 

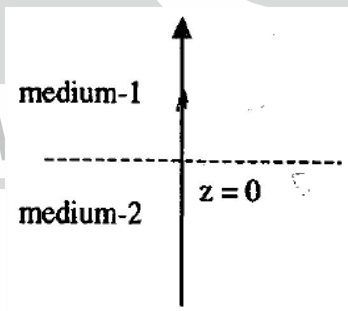
(c) 

(d) 

38. Consider a system whose three energy levels are given by 0, ε and 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta = \dfrac{1}{k_B T}$ is given by

(a) $1+2e^{-\beta\epsilon}$ (b) $2e^{-\beta\epsilon}+e^{-2\beta\epsilon}$ (c) $\left(1+e^{-\beta\epsilon}\right)^2$ (d) $1+e^{-\beta\epsilon}+e^{-2\beta\epsilon}$

39. Two infinitely extended homogeneous isotropic dielectric media (medium -1 and medium – 2 with dielectric constants $\dfrac{\varepsilon_1}{\varepsilon_0}=2$ and $\dfrac{\varepsilon_2}{\varepsilon_0}=5$, respectively) meet at the z = 0 plane as shown in the figure. A uniform electric field exists everywhere. For $z \geq 0$, the electric field is given by $\vec{E}_1 = 2\hat{i}-3\hat{j}+5\hat{k}$. The interface separating the two media is charge free.
The electric displacement vector in the medium-2 is given by

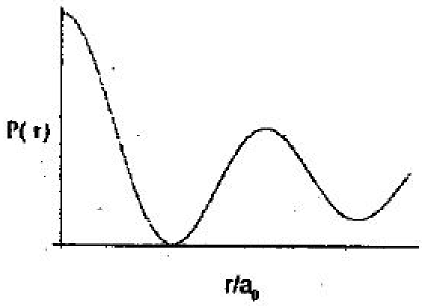


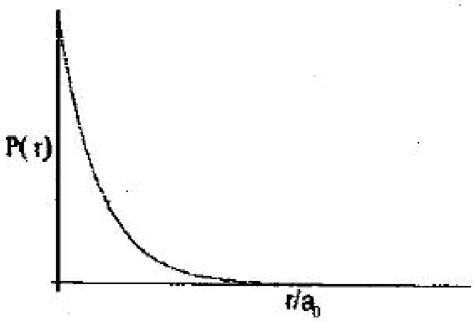
(a) $\overrightarrow{D_2} = \varepsilon_0\left[10\hat{i}+15\hat{j}+10\hat{k}\right]$ (b) $\overrightarrow{D_2} = \varepsilon_0\left[10\hat{i}-15\hat{j}+10\hat{k}\right]$

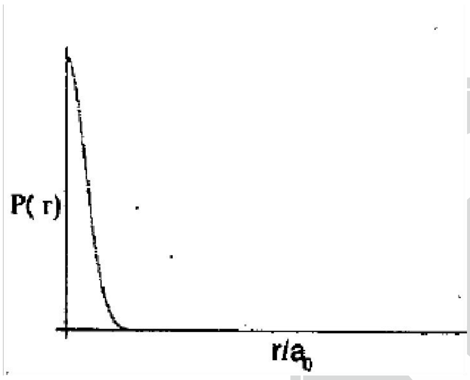
(c) $\overrightarrow{D_2} = \varepsilon_0\left[4\hat{i}-6\hat{j}+10\hat{k}\right]$ (d) $\overrightarrow{D_2} = \varepsilon_0\left[4\hat{i}+6\hat{j}+10\hat{k}\right]$

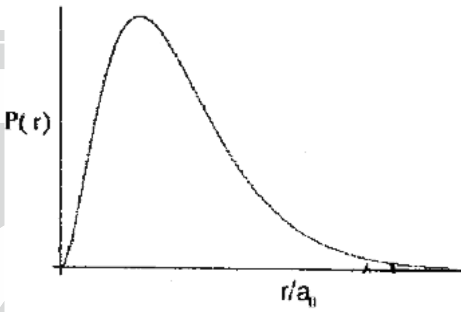
40. The ground state wavefunction for the hydrogen atom is given by $\psi_{100} = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$, where a_0 is the Bohr radius.

The plot of the radial probability density, P(r) for the hydrogen atom in the ground state is

(a) 

(b) 

(c) 

(d) 

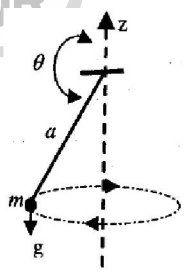
41. Total binding energies of O^{15}, O^{16} and O^{17} are 111.96 MeV, 127.62 MeV and 131.76 MeV, respectively. The energy gap between ${}^1p_{1/2}$ and ${}^1d_{5/2}$ neutron shells for the nuclei whose mass number is close to 16, is:

(a) 4.1 MeV (b) 11.5 MeV (c) 15.7 MeV (d) 19.8 MeV

42. A particle of mass m is attached to a fixed point 'O' by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure.

CAREER ENDEAVOUR

The Lagrangian of the particle is



$$L(\theta,\phi) = \frac{1}{2}ma^2\left(\dot\theta^2 + \sin^2\theta\dot\phi^2\right) - mga\cos\theta$$

where θ and ϕ are the polar angles.

The Hamiltonian of the particle is:

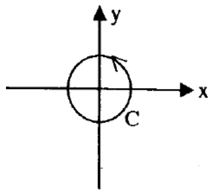
(a) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) - mga\cos\theta$

(b) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$

(c) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) - mga\cos\theta$

(d) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) + mga\cos\theta$

43. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0\left(\hat{i} + \hat{j} + k\right)$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F}.d\vec{r}$, where C is a circle of unit radius centered at origin is,



(a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

44. The value of the integral $\oint_C e^{1/z} dz$, using the contour C of circle with unit radius |z| = 1 is:

(a) 0 (b) $1-2\pi i$ (c) $1+2\pi i$ (d) $2\pi i$

45. A paramagnetic system consisting of N spin-half particles, is placed in an external magnetic field. It is found that N/2 spins are aligned parallel and the remaining N/2 spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is,

(a) $2Nk_B \ell n\ 2$ (b) $2\dfrac{N}{2}k_B\ \ell n\ 2$ (c) $\dfrac{3N}{2}k_B\ \ell n\ 2$ (d) $Nk_B\ \ell n\ 2$

46. The equilibrium vibration frequency for a oscillator is observed at 2990 cm^{-1}. The ratio of the frequencies corresponding to the first and the fundamental spectral lines is 1.96. Considering the oscillator to be anharmonic, the anharmonicity constant is

(a) 0.005 (b) 0.02 (c) 0.05 (d) 0.1

47. At a certain temperature T, the average speed of nitrogen molecules in air is found to be 400 m/s. The most probable and the root mean square speeds of the molecules are, respectively,

(a) 355 m/s, 434 m/s (b) 820 m/s, 917 m/s
(c) 152 m/s, 301 m/s (d) 422 m/s, 600 m/s

Common data for Q.48 and Q.49

The wavefunction of a particle moving in free space is given by, $\psi = e^{ikx} + 2e^{-ikx}$

48. The energy of the particle is

(a) $\dfrac{5\hbar^2 k^2}{2m}$ (b) $\dfrac{3\hbar^2 k^2}{4m}$ (c) $\dfrac{\hbar^2 k^2}{2m}$ (d) $\dfrac{\hbar^2 k^2}{m}$

49. The probability current density for the real part of the wavefunction is

(a) 1 (b) $\dfrac{\hbar k}{m}$ (c) $\dfrac{\hbar k}{2m}$ (d) 0

Common data for Q.50 and Q. 51

The dispersion relation for a one dimensional monatomic crystal with lattice spacing a, which interacts via nearest neighbour harmonic potential is given by

$$\omega = A\left|\sin\frac{Ka}{2}\right|,$$

where A is a constant of a appropriate unit.

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50. The group velocity at the boundary of the first Brillouin zone is

(a) 0 (b) 1 (c) $\sqrt{\frac{Aa^2}{2}}$ (d) $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

51. The force constant between the nearest neighbour of the lattice is (M is the mass of the atom)

(a) $\frac{MA^2}{4}$ (b) $\frac{MA^2}{2}$ (c) MA^2 (d) $2MA^2$

Statement for Linked Answer Q.52 and Q.53

In a hydrogen atom, consider that the electronic charge is uniformly distributed in a spherical volume of radius $a\left(=0.5\times10^{-10}\,\text{m}\right)$ around the proton. The atom is placed in a uniform electric field $E = 30\times10^5\ \text{V/m}$. Assume that the spherical distribution of the negative charge remains undistorted under the electric field.

52. In the equilibrium condition, the separation between the positive and the negative charge centers is

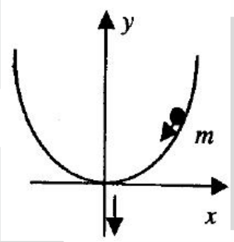
(a) $8.66\times10^{-16}\,\text{m}$ (b) $2.60\times10^{-15}\,\text{m}$ (c) $2.60\times10^{-16}\,\text{m}$ (d) $8.66\times10^{-15}\,\text{m}$

53. The polarizability of the hydrogen atom in unit of $\left(C^2 m / N\right)$ is

(a) 2.0×10^{-40} (b) 1.4×10^{-41} (c) 1.4×10^{-40} (d) 2.0×10^{-39}

Statement for Linked Answer Q.54 and Q.55

A particle of mass m slides under the gravity without friction along parabolic path $y = ax^2$ as shown in the figure. Here a is a constant.



54. The Lagrangian for this particle is given by,

(a) $L = \frac{1}{2}m\dot{x}^2 - mgax^2$ (b) $L = \frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2 - mgax^2$

(c) $L = \frac{1}{2}m\dot{x}^2 + mgax^2$ (d) $L = \frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2 + mgax^2$

55. The Lagrange's equation of motion of the particle is

(a) $\ddot{x} = 2gax$ (b) $m\left(1+4a^2x^2\right)\ddot{x} = -2mgax - 4ma^2x\,\dot{x}^2$

(c) $m\left(1+4a^2x^2\right)\ddot{x} = 2mgax + 4ma^2x\,\dot{x}^2$ (d) $\ddot{x} = -2gax$

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