TEST SERIES CSIR-NET/JRF JUNE 2018

BOOKLET SERIES A

Paper Code 05

Test Type: TEST SERIES

PHYSICAL SCIENCES

Duration: 02:00 Hours

Date: 21-05-2018

Maximum Marks: 120

Read the following instructions carefully:

* Single Paper Test is divided into **TWO** Parts.

Part - A: This part shall carry **10** questions. Each question shall be of **2 marks.**

Part - B: This part shall contain **50** questions. Each question shall be of **2 marks**.

* Darken the appropriate bubbles with HB pencil/Ball Pen to write your answer.

* There will be negative marking @25% for each wrong answer.

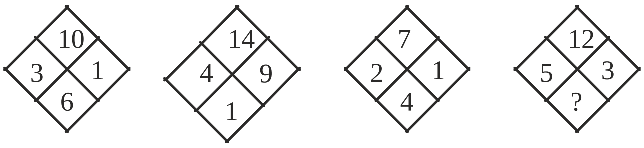
* The candidates shall be allowed to carry the Question Paper Booklet after completion of the exam.

* For rough work, blank sheet is attached at the end of test booklet.

PART-A : GENERAL APTITUDE

1. Two trains running in opposite directions cross a man standing on the platform in 27 seconds and 17 seconds respectively and they cross each other in 23 seconds. The ratio of their speed is
(a) 1 : 3 (b) 3 : 2 (c) 3 : 4 (d) 2 : 3

2. Which number replaces the question mark?



(a) 2 (b) 3 (c) 8 (d) 4

3. 8 litres are drawn from a cask full of wine and then is filled with water. The operation is performed three more times. The ratio of the quantity of wine now left in cask to that of water is 16 : 65. How much wine did the cask hold originally?
(a) 18 litres (b) 24 litres (c) 32 litres (d) 42 litres

4. A motorboat, whose speed is 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes. The speed of the stream (in km/hr) is
(a) 4 (b) 5 (c) 6 (d) 10

5. Let N be the greatest number that will divide 1305, 4665 and 6905, leaving the same remainder in each case. The sum of the digits in N is
(a) 4 (b) 5 (c) 6 (d) 8

6. A can do a certain work in the same time in which B and C together can do it. If A and B together could do it in 10 days and C alone in 50 days, then B alone could do it in
(a) 15 days (b) 20 days (c) 25 days (d) 30 days

7. Some articles were bought at 6 article for Rs. 5 and sold at 5 article for Rs. 6. Gain percentage is
(a) 30% (b) $33\frac{1}{3}\%$ (c) 35% (d) 44%

8. Usain runs 100 m south from his house, turns left and runs 250 m, again turns left and runs 400 m, then turns right and runs 50m to reach to the stadium. In which direction is the stadium from this house?
(a) South West (b) North east (c) East (d) North

9. In this series AABABCABCDABCDE ....
which letter occupies the 100^{th} position?
(a) H (b) I (c) J (d) K

10. A park is in shape of a circle. A man crossed the park across its diameter. What percentage of distance is saved by not walking along the circumference?
(a) 31.4% (b) 11.4% (c) 57% (d) None of these

PART-B : ELECTROMAGNETIC THEORY, SOLID STATE & QUANTUM MECHANICS

11. An electric charge Q is given to a conducting sphere of radius R. The force on a small charge element ΔQ on its surface is

(a) $\frac{Q^2}{4\pi\varepsilon_0 R^2}$ (b) $\frac{Q\Delta Q}{8\pi\varepsilon_0 R^2}$ (c) $\frac{Q\Delta Q}{4\pi\varepsilon_0 R^2}$ (d) $-\frac{Q\Delta Q}{4\pi\varepsilon_0 R^2}$

12. A long solenoid is driven by an alternating current so that the axial field inside varies with time according to $B(t) = B_0 \cos(\omega t)$. A small circular loop of radius a and resistance R is placed coaxial inside the solenoid. The average power dissipated in the loop is

(a) $\frac{B_0^2\omega^2\pi^2 a^4}{2R}$ (b) $\frac{B_0^2\omega^2\pi^2 a^2}{2R}$ (c) $\frac{B_0^2\omega^2\pi^2 a^2}{R}$ (d) zero

13. A left circularly polarised electromagnetic wave with wavelength $\lambda = 5000\mathring{A}$ is incident normally on a calcite crystal. The refractive index for E-wave and O-wave are $n_e = 1.554$ and $n_0 = 1.544$ respectively. The thickness of the crystal so that emergent light is right circularly polarised is:

(a) 2.5μm (b) 12.5 μm (c) 1.25 μm (d) 25 μm

14. Consider a square waveguide of sides 'a'. It is transmit an electromagnetic wave of wavelength λ in TE_{10} mode but not in the TE_{11} or TM_{11} modes. The limiting range of values of a is

(a) $\frac{\lambda}{2} < a < \lambda$ (b) $\frac{\lambda}{\sqrt{2}} < a < \lambda$ (c) $\frac{\lambda}{2} < a < \frac{\lambda}{\sqrt{2}}$ (d) $2\lambda < a < 3\lambda$

15. In Bohr's theory of hydrogen, the electron in its ground state was supposed to travel in a circle of radius r_0, held in orbit by the coulomb attration of the proton. According to classical electrodynamics, this electron should radiate and hence spiral into the nucleus. The lifespan of Bohr's atom is

(a) $\frac{4\pi^2\varepsilon_0^2 m^2 c^3}{e^4} r_0^3$ (b) $\frac{8\pi^2\varepsilon_0^2 m^2 c^3}{e^3} r_0^2$ (c) $\frac{16\pi^2\varepsilon_0^2 m^3 c^3}{e^3} r_0^3$ (d) ∞

16. Consider the $_{92}U^{238}$ nucleus having a radius $R = 6.63\times10^{-15}$m. The charge distribution inside the nucleus is given by $\rho(r) = \alpha r$. The value of α is (in C/m^4)

(a) 2.43×10^{39} (b) -2.43×10^{39} (c) 2.43×10^{-39} (d) zero

17. A long solenoid having n turns per unit length and radius R carries a current $I = I_0 \sin\omega t$. The displacement current density at a distance r from the axis with in solenoid is given by

(a) $\frac{r\omega}{2c^2} nI_0 \sin(\omega t)\hat{\phi}$ (b) $\frac{r\omega^2}{2c^2} nI_0 \sin(\omega t)\hat{\phi}$ (c) $\frac{\omega^2}{2c^2} nI_0 \cos(\omega t)\hat{\phi}$ (d) $\frac{\omega^2 r}{2c^2} nI_0 \cos(\omega t)\hat{\phi}$

18. Consider an infinite solenoid with circular cross-section of radius R with N turns per unit length carrying a steady current I. The magnetic vector potential is given by

$$\begin{cases} A \propto r^m & for\ r > R \\ \propto r^n & for\ r < R \end{cases}$$

The value of m and n are

(a) $m = -1, n = -1$ (b) $m = 0, n = 0$ (c) $m = -1, n = 1$ (d) $m = -1, n = 0$

19. Consider an infinitely long wire carrying λ_0 charge per unit length. An observer moving with velocity 0.6c parallel to length of the wire at a perpendicular distance *d* from the wire. The observer will measure the electric field as

(a) $\frac{5\lambda_0}{8\pi\varepsilon_0 d}\hat{d}$ (b) $\frac{\lambda_0}{2\pi\varepsilon_0 d}\hat{d}$ (c) $\frac{4\lambda_0}{10\pi\varepsilon_0 d}\hat{d}$ (d) zero

20. Consider a y-polarized electromagnetic wave $\vec{E} = \hat{y}E_0\cos(\omega t - kx)$ incident normally on the interface of a dielectric and a perfect conductor $(\sigma = \infty)$. The standing wave due to superposition of reflected and incident wave is given by

(a) $\hat{z}2E_0\cos(kx)\cos(\omega t)$ (b) $\hat{y}2E_0\cos(kx)\cos(\omega t)$

(c) $\hat{y}2E_0\sin(kx)\cos(\omega t)$ (d) $\hat{y}2E_0\sin(kx)\sin(\omega t)$

21. A point charge *q* is situated at a large distance *r* from a neutral atom of polarizability α. The force of attraction between them will be

(a) $\frac{q}{4\pi\varepsilon_0 r^2}$ (b) $\frac{q\alpha}{4\pi\varepsilon_0 r^6}$ (c) $\frac{q^2 2\alpha}{16\pi^2\varepsilon_0^2 r^5}$ (d) $\frac{\alpha q^2}{4\pi\varepsilon_0 r^2}$

22. Consider $\vec{A}$ and V are vector and scalar potential $\vec{A}(r,t) = -\frac{qt\hat{r}}{4\pi\varepsilon_0 r^3}, V(r,t) = 0$ and a scalar function $\lambda = -\frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}$. Due to Gauge transformation, the transformed potential are

(a) $V'(r,t) = 0, \vec{A}'(r,t) = \frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}\hat{r}$

(b) $V'(r,t) = \frac{q}{4\pi\varepsilon_0 r^2}, \vec{A}'(r,t) = \frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}\hat{r}$

(c) $V'(r,t) = -\frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}, \vec{A}'(r,t) = -\frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}\hat{r}$

(d) $V'(r,t) = \frac{q}{4\pi\varepsilon_0 r^2}, \vec{A}'(r,t) = \frac{q}{4\pi\varepsilon_0}\frac{t}{r^3}\hat{r}$

23. A parallel plate capacitor of plate area A and plate separation *d* is charged to potential difference of V_0. If now disconnected from source and then its plate separation is increased to 3d. The work required to increase the distance is

(a) $\frac{3\varepsilon_0 AV_0^2}{d}$ (b) $\frac{\varepsilon_0 AV_0^2}{2d}$ (c) $\frac{\varepsilon_0 AV_0^2}{3d}$ (d) $\frac{\varepsilon_0 AV_0^2}{d}$

24. In a single-slit diffraction, the second-order bright fringe is at a distance 1.40 mm from the centre of the central maximum. The screen is 80.0 cm from a slit of width 0.800 mm. Assuming that the incident light is monochromatic, the approximate wavelength of the incident light is

(a) 5.60×10^{-7} m (b) 5.60×10^{7} m (c) 7.90×10^{9} m (d) 7.90×10^{-9} m

25. Certain volume current distribution $\vec{J}$ gives arise to the magnetic vector potential $\vec{A}(r,\phi,z)=\frac{k}{r^2}\hat{z}$. The corresponding current distribution $\vec{J}$ is given by

(a) $-\frac{4k}{\mu_0 r}\hat{z}$ (b) $-\frac{4k}{\mu_0 r^4}\hat{z}$ (c) $-\frac{4k}{\mu_0 r^4}\hat{\phi}$ (d) $\frac{4k}{\mu_0 r^2}\hat{\phi}$

26. Consider two coaxial circular loops of wire of radii a and b separated by a distance x and carrying I_1 and I_2 respectively. Assuming $b << a$. The force between two loops is

(a) $\frac{3\pi\mu_0 a^2 b^2 I_1 I_2 x}{2\left(a^2+x^2\right)^{5/2}}$ (b) $\frac{\pi\mu_0 a^2 b^2 I_1 I_2 x}{\left(a^2+x^2\right)^{5/2}}$ (c) $\frac{3\pi\mu_0 I_1^2 a^2 x}{2\left(a^2+x^2\right)^{5/2}}$ (d) zero

27. An electromagnetic wave propagating in a medium which propagation vector is given $k=\left(3+i\sqrt{3}\right)m^{-1}$. Which of the following state is correct.

(a) $\vec{E}$ lags behind $\vec{H}$ by an phase angle $\frac{\pi}{6}$ with skin depth $\delta=\frac{1}{\sqrt{3}}m$

(b) $\vec{H}$ lags behind $\vec{E}$ by an phase angle $\frac{\pi}{6}$ with skin depth $\delta=\frac{1}{\sqrt{3}}m$

(c) $\vec{E}$ lags behind $\vec{H}$ by an phase angle $\frac{\pi}{6}$ with skin depth $\delta=\sqrt{3}\ m$

(d) $\vec{H}$ lags behind $\vec{E}$ by an phase angle $\frac{\pi}{6}$ with skin depth $\delta=\sqrt{3}\ m$

28. The tight binding energy dispersion (E–k) relation for electrons in a one-dimensional array of atoms having lattice constant a and total length L is:

$$E=2E_0\left[\sin^2\left(\frac{ka}{2}\right)-\frac{1}{6}\sin^2(ka)\right]$$

Where E_0 is constant and k is the wave-vector. The effective mass (m*) of electron at $k=\frac{\pi}{2a}$ is

(a) $-\frac{\hbar^2}{E_0 a^2}$ (b) $\frac{\hbar^2}{E_0 a^2}$ (c) $\frac{3\hbar^2}{2E_0 a^2}$ (d) $-\frac{3\hbar^2}{2E_0 a^2}$

29. The electrons moves with relativistic speeds on graphene sheet and assumed to follow the dispersion relation $\varepsilon(k)=vk$ (where v is constant) over the entire k-space, then the dependence of Fermi temperature (T_F) on electron density (n) is

(a) $T_F \propto n^{1/2}$ (b) $T_F \propto n$ (c) $T_F \propto n^{2/3}$ (d) $T_F \propto n^{1/3}$

30. The energy of an electron in a band as a function of its wave vector **k** is given by

$E(k)=E_0-B(\cos k_x a+\cos k_y a+\cos k_z a)$, where E_0, B and a are constants. The group velocity $\left(v_g\right)$ of electron at centre of BZ is

(a) 0 (b) $\frac{Ba}{\hbar}$ (c) $-\frac{Ba}{\hbar}$ (d) $\frac{3Ba}{\hbar}$

31. The dispersion relation for an electron in a solid is $\omega\ (k) = \omega_0 \left(3 - \cos k_x a - \cos k_y a - \cos k_z a\right)$

The effective mass (m*) at $\left(0, \frac{\pi}{a}, 0\right)$

(a) $\frac{\hbar}{\omega_0 a^2}$ (b) $-\frac{\hbar}{\omega_0 a^2}$ (c) $\frac{2\hbar}{\omega_0 a^2}$ (d) ∞

32. Iron (Fe) has fcc lattice with lattice parameter a = 1.0 Å. A beam of electrons 150 eV energy falls on powder Iron (Fe) sample. The angle of diffraction for 2nd XRD peak is

(a) 30º (b) 60º (c) 90º (d) 180º

33. A monoatomic monovalent metal has an fcc structure of lattice constant *a*. The metal consists of N primitive unit cell (N is very large). Take the conduction electron in the metal as free independent electron. Then the ratio of the volume of Fermi sphere to the volume of the first Brillion zone is

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 3

34. From the phonon dispersion Relation, $\omega(k) = \sqrt{\frac{4c}{M}} \sin\frac{ka}{2}$

in a monoatomic linear lattice of N with extending 0 to L atoms with nearest neighbour interaction, then the density of the vibrational states, if ω_m is the maximum frequency (where *a* is the lattice constant, *c* is the force constant)

(a) $\frac{2N}{a}\frac{1}{\sqrt{\omega_m^2 - \omega^2}}$ (b) $\frac{N}{a}\frac{1}{\sqrt{\omega_m^2 - \omega^2}}$ (c) $\frac{N}{2a}\frac{1}{\sqrt{\omega_m^2 - \omega^2}}$ (d) $\frac{2N}{a}\frac{1}{\sqrt{\omega_m^2 + \omega^2}}$

35. A long thin super conducting wire of lead having radius of 1mm at 4.2K. The critical temperature for lead is 7 K and critical magnetic field is 1×10^5 A/m then the critical current density

(a) 0.71×10^6 A/m^2 (b) 3.84×10^7 A/m^2 (c) 1.71×10^8 A/m^2 (d) 12.8×10^9 A/m^2

36. A hypothetical semiconductor has a conduction band that can be described by $E_c(k) = E_1 - E_2 \cos ka$ and a valence band that can be described by $E_v(k) = E_3 - E_4 \sin^2 ka$ where $E_i > 0\ \ (i = 1, 2, 3, 4)$ and $-\frac{\pi}{a} \le k \le \frac{\pi}{a}$ than the band width of the conduction band anad band gap between them

(a) $E_2, E_3 + E_1 + E_2$ (b) $2E_2, E_1 - E_2 - E_3$ (c) $2E_2, E_1 - E_2$ (d) $E_3, E_2 - E_3$

37. The energy dispersion relation of the conduction band of 1-D metal is given by

$$E_c(k_x) = E_g + E_1 \sin^2\left(\frac{k_x a}{2}\right)$$

where $\hbar k_x$ is the momentum, *a* is the lattice constant and E_g and E_1 are the constant. Then the group velocity of electron using the semi classical equation of motion has

(a) Non-zero value at the centre and the boundary of first Brillian zone
(b) Vanishes at the centre and the Boundary of first Brillion zone
(c) Non-zero value at the centre and vanishes at the boundary
(d) Vanishes at the centre and non-zero value at the boundary of first Brillion zone.

38. The Lennard-Jones potential

$$U_{LJ}(R) = 4\varepsilon\left[\left(\frac{\sigma}{R}\right)^{12} - \left(\frac{\sigma}{R}\right)^{6}\right]$$

Consider a diatomic molecules due to the vander waals bond. Then the equilibrium separation and the bond strength is

(a) $2^{1/6}\sigma, 2\varepsilon$ (b) $2^{1/6}\sigma, \varepsilon$ (c) $2^{1/6}\sigma^2, \varepsilon$ (d) $2^{1/6}\sigma^2, 2\varepsilon$

39. The Fermi vector of the Fermi sphere in three dimension is $k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$ then the number of orbitals per unit energy $D(\varepsilon)$ is

(a) $\frac{2}{3}\frac{E}{N}$ (b) $\frac{5}{2}\frac{E}{N}$ (c) $\frac{3}{2}\frac{N}{E}$ (d) $\frac{5}{2}\frac{N}{E}$

40. Conduction electron in a metal can be modeled as an ideal Fermi gas with the Fermions effective mass m*. If the effective mass of the Fermi gas is increased by the factor four (say due to effect of strong crystalline potential) then what will be the change in specific heat capacity at a given temperature

(a) decrease by the four times (b) decrease by the sixteen times
(c) increased by the four times (d) increased by the two times

41. A hypothetical diatomic substance crstallizes in a centred tetragonal structure. The conventional unit cell can described by the primitive vectors (a, 0, 0), (0, a, 0), (0, 0, c) with $c = \frac{3a}{2}$ and a basis consists of two atoms at position (0 0 0) and $\left(\frac{a}{2}, \frac{a}{2}, \frac{c}{2}\right)$. The lattice constant is (a = 4.2 Å) then the max. space filling the lattice

(a) 78.8% (b) 69.8% (c) 54% (d) 59.8%

42. The wavelength associated with an electron having an energy equal to the Fermi energy is (where n is the electron density)

(a) $\left(\frac{\pi}{3n}\right)^{1/3}$ (b) $2\left(\frac{\pi}{3n}\right)^{2/3}$ (c) $\left(\frac{\pi}{3n}\right)^{2/3}$ (d) $2\left(\frac{\pi}{3n}\right)^{1/3}$

43. Consider the following three operators on the 2-dimensional Hilbert space of complex-valued column vectors

$$L_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; L_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; L_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Assume L_z is measured and one finds the value –1. Afterwards L_x and L_x^2 are measured. The value of ΔL_x

(a) $\frac{1}{2}$ (b) 2 (c) 0 (d) 1

44. Consider a system whose state and two observables A and B are given by

$$|\psi\rangle = \frac{1}{6}\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, A = \frac{1}{\sqrt{2}}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & i \\ 0 & -i & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

If we measure B first then, immediately afterwards A. The probability of obtaining value of 1 for B and a value of 1 for A is

(a) $\frac{9}{17}$ (b) $\frac{8}{9}$ (c) 1 (d) 0

45. Consider operator $\hat{A} = \hat{x}\dfrac{d}{dx}$ and $\hat{B} = \hat{A}^{\dagger}$. The commutator $\left[\hat{B}, \sin x\right]$ is

(a) $-x\cos x$ (b) $-\sin x$

(c) $-x\sin x$ (d) $x\cos x$

46. The expectation value of the energy in the ground state for a particle of mass m whose Hamiltonian is given by

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 - \lambda\hat{x}^4$$, where ω is angular frequency and $\lambda << 1$

is

(a) $\frac{1}{2}\hbar\omega - \frac{3\hbar^2\lambda}{4m^2\omega^2}$ (b) $\frac{1}{2}\hbar\omega - \frac{3\hbar^2\lambda}{2m^2\omega^2}$ (c) $\frac{1}{2}\hbar\omega + \frac{3\hbar^2\lambda}{4m^2\omega^2}$ (d) $\frac{1}{2}\hbar\omega + \frac{3\hbar^2\lambda}{2m^2\omega^2}$

47. If the operators A and B satisfy the commutation relation $[A, B] = i$, where $i = \sqrt{-1}$, then $\left[e^{iA^2}, B\right]$ equals

(a) $-2B\left[1 + \sum_{n=1}^{\infty}\frac{i^{n-1}B^{2n}}{(n-1)!}\right]$ (b) $2B\left[1 + \sum_{n=1}^{\infty}\frac{i^{n-1}B^{2n}}{(n-1)!}\right]$

(c) $-2A\left[1 + \sum_{n=1}^{\infty}\frac{i^{n-1}A^{2n}}{(n-1)!}\right]$ (d) $2A\left[1 + \sum_{n=1}^{\infty}\frac{i^{n-1}A^{2n}}{(n-1)!}\right]$

48. The differential cross-section for neutron-neutron scattering in the case, where the potential is approximated by $V(r) = V_0\vec{S}_1 \cdot \vec{S}_2\, e^{-r/a}$, where $\vec{S}_1$ and $\vec{S}_2$ -are the spin vector operators of the two neutrons, is

(a) $\frac{d\sigma}{d\Omega} = \frac{3}{4}\left|f(\theta) + f(\pi - \theta)\right|^2 + \frac{1}{4}\left|f(\theta) - f(\pi - \theta)\right|^2$

(b) $\frac{d\sigma}{d\Omega} = \frac{3}{4}\left|f(\theta) - f(\pi - \theta)\right|^2 + \frac{1}{4}\left|f(\theta) + f(\pi - \theta)\right|^2$

(c) $\frac{d\sigma}{d\Omega} = \frac{3}{4}\left|f(\theta) + f(\pi - \theta)\right|^2 - \frac{1}{4}\left|f(\theta) - f(\pi - \theta)\right|^2$

(d) $\frac{d\sigma}{d\Omega} = \frac{3}{4}\left|f(\theta) - f(\pi - \theta)\right|^2 - \frac{1}{4}\left|f(\theta) + f(\pi - \theta)\right|^2$

49. A particle is initially (i.e., $t < 0$) in its ground state in a one dimensional harmonic oscillator potential with angular frequency ω. At $t = 0$, a time depependent perturbation $\hat{V}(x, t) = V_0\hat{x}^3\, e^{-t/\tau}$ is turned on, where V_0 and τ are positive constants. After long duration of time, particle will be found

(a) only in the ground state (b) only in the first excited state

(c) only in the second excited state (d) both in the first and third excited state

50. A particle is in the ground state of a 1-D potential box of width a. Now, if we suddenly changed the width of the potential box from a to $8a$. The ground state energy in the new potential box and the probability of finding the particle in the new ground state are

(a) $\frac{\hbar^2\pi^2}{128ma^2}$, 17% (b) $\frac{\hbar^2\pi^2}{64ma^2}$, 17% (c) $\frac{\hbar^2\pi^2}{64ma^2}$, 0.7% (d) $\frac{\hbar^2\pi^2}{128ma^2}$, 0.7%

51. A particle of mass m moves in the one dimensional harmonic oscillator potential $V(x)=\frac{1}{2}m\omega^2x^2$. Assuming $\psi(x)=\frac{1}{x^2+a^2}$ as a trial wavefunction, the ground state energy of the particle can be estimated to be (where a is a variational parameter)

(a) $\frac{\hbar\omega}{2}$ (b) $\sqrt{2}\,\hbar\omega$ (c) $\frac{1}{\sqrt{2}}\hbar\omega$ (d) $\frac{\sqrt{3}}{2}\hbar\omega$

52. A particle is prepared in a simultaneous eigenstate of J^2 & J_z. If $j(j+1)\hbar^2$ and $m_j\hbar$ are respectively the eigenvalues of J^2 & J_z. The maximum value of the expectation value $\langle J_y^2\rangle$, is

(a) $j^2\hbar^2$ (b) $j(j+1)\hbar^2$ (c) $\frac{1}{2}j(j+1)\hbar^2$ (d) $\frac{1}{3}j(j+1)\hbar^2$

53. Consider a spinless charged $(-q)$ particle of mass m, which is moving in a three dimensional potential

$$V(x,y,z)=\begin{cases}\frac{1}{2}m\omega^2z^2 & 0<x<a,\ 0<y<a\\ \infty & \text{elsewhere}\end{cases}$$

and it subjected to a constant electric field ε directed along the z-axis. (Assuming $\hbar\omega>\frac{5\pi^2\hbar^2}{2ma^2}$).

The first excited energy of the particle, will be

(a) $\frac{\pi^2\hbar^2}{ma^2}+\frac{3}{2}\hbar\omega-\frac{q^2\varepsilon^2}{2m\omega^2}$ (b) $\frac{5\pi^2\hbar^2}{2ma^2}+\frac{1}{2}\hbar\omega-\frac{q^2\varepsilon^2}{2m\omega^2}$

(c) $\frac{\pi^2\hbar^2}{ma^2}+\frac{1}{2}\hbar\omega-\frac{q^2\varepsilon^2}{2m\omega^2}$ (d) $\frac{\pi^2\hbar^2}{2ma^2}+\frac{3}{2}\hbar\omega-\frac{q^2\varepsilon^2}{2m\omega^2}$

54. The expectation value of x-coordinate in the ground state of hydrogen atom is

(a) zero (b) a_0 (c) $\frac{3a_0}{2}$ (d) $\frac{a_0}{2}$

55. A spin-$\frac{1}{2}$ particle is in the spin state

$$|\psi\rangle=\cos\left(\frac{\theta}{2}\right)e^{-i\varphi}\left|\frac{1}{2},\frac{1}{2}\right\rangle+\sin\left(\frac{\theta}{2}\right)\left|\frac{1}{2},-\frac{1}{2}\right\rangle,$$

where, $\left|\frac{1}{2},\frac{1}{2}\right\rangle$ and $\left|\frac{1}{2},-\frac{1}{2}\right\rangle$ are the eigenstates of S_z, the operator for the z-component of spin angular momentum, with eigenvalues of $\hbar/2$ and $-\hbar/2$, respectively. Now, if z-component of spin is measured, then the probability of getting the result $-\hbar/2$, will be

(a) $\cos^2\left(\frac{\theta}{2}\right)$ (b) $\sin^2\left(\frac{\theta}{2}\right)$ (c) $\cos\left(\frac{\theta}{2}\right).\sin\left(\frac{\theta}{2}\right)$ (d) 1

56. Consider a rotational operator $\hat{R}_z(\alpha)$

$$\hat{R}_z(\alpha)\psi(r,\theta,\phi)=\psi(r,\theta,\phi-\alpha)$$

The value of $\left[\hat{R}_z(\alpha), x\right]$ is

(a) zero (b) $L_x\, e^{-\frac{i\alpha L_z}{\hbar}}$ (c) $-\alpha L_y e^{-\frac{i\alpha L_z}{\hbar}}$ (d) $\alpha\, y\, e^{-\frac{i\alpha L_z}{\hbar}}$

57. Consider 3 spin-1 particles, called 1, 2, 3 with spin operators $\vec{S}_1, \vec{S}_2$ and $\vec{S}_3$, respectively. The particles are placed along a circle. The Hamiltonian takes the form

$$H=\frac{\Delta}{\hbar^2}\left(\vec{S}_1\cdot\vec{S}_2+\vec{S}_2\cdot\vec{S}_3+\vec{S}_3\cdot\vec{S}_1\right)$$

with $\Delta>0$ a constant with units of energy. The number of degeneracy corresponding energy -2Δ is

(a) 1 (b) 3 (c) 4 (d) 5

58. For a particle confined in three dimensional cubical potential well of width a, the momentum of the particle for the state (2, 2, 1) is

(a) $\frac{\pi\hbar}{3a}$ (b) $\frac{\pi\hbar}{a}$ (c) $\frac{3\pi\hbar}{a}$ (d) $\frac{\pi a}{3\hbar}$

59. Consider a system which is initially in the following state:

$$\psi(\theta,\varphi)=\frac{1}{\sqrt{5}}Y_{1,-1}(\theta,\varphi)+\sqrt{\frac{3}{5}}Y_{1,0}(\theta,\varphi)+\frac{1}{\sqrt{5}}Y_{1,1}(\theta,\varphi),$$

where $Y_{\ell,m}$ are simultaneous eigenstate of L^2 & L_z. The value of $\Delta L_x\,\Delta L_y$ is

(a) $\frac{\hbar}{2}$ (b) $\frac{\hbar}{4}$ (c) $\hbar$ (d) zero

60. A rigid rotator is in a quantum state described by the wave function

$$\psi(\theta,\phi)=\sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{-2i\phi}$$

where θ and ϕ are the usual polar angles. The degeneracy of the given state is

(a) 3 (b) 4 (c) 5 (d) 10

Space for rough work

Physical Sciences (NET-JRF/GATE)

Test Series- I

Date: 21-05-2018

ANSWER KEY

PART-A

1. (b)	2. (d)	3. (b)	4. (b)	5. (a)	6. (c)	7. (d)
8. (b)	9. (b)	10. (c)				

PART-B

11. (b)	12. (a)	13. (d)	14. (c)	15. (a)	16. (a)	17. (b)
18. (c)	19. (a)	20. (d)	21. (c)	22. (d)	23. (d)	24. (a)
25. (b)	26. (a)	27. (b)	28. (c)	29. (a)	30. (a)	31. (d)
32. (d)	33. (b)	34. (d)	35. (d)	36. (b)	37. (b)	38. (b)
39. (c)	40. (c)	41. (b)	42. (d)	43. (d)	44. (d)	45. (a)
46. (a)	47. (c)	48. (b)	49. (d)	50. (d)	51. (c)	52. (c)
53. (b)	54. (a)	55. (b)	56. (d)	57. (b)	58. (c)	59. (d)
60. (c)						