PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. $f(x)$ is a symmetric periodic function of x i.e. $f(x)=f(-x)$. Then, in general, the Fourier series of the function $f(x)$ will be of the form

(a) $f(x)=\sum_{n=1}^{\infty}(a_n\cos(nkx)+b_n\sin(nkx))$ (b) $f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos(nkx))$

(c) $f(x)=\sum_{n=1}^{\infty}(b_n\sin(nkx))$ (d) $f(x)=a_0+\sum_{n=1}^{\infty}(b_n\sin(nkx))$

2. In the most general case, which one of the following quantities is NOT a second order tensor?

(a) Stress (b) Strain (c) Moment of inertia (d) Pressure

3. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is

(a) c (b) $-c$ (c) $0.15c$ (d) $-0.15c$

4. If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature T would be (k is the Boltzmann constant and T is non zero)

(a) Zero (b) Infinite (c) $\frac{3}{2}kT$ (d) kT

5. Across a first order phase transition, the free energy is

(a) proportional to the temperature
(b) a discontinuous function of the temperature
(c) a continuous function of the temperature but its derivative is discontinuous
(d) such that the first derivative with respect to temperature is continuous

6. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same

(a) pressure and temperature (b) volume and temperature
(c) pressure and volume (d) volume and energy

7. The entropy function of a system is given by $S(E)=aE(E_0-E)$ where a and E_0 are positive constants. The temperature of the system is

(a) negative for some energies (b) increases monotonically with energy
(c) decreases monotonically with energy (d) zero

8. Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is (k is the Boltzmann constant)

(a) Zero (b) Nk (c) $\frac{1}{2}Nk$ (d) $Nk\ ln(2)$

9. The decay process $n\to p^{+}+e^{-}+\bar{\nu}_e$ violates

(a) baryon number (b) lepton number (c) isospin (d) strangeness

10. The isospin (I) and baryon number (B) of the up quark is

(a) I = 1, B = 1 (b) I = 1, B = 1/3 (c) I = 1/2, B = 1 (d) I = 1/2, B = 1/3

11. Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range r_0. Given that, $\cot(kr_0+\delta)\approx-\frac{\gamma}{k}$ where δ is the phase shift, k the wave number and $(-\gamma)$ the logarithmic derivative of the deuteron ground state wave function, the phase shift is

(a) $\delta\approx-\frac{k}{\gamma}-kr_0$ (b) $\delta\approx-\frac{\gamma}{k}-kr_0$ (c) $\delta\approx\frac{\pi}{2}-kr_0$ (d) $\delta\approx-\frac{\pi}{2}-kr_0$

12. In the β decay process, the transition $2^+ \to 3^+$, is

(a) allowed both by Fermi and Gamow-Teller selection rule
(b) allowed by Fermi and but not by Gamow-Teller selection rule
(c) not allowed by Fermi but allowed by Gamow-Teller selection rule
(d) not allowed both by Fermi and Gamow-Teller selection rule

13. At a surface current, which one of the magnetostatic boundary condition is NOT CORRECT?

(a) Normal component of the magnetic field is continuous.
(b) Normal component of the magnetic vector potential is continuous.
(c) Tangential component of the magnetic vector potential is continuous.
(d) Tangential component of the magnetic vector potential is not continuous.

14. Interference fringes are seen at an observation plane $z = 0$, by the superposition of two plane waves $A_1 \exp[i(\vec{k}_1 \cdot \vec{r} - \omega t)]$ and $A_2 \exp[i(\vec{k}_2 \cdot \vec{r} - \omega t)]$, where A_1 and A_2 are real amplitudes. The condition for interference maximum is

(a) $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = (2m+1)\pi$ (b) $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = 2m\pi$ (c) $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = (2m+1)\pi$

(d) $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = 2m\pi$

15. For a scalar function φ satisfying the Laplace equation, $\nabla\varphi$ has

(a) zero curl and non-zero divergence (b) non-zero curl and zero divergence
(c) zero curl and zero divergence (d) non-zero curl and non-zero divergence

16. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which one of the following statements is CORRECT?

(a) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.
(b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.
(c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.
(d) There will be no reflected light and the transmitted light is circularly polarized.

17. Which one of the following commutation relations is NOT CORRECT? Here, symbols have their usual meanings.

(a) $[L^2, L_z] = 0$ (b) $[L_x, L_y] = i\hbar L_z$ (c) $[L_z, L_+] = \hbar L_+$ (d) $[L_z, L_-] = \hbar L_-$

18. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

(a) $p_q = 2\beta q$ and it is a conserved quantity. (b) $p_q = 2\beta q$ and it is not a conserved quantity.

(c) $p_q = 2\alpha\dot{q}$ and it is a conserved quantity. (d) $p_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

19. What should be the clock frequency of a 6-bit A/D converter so that its maximum conversion time is $32\mu s$?

(a) 1 MHz (b) 2 MHz (c) 0.5 MHz (d) 4 MHz

20. A phosphorous doped silicon semiconductor (doping density: 10^{17} / cm^3) is heated from 100ºC to 200ºC. Which one of the following statements is CORRECT?

(a) Position of Fermi level moves towards conduction band
(b) Position of dopant level moves towards conduction band
(c) Position of Fermi level moves towards middle of energy gap
(d) Position of dopant level moves towards middle of energy gap

21. Considering the BCS theory of superconductors, which one of the following statements is NOT CORRECT? (h is the Planck's constant and e is the electronic charge)

(a) Presence of energy gap at temperatures below the critical temperature

(b) Different critical temperatures or isotopes

(c) Quantization of magnetic flux in superconducting ring in the unit of $\left(\frac{h}{e}\right)$

(d) Presence of Meissner effect

22. Group I contains elementary excitations in solids. Group II gives the associated fields with these excitations. MATCH the excitations with their associated field and select your answer as per codes given below.

Group-I	**Group-II**
(P) phonon	(i) photon + lattice vibration
(Q) plasmon	(ii) electron + elastic deformation
(R) polaron	(iii) collective electron oscillations
(S) polariton	(iv) elastic wave

Codes:

(a) (P–iv), (Q-iii), (R-i), (S-ii)

(b) (P–iv), (Q-iii), (R-ii), (S-i)

(c) (P–i), (Q-iii), (R-ii), (S-iv)

(d) (P–iii), (Q-iv), (R-ii), (S-i)

23. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is ______

24. A voltage regulator has ripple rejection of –50dB. If input ripple is 1mV, what is the output ripple voltage in μV? The answer should be up to two decimal places _______

25. The number of spectral lines allowed in the spectrum for the $3\,^2D \rightarrow 3\,^2P$ transition in sodium is ______

Q.26 – Q.55 : Carry TWO marks each.

26. Which of the following pairs of the given function $F(t)$ and its Laplace transform $f(s)$ is NOT CORRECT?

(a) $F(t) = \delta(t),\ f(s) = 1,$ (Singularity at +0)

(b) $F(t) = 1,\ f(s) = \frac{1}{s}$ $(s > 0)$

(c) $F(t) = \sin kt,\ f(s) = \frac{s}{s^2 + k^2}, (s > 0)$

(d) $F(t) = te^{kt},\ f(s) = \frac{1}{(s-k)^2}, (s > k, s > 0)$

27. If $\vec{A}$ and $\vec{B}$ are constant vectors, then $\nabla(\vec{A}\cdot\vec{B}\times\vec{r})$ is

(a) $\vec{A}\cdot\vec{B}$ (b) $\vec{A}\times\vec{B}$ (c) $\vec{r}$ (d) zero

28. $\Gamma\left(n+\frac{1}{2}\right)$ is equal to [Given $\Gamma(n+1) = n\Gamma(n)$ and $\Gamma(1/2) = \sqrt{\pi}$]

(a) $\frac{n!}{2^n}\sqrt{\pi}$ (b) $\frac{2n!}{n!2^n}\sqrt{\pi}$ (c) $\frac{2n!}{n!2^{2n}}\sqrt{\pi}$ (d) $\frac{n!}{2^{2n}}\sqrt{\pi}$

29. The relativistic form of Newton's second law of motion is

(a) $F = \frac{mc}{\sqrt{c^2 - v^2}}\frac{dv}{dt}$ (b) $F = \frac{m\sqrt{c^2 - v^2}}{c}\frac{dv}{dt}$ (c) $F = \frac{mc^2}{c^2 - v^2}\frac{dv}{dt}$ (d) $F = m\frac{c^2 - v^2}{c^2}\frac{dv}{dt}$

30. Consider a gas of atoms obeying Maxwell-Boltzmann statistics. The average value of $e^{i\vec{a}.\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and a is its magnitude, m is the mass of each atom, T is temperature and k is Boltzmann's constant) is,

(a) One (b) Zero (c) $e^{-\frac{1}{2}a^2mkT}$ (d) $e^{-\frac{3}{2}a^2mkT}$

31. The electromagnetic form factor $F(q^2)$ of a nucleus is given by, $F(q^2) = \exp\left|-\dfrac{q^2}{2Q^2}\right|$

where Q is a constant. Given that $F(q^2) = \dfrac{4\pi}{q}\int_0^\infty rdr\,\rho(r)\sin qr \quad \int d^3r\,\rho(r) = 1$

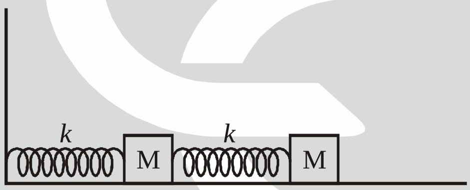
where $\rho(r)$ is the charge density, the root mean square radius of the nucleus is given by

(a) $1/Q$ (b) $\sqrt{2}/Q$ (c) $\sqrt{3}/Q$ (d) $\sqrt{6}/Q$

32. A uniform circular disk of radius R and mass M is rotating with angular speed ω about an axis, passing through its center and inclined at an angle 60 degrees with respect to its symmetry axis. The magnitude of the angular momentum of the disk is,

(a) $\dfrac{\sqrt{3}}{4}\omega MR^2$ (b) $\dfrac{\sqrt{3}}{8}\omega MR^2$ (c) $\dfrac{\sqrt{7}}{8}\omega MR^2$ (d) $\dfrac{\sqrt{7}}{4}\omega MR^2$

33. Consider two small blocks, each of mass M, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is k. The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is,



(a) $\sqrt{\dfrac{3+\sqrt{2}}{2}}\sqrt{\dfrac{k}{M}}$ (b) $\sqrt{\dfrac{3+\sqrt{3}}{2}}\sqrt{\dfrac{k}{M}}$ (c) $\sqrt{\dfrac{3+\sqrt{5}}{2}}\sqrt{\dfrac{k}{M}}$ (d) $\sqrt{\dfrac{3+\sqrt{6}}{2}}\sqrt{\dfrac{k}{M}}$

34. A charge distribution has the charge density given by $\rho = Q\{\delta(x-x_0)-\delta(x+x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$

(a) $\dfrac{2Q\hat{x}}{9\pi\epsilon_0 x_0^2}$ (b) $\dfrac{Q\hat{x}}{4\pi\epsilon_0 x_0^3}$ (c) $\dfrac{Q\hat{x}}{4\pi\epsilon_0 x_0^2}$ (d) $\dfrac{Q\hat{x}}{16\pi\epsilon_0 x_0^2}$

35. A monochromatic plane wave at oblique incidence undergoes reflection at a dielectric interface. If $\hat{k}_i$, $\hat{k}_r$ and $\hat{n}$ are the unit vectors in the directions of incident wave, reflected wave and the normal to the surface respectively, which one of the following expression is correct?

(a) $(\hat{k}_i - \hat{k}_r)\times\hat{n} \neq 0$ (b) $(\hat{k}_i - \hat{k}_r)\cdot\hat{n} = 0$ (c) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r = 0$ (d) $(\hat{k}_i \times \hat{n})\cdot\hat{k}_r \neq 0$

36. In a normal Zeeman effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition $5\,^1D_2 \rightarrow 5\,^1P_1$ of cadmium atom is to be observed. The spectrometer has a resolution of 0.01 nm. The minimum magnetic field needed to observe this is ($m_e = 9.1\times10^{-31}$ kg, $e = 1.6\times10^{-19}$ C, $c = 3\times10^8$ m/s)

(a) 0.26 T (b) 0.52 T (c) 2.6 T (d) 5.2 T

37. The spacing between vibrational energy levels in CO molecule is found to be 8.44l × 10^{-2} eV. Given that the reduced mass of CO is 1.14 × 10^{-26} kg, Planck's constant is 6.626 × 10^{-34} Js and 1eV = 1.6 × 10^{-19}J. The force constant of the bond is CO molecule is

(a) 1.87 N/m (b) 18.7 N/m (c) 187 N/m (d) 1870 N/m

38. A lattice has the following primitive vectors (in A): = $\vec{a} = 2(\hat{j}+\hat{k}), \vec{b} = 2(\hat{k}+\hat{i}), \vec{c} = 2(\hat{i}+\hat{j})$. The reciprocal lattice corresponding to the above lattice is

(a) BCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$ (b) BCC lattice with cube edge of $(2\pi)A^{-1}$

(c) FCC lattice with cube edge of $\left(\frac{\pi}{2}\right)A^{-1}$ (d) FCC lattice with cube edge of $(2\pi)A^{-1}$

39. The total energy of an ionic solid is given by an expression $E = -\frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^9}$ where α is Madelung constant, r is the distance between the nearest neighbours in the crystal and B is a constant. If r_0 is the equilibrium separation between the nearest neighbours then the value of B is

(a) $\frac{\alpha e^2 r_0^8}{36\pi\epsilon_0}$ (b) $\frac{\alpha e^2 r_0^8}{4\pi\epsilon_0}$ (c) $\frac{2\alpha e^2 r_0^{10}}{9\pi\epsilon_0}$ (d) $\frac{\alpha e^2 r_0^{10}}{36\pi\epsilon_0}$

40. A proton is confined to a cubic box, whose sides have length 10^{-12} *m*. What is the minimum kinetic energy of the proton? The mass of proton is 1.67 × 10^{-27} *kg* and Planck's constant is 6.63 × 10^{-34} *Js*.

(a) 1.1 × 10^{-17} *J* (b) 3.3 × 10^{-17} *J* (c) 9.9 × 10^{-17} *J* (d) 6.6 × 10^{-17} *J*

41. For the function $f(z) = \frac{16z}{(z+3)(z-1)^2}$, the residue at the pole $z = 1$ is (your answer should be an integer) ________.

42. The degenerate eigenvalue of the matrix

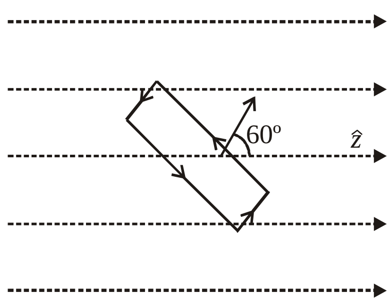
$$\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$$

is (your answer should be an integer) ________________

43. Consider the decay of a pion into a muon and an anti-neutrino $\pi^- \to \mu^- + \bar{\nu}_\mu$ in the pion rest frame.

$m_\pi = 139.6\, MeV/c^2$, $m_\mu = 105.7\, MeV/c^2$, $m_\nu \approx 0$. The energy (in MeV) of the emitted neutrino, to the nearest integer is ________

44. In a constant magnetic field of 0.6 Tesla along the *z* direction, find the value of the path integral $\oint \vec{A}.\vec{dl}$ in the units of (Tesla m^2) on a square loop of side length $(1/\sqrt{2})$ meters. The normal to the loop makes an angle of 60° to the z-axis, as shown in the figure. The answer should be up to two decimal places. ____



45. A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigen states of σ_z then what is the expectation value, up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? Here, symbols have their usual meanings _________

46. Consider the wave function $Ae^{ikr}(r_0/r)$, where A is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2\hbar k/m)$, is _______.

47. An n-channel junction field effect transistor has 5 mA source to drain current at shorted gate (I_{DSS}) and 5V pinch off voltage (V_P). Calculate the drain current in mA for a gate-source voltage (V_{GS}) of –2.5V. The answer should be up to two decimal places ________

Common Data Questions

Common Data for Questions 48 and 49: There are four energy levels E, $2E$, $3E$ and $4E$ (where $E > 0$). The canonical partition function of two particles is, if these particles are

48. Two identical fermions

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$
(b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$
(c) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$
(d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

49. Two distinguishable particles

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$
(b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$
(c) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$
(d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

Common Data for Questions 50 and 51: To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

we add a small perturbation given by

$$\varepsilon\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

where ε is a small quantity.

50. The ground state eigen vector of the unperturbed Hamiltonian is

(a) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ (b) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$ (c) $(0, 0, 1)$ (d) $(1, 0, 0)$

51. A pair of eigen values of the perturbed Hamiltonian, using first order perturbation theory, is

(a) $3+2\varepsilon, 7+2\varepsilon$ (b) $3+2\varepsilon, 2+\varepsilon$ (c) $3, 7+2\varepsilon$ (d) $3, 2+2\varepsilon$

Linked Answer Questions

Statement for Linked Answer Q.52 and Q.53: In the Schmidt model of nuclear magnetic moments, We have,

$$\vec{\mu} = \frac{e\hbar}{2Mc}(g_\ell \vec{l} + g_s \vec{S})$$

where the symbols have their usual meaning

52. For the case $J = l + 1/2$, where J is the total angular momentum, the expectation value of $\vec{S}\cdot\vec{J}$ in the nuclear ground state is equal to,

(a) $(J-1)/2$ (b) $(J+1)/2$ (c) $J/2$ (d) $-J/2$

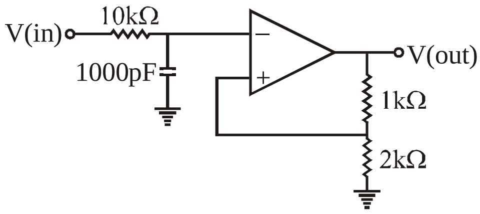
53. For the O^{17} nucleus (A = 17, Z = 8), the effective magnetic moment is given by,

$$\vec{\mu}_{eff} = \frac{e\hbar}{2Mc} g\vec{J}$$

where g is equal to, (g_s = 5.59 for proton and –3.83 for neutron)

(a) 1.12 (b) –0.77 (c) –1.28 (d) 1.28

Statement for Linked Answer Questions 54 and 55: Consider the following circuit



54. For this circuit the frequency above which the gain will decrease by 20 dB per decade is

(a) 15.9 kHz (b) 1.2 kHz (c) 5.6 kHz (d) 22.5 kHz

55. At 1.2 kHz the closed loop gain is

(a) 1 (b) 1.5 (c) 3 (d) 0.5