PHYSICS-PH

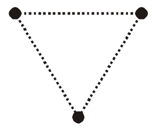
Q.1 – Q.25 : Carry ONE mark each.

1. In the nuclear reaction ${}^{13}C_6 + \nu_e \to {}^{13}N_7 + X$, the particle X is

(a) an electron (b) an anti-electron (c) a muon (d) a pion

2. Two identical masses of 10 gm each are connected by a massless spring of spring constant 1 N/m. The non-zero angular eigen frequency of the system is _______ rad/s. (up to two decimal places).

3. Consider a triatomic molecule of the shape shown in the figure below in three dimensions. The heat capacity of this molecule at high temperature (temperature much higher than the vibrational and rotational energy scales of the molecule but lower than its bond dissociation energies) is:



(a) $\frac{3}{2}k_B$ (b) $3k_B$ (c) $\frac{9}{2}k_B$ (d) $6k_B$

4. For the Hamiltonian $H = a_0 I + \vec{b}.\vec{\sigma}$ where $a_0 \in R, \vec{b}$ is a real vector, I is the 2×2 identity matrix and $\vec{\sigma}$ are the Pauli matrices, the ground state energy is

(a) $|b|$ (b) $2a_0 - |b|$ (c) $a_0 - |b|$ (d) a_0

5. The Poisson bracket $[x, xp_y + yp_x]$ is equal to

(a) $-x$ (b) y (c) $2p_x$ (d) p_y

6. The wavefunction of which orbital is spherically symmetric:

(a) p_x (b) p_y (c) s (d) d_{xy}

7. A monochromatic plane wave in free space with electric field amplitude of 1 V/m is normally incident on a fully reflecting mirror. The pressure exerted on the mirror is _______ $\times 10^{-12}$ Pa. (up to two decimal places) $(\varepsilon_0 = 8.854\times 10^{-12}\text{ F/m})$.

8. The electronic ground state energy of the Hydrogen atom is –13.6 eV. The highest possible electronic energy eigenstate has an energy equal to

(a) 0 (b) 1 eV (c) +13.6 eV (d) ∞

9. Consider a one-dimensional lattice with a weak periodic potential $U(x) = U_0 \cos\left(\frac{2\pi x}{a}\right)$. The gap at the edge of the Brillouin zone $\left(k = \frac{\pi}{a}\right)$ is:

(a) U_0 (b) $\frac{U_0}{2}$ (c) $2U_0$ (d) $\frac{U_0}{4}$

10. Identical charge q are placed at five vertices of a regular hexagon of side a. The magnitude of the electric field and the electrostatic potential at the centre of the hexagon are respectively

(a) 0, 0 (b) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{q}{4\pi\varepsilon_0 a}$ (c) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{5q}{4\pi\varepsilon_0 a}$ (d) $\frac{\sqrt{5}q}{4\pi\varepsilon_0 a^2}, \frac{\sqrt{5}q}{4\pi\varepsilon_0 a}$

11. A reversible Carnot engine is operated between temperatures T_1 and $T_2\ (T_2 > T_1)$ with a photon gas as the working substance. The efficiency of the engine is

(a) $1-\frac{3T_1}{4T_2}$ (b) $1-\frac{T_1}{T_2}$ (c) $1-\left(\frac{T_1}{T_2}\right)^{3/4}$ (d) $1-\left(\frac{T_1}{T_2}\right)^{4/3}$

12. The best resolution that a 7 bit A/D converter with 5V full scale can achieve is _______ mV. (up to two decimal places).

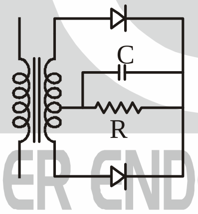
13. If the Lagrangian $L_0=\frac{1}{2}m\left(\frac{dq}{dt}\right)^2-\frac{1}{2}m\omega^2q^2$ is modified to $L=L_0+aq\left(\frac{dq}{dt}\right)$, which one of the following is TRUE?

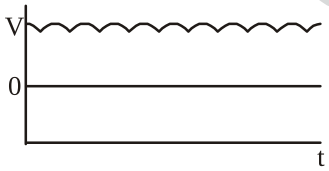
(a) Both the canonical momentum and equation of motion do not change
(b) Canonical momentum changes, equation of motion does not change
(c) Canonical momentum does not change, equation of motion changes
(d) Both the canonical momentum and equation of motion change

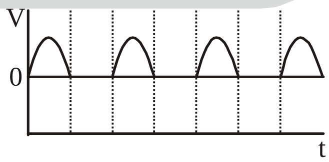
14. A parallel plate capacitor with square plates of side 1 m separated by 1 micro meter is filled with a medium of dielectric constant of 10. If the charges on the two plates are 1 C and -1 C, the voltage across the capacitor is ______kV. (up to two decimal places). $(\varepsilon_0 = 8.854\times10^{-12}\ \text{F/m})$

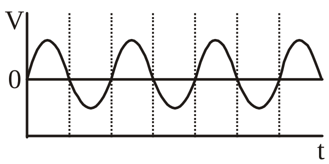
15. The contour integral $\oint\frac{dz}{1+z^2}$ evaluated along a contour going from $-\infty$ to $+\infty$ along the real axis and closed in the lower half-plane by a half circle is equal to ________. (up to two decimal places).

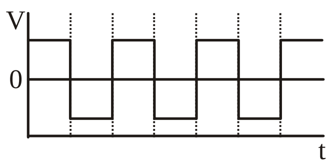
16. In the figure given below, the input to the primary of the transformer is a voltage varying sinusoidally with time. The resistor R is connected to the centre tap of the secondary. Which one of the following plots represents the voltage across the resistor R as a function of time?



(a)


(b)


(c)


(d)


17. Light is incident from a medium of refractive index $n=1.5$ onto vacuum. The smallest angle of incidence for which the light is not transmitted into vacuum is ________ degrees. (up to two decimal places).

18. Electromagnetic interactions are :

(a) C conserving (b) C non-conserving but CP conserving
(c) CP non-conserving but CPT conserving (d) CPT non-conserving

19. The Compton wavelength of a proton is _____ fm. (up to two decimal places).
$(m_p = 1.67\times10^{-27}\,\text{kg},\ h = 6.626\times10^{-34}\,\text{Js},\ e = 1.602\times10^{-19}\,\text{C},\ c = 3\times10^{8}\,\text{ms}^{-1})$

20. Which one of the following conservation laws is violated in the decay $\tau^+ \to \mu^+ + \mu^+ + \mu^-$
(a) Angular momentum (b) Total Lepton number
(c) Electric charge (d) Tau number

21. The coefficient of e^{ikx} in the Fourier expansion of $u(x) = A\sin^2(ax)$ for $k = -2a$ is
(a) $A/4$ (b) $-A/4$ (c) $A/2$ (d) $-A/2$

22. The phase space trajectory of a free particle bouncing between two hard walls elastically in one dimension is a
(a) straight line (b) parabola (c) rectangle (d) circle

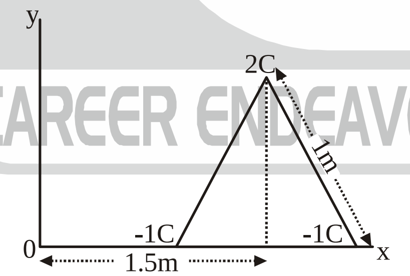
23. The atomic mass and mass density of Sodium are 23 and $0.968\ \text{g cm}^{-3}$, respectively. The number density of valence electrons is ______$\times10^{22}\ \text{cm}^{-3}$. (Up to two decimal places.)
(Avogadro number, $N_A = 6.022\times10^{23}$).

24. The degeneracy of the third energy level of a 3-dimensional isotropic quantum harmonic oscillator is
(a) 6 (b) 12 (c) 8 (d) 10

25. A one dimensional simple harmonic oscillator with Hamiltonian $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2$ is subjected to a small perturbation, $H_1 = \alpha x + \beta x^3 + \gamma x^4$. The first order correction to the ground state energy is dependent on
(a) only β (b) α and γ (c) α and β (d) only γ

Q.26 – Q.55 : Carry TWO marks each.

26. Three charges (2 C, –1 C, –1 C) are placed at the vertices of an equilateral triangle of side 1m as shown in the figure. The component of the electric dipole moment about the marked origin along the $\hat{y}$ direction is ___ cm.



27. An object travels along the x-direction with velocity $c/2$ in a frame O. An observer in a frame O' sees the same object travelling with velocity $c/4$. The relative velocity of O' with respect to O in units of c is _______. (up to two decimal places).

28. The energy density and pressure of a photon gas are given by $u = aT^4$ and $P = u/3$, where T is the temperature and a is the radiation constant. The entropy per unit volume is given by $\alpha a T^3$. The value of α is _____. (up to two decimal places).

29. A person weighs w_p at Earth's north pole and w_e at the equator. Treating the Earth as a perfect sphere of radius 6400 km, the value $100\times(w_p - w_e)/w_p$ is _____. (up to two decimal places).
(Take $g = 10\ \text{ms}^{-2}$).

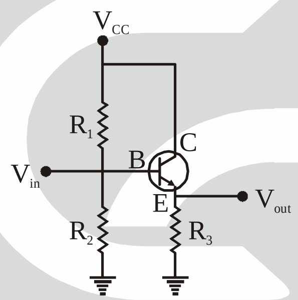
30. The minimum number of NAND gates required to construct an OR gate is:
(a) 2 (b) 4 (c) 5 (d) 3

31. The total energy of an inert-gas crystal is given by $E(R) = \frac{0.5}{R^{12}} - \frac{1}{R^6}$ (in eV), where R is the inter-atomic spacing in Angstroms. The equilibrium separation between the atoms is ________ Angstroms. (up to two decimal places).

32. The imaginary part of an analytic complex function is $v(x, y) = 2xy + 3y$. The real part of the function is zero at the origin. The value of the real part of the function at $1+i$ is ______. (up to two decimal places).

33. Consider N non-interacting, distinguishable particles in a two-level system at temperature T. The energies of the levels are 0 and ε, where $\varepsilon > 0$. In the high temperature limit $(k_B T >> \varepsilon)$, what is the population of particles in the level with energy ε?
(a) $\frac{N}{2}$ (b) N (c) $\frac{N}{4}$ (d) $\frac{3N}{4}$

34. For the transistor amplifier circuit shown below with $R_1 = 10\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, $R_3 = 1\ \text{k}\Omega$, and $\beta = 99$. Neglecting the emitter diode resistance, the input impedance of the amplifier looking into the base for small ac signal is _______ $\text{k}\Omega$. (up to two decimal places).



35. Which one of the following gases of diatomic molecules is Raman, infrared, and NMR active?
(a) ${}^{1}H-{}^{1}H$ (b) ${}^{12}C-{}^{16}O$ (c) ${}^{1}H-{}^{35}Cl$ (d) ${}^{16}O-{}^{16}O$

36. Let X be a column vector of dimension $n > 1$ with at least one non-zero entry. The number of non-zero eigenvalues of the matrix $M = XX^T$ is
(a) 0 (b) n (c) 1 (d) $n-1$

37. A free electron of energy 1 eV is incident upon a one-dimensional finite potential step of height 0.75 eV. The probability of its reflection from the barrier is _______ (up to two decimal places).

38. An infinite solenoid carries a time varying current $I(t) = At^2$, with $A \neq 0$. The axis of the solenoid is along the $\hat{z}$ direction. $\hat{r}$ and $\hat{\theta}$ are the usual radial and polar directions in cylindrical polar coordinates. $\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} + B_z\hat{z}$ is the magnetic field at a point outside the solenoid. Which one of the following statements is true?
(a) $B_r = 0, B_\theta = 0, B_z = 0$ (b) $B_r \neq 0, B_\theta \neq 0, B_z = 0$
(c) $B_r \neq 0, B_\theta \neq 0, B_z \neq 0$ (d) $B_r = 0, B_\theta = 0, B_z \neq 0$

39. Consider two particles and two non-degenerate quantum levels 1 and 2. Level 1 always contains a particle. Hence, what is the probability that level 2 also contains a particle for each of the two cases:
(i) when the two particles are distinguishable and (ii) when the two particles are bosons?
(a) (i) 1/2 and (ii) 1/3 (b) (i) 1/2 and (ii) 1/2 (c) (i) 2/3 and (ii) 1/2 (d) (i) 1 and (ii) 0

40. The real space primitive lattice vectors are $\vec{a}_1 = a\hat{x}$ and $\vec{a}_1 = \frac{a}{2}\left(\hat{x}+\sqrt{3}\hat{y}\right)$. The reciprocal space unit vectors $\vec{b}_1$ and $\vec{b}_2$ for this lattice are, respectively

(a) $\frac{2\pi}{a}\left(\hat{x}-\frac{\hat{y}}{\sqrt{3}}\right)$ and $\frac{4\pi}{a\sqrt{3}}\hat{y}$ (b) $\frac{2\pi}{a}\left(\hat{x}+\frac{\hat{y}}{\sqrt{3}}\right)$ and $\frac{4\pi}{a\sqrt{3}}\hat{y}$

(c) $\frac{2\pi}{a\sqrt{3}}\hat{x}$ and $\frac{4\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}}+\hat{y}\right)$ (d) $\frac{2\pi}{a\sqrt{3}}\hat{x}$ and $\frac{4\pi}{a}\left(\frac{\hat{x}}{\sqrt{3}}-\hat{y}\right)$

41. The geometric cross-section of two colliding protons at large energies is very well estimated by the product of the effective sizes of each particle. This is closest to
(a) 10 b (b) 10 mb (c) 10 μb (d) 10 pb

42. A uniform volume charge density is placed inside a conductor (with resistivity $10^{-2}\,\Omega\text{m}$). The charge density becomes 1/(2.718) of its original value after time ______ femto seconds. (up to two decimal places) $(\varepsilon_0 = 8.854\times10^{-12}\,\text{F/m})$

43. Water freezes at 0ºC at atmospheric pressure $(1.01\times10^5\ \text{Pa})$. The densities of water and ice at this temperature and pressure are 1000 kg/m³ and 934 kg/m³ respectively. The latent heat of fusion is $3.34\times10^5\ \text{J/kg}$. The pressure required for depressing the melting temperature of ice by 10ºC is ____ GPa. (up to two decimal places)

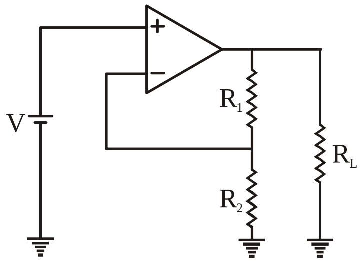
44. The integral $\int_0^\infty x^2 e^{-x^2}dx$ is equal to _______. (up to two decimal places).

45. J^P for the ground state of the $^{13}C_6$ nucleus is
(a) 1^+ (b) $\frac{3^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{1^-}{2}$

46. A uniform solid cylinder is released on a horizontal surface with speed 5 m/s without any rotation (slipping without rolling). The cylinder eventually starts rolling without slipping. If the mass and radius of the cylinder are 10 gm and 1 cm respectively, the final linear velocity of the cylinder is _______ m/s. (up to two decimal places)

47. Consider a one-dimensional potential well of width 3 nm. Using the uncertainty principle $\left(\Delta x.\Delta p \geq \hbar/2\right)$, an estimate of the minimum depth of the well such that it has at least one bound state for an electron is $(m_e = 9.31\times10^{-31}kg,\ h = 6.626\times10^{-34}\,J\ s,\ e = 1.602\times10^{-19}C)$:
(a) 1 μeV (b) 1 meV (c) 1 eV (d) 1 MeV

48. Consider an ideal operational amplifier as shown in the figure below with $\text{R}_1 = 5\ \text{k}\Omega$, $\text{R}_2 = 1\ \text{k}\Omega$, $\text{R}_L = 100\ \text{k}\Omega$. For an applied input voltage $V = 10\ \text{mV}$, the current passing through R_2 is _______ μA. (up to two decimal places).



49. The π^+ decays at rest to μ^+ and ν_π. Assuming the neutrino to be massless, the momentum of the neutrino is ________ MeV/c. (up to two decimal places)
$\left(m_x = 139\text{MeV}/c^2,\ m_\mu = 105\ \text{MeV}/c^2\right)$.

50. Consider the differential equation $dy/dx + y\tan(x) = \cos(x)$. If $y(0) = 0,\ y(\pi/3)$ is ______. (up to two decimal places).

51. Consider a metal with free electron density of $6\times10^{22}\ \text{cm}^{-3}$. The lowest frequency electromagnetic radiation to which this metal is transparent is $1.38\times10^{16}\ \text{Hz}$. If this metal had a free electron density of $1.8\times10^{23}\ \text{cm}^{-3}$ instead, the lowest frequency electromagnetic radiation to which it would be transparent is _______$\times10^{16}\ \text{Hz}$. (up to two decimal places).

52. Using Hund's rule, the total angular momentum quantum number *J* for the electronic ground state of the nitrogen atom is
(a) 1/2 (b) 3/2 (c) 0 (d) 1

53. Consider a 2-dimensional electron gas with a density of $10^{19}\ \text{m}^{-2}$. The Fermi energy of the system is _____eV. (up to two decimal places).
$\left(m_e = 9.31\times10^{-31}\text{kg},\ h = 6.626\times10^{-34}\text{Js},\ e = 1.602\times10^{-19}\text{C}\right)$

54. Which one of the following operators is Hermitian?
(a) $i\dfrac{\left(p_x x^2 - x^2 p_x\right)}{2}$ (b) $i\dfrac{\left(p_x x^2 + x^2 p_x\right)}{2}$ (c) $e^{ip_x a}$ (d) $e^{-ip_x a}$

55. Positronium is an atom made of an electron and a positron. Given that the Bohr radius for the ground state of the Hydrogen atom to be 0.53 Angstroms, the Bohr radius for the ground state of positronium is ______ Angstroms. (up to two decimal places).