CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED
Best Institute for IIT-JAM, NET/JRF & GATE Exams

Physical Sciences

Test-II: Mathematical Physics

Time : 01:00 Hour **Date : 06-03-2014**
M.M. : 70

INSTRUCTIONS

- **This question booklet contains 10 questions in Part-A + 7 questions Part-B only. You have to answer all questions from Part-A and Part-B.**
- **In Part-A, 3.5 marks will be awarded for a right answer and (–1) marks will be awarded for a wrong answer.**
- **In Part-B, 5 marks will be awarded for a right answer and (–1.5) marks will be awarded for a wrong answer.**

Part-A

1. The particular integral to the differential equation $\frac{d^2y}{dx^2}+64y=\sin 6x\cos 2x$ is equal to

(a) $-\frac{1}{16}\cos 8x+\frac{1}{48}\sin 4x$ (b) $-\frac{1}{32}\cos 8x+\frac{1}{96}\sin 4x$

(c) $\frac{1}{16}\cos 8x-\frac{1}{48}\sin 4x$ (d) $\frac{1}{32}\cos 8x-\frac{1}{96}\sin 4x$

2. If the polynomial $4x^3+6x^2+7x+2$ can be wriiten as $\sum_n a_n P_n(x)$, [where $P_n(x)$ is Legendre Polynomial of order 'n' and a_n are constant co-effiecients] then which of the following statement is true?

(a) $a_2=a_1$ (b) $a_2=-a_1$ (c) $a_2=-a_0$ (d) $a_2=a_0$

3. For the differential equation $z^3(z-4)\frac{d^2y}{dz^2}+z\frac{dy}{dz}+\frac{1}{(z-4)}y=0$, which of the following statement is true?

(a) z = 0 is an ordinary point (b) z = 0 is regular singular point
(c) z =0 and z = 4 are regular singular points
(d) z = 0 is an irregular singular point and z = 4 is a regular singular point

4. If the uncertainity in a descrete random variable 'x' and it's mean value are measured to be 0.1 and 0.5 respectively, then the value of $\langle x^2\rangle$ is equal to

(a) 0.51 (b) 0.49 (c) 0.26 (d) 0.24

5. The probability that Rohit, Sanjay and Aman will solve a problem are $\frac{1}{6},\frac{1}{4},\frac{1}{3}$ respectively. The probability that exactly one of them will solve the problem, will be

(a) $\frac{23}{96}$ (b) $\frac{41}{72}$ (c) $\frac{19}{48}$ (d) $\frac{31}{72}$

6. There are 100 errors, randomly distributed in a Physics book of 600 pages and six pages are picked at random. The probability that there will be no errors in those pages, will be

(a) $\frac{1}{e}$ (b) $\frac{2}{e}$ (c) $\frac{1}{e^6}$ (d) $\frac{2}{e^6}$

7. Suppose 'x' is a continuous random variable with probability density function given by

$$\begin{aligned} f(x) &= cx^2 && ;(0 \le x < 1) \\ &= c+3 && ;(1 \le x < 2) \\ &= -cx^3 + 6cx && ;(2 \le x < 3) \\ &= 0 && ; \text{otherwise} \end{aligned}$$

The value of 'c' will be

(a) 24 (b) -24 (c) 37 (d) none of these

8. Consider $J_1(x)$ denotes the bessel function of first order, then laplace tranform of $J_1(x)$ is

(a) $\frac{t}{\sqrt{1+t^2}}$ (b) $1-\frac{t}{\sqrt{1+t^2}}$ (c) $1+\frac{t}{\sqrt{1+t^2}}$ (d) $1+\frac{t^{1/2}}{\sqrt{1+t^2}}$

[Given: $(1+x)^{-1/2} = 1-\frac{1}{2}x+\frac{1.3}{2.4}x^2-\frac{1.3.5}{2.4.6}x^3+.....]$

9. Given the following relation for Hermite polynomial of order 'n' i.e. $H_n(x)$:

$$e^{2zx-z^2} = \sum_{n=0}^{\infty} H_n(x)\frac{z^n}{n!}$$

The value of $H_6(0)$ will be

(a) 120 (b) -120 (c) 24 (d) 72

10. Given the recurrence relation for Bessel function of order 'n':

$$2nJ_n(x) = x\left[J_{n+1}(x)+J_{n-1}(x)\right]$$

The expression $\frac{d}{dx}\left[J_4\left\{x^4+\frac{1}{x^4}\right\}\right]$ (where $J_4(x)$ is the bessel function of order '4') can be written as

(a) $\left(x^4+\frac{1}{x^4}\right)J_3-\frac{8}{x^5}J_4$ (b) $\left(x^4+\frac{1}{x^4}\right)J_3+\frac{8}{x^5}J_4$

(c) $\left(x^4-\frac{1}{x^4}\right)J_3+\frac{8}{x^5}J_4$ (d) $\left(x^4-\frac{1}{x^4}\right)J_3-\frac{8}{x^5}J_4$

PART-B

11. Consider the variable 'y' satisfies the equation: $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+6\frac{dy}{dx}-18y=0$ subjected to the conditions $y(\infty)=0$ and $y'(0)=\sqrt{6}i$, then the value of $y\left(\pi/\sqrt{6}\right)$ is equal to

(a) 0 (b) 1 (c) -1 (d) $\sqrt{2}$

12. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(a) $\left(y^2 - x^2\right) dx / dy = 2xy$ (b) $\left(x^2 - y^2\right) dy / dx = 2xy$

(c) $dx / dy = 2xy\left(x^2 + y^2\right)$ (d) $dy / dx = 2xy\left(x^2 - y^2\right)$

13. A cricket umpire will make a correct LBW decision in 80% of the appeals made by the bowler. What is the probability that he will make atmost 2 incorrect decisions in a cricket match in which 10 appeals are made?

(a) $101\frac{4^8}{5^{10}}$ (b) $101\frac{4^9}{5^{10}}$ (c) $99\frac{4^9}{5^{10}}$ (d) $99\frac{4^8}{5^{10}}$

14. Three persons play a game by tossing a fair coin each independently. The gane ends in a trial if two of them get the same outcome in that trial, otherwise they continue to the next trial. What is the probability that the game ends in an odd number of trials?

(a) 3/5 (b) 4/5 (c) 3/4 (d) 2/3

15. The complex fourier transform of the function $f(x) = e^{-|x-2|}$ is

(a) $\frac{e^{2is}}{1+s^2}$ (b) $\frac{e^{2is}}{2\left(1+s^2\right)}$ (c) $\frac{2e^{2is}}{\left(1+s^2\right)}$ (d) $\frac{2e^{2is}}{\left(1-s^2\right)}$

16. Given the recurrence relation:

$$(n+1) P_{n+1}(x) = (2n+1) x P_n(x) - n P_{n-1}(x)$$

(where $P_n(x)$ is the Legendre polynomial of order 'n')

The value of the integral $\int_{-1}^{1} x^2 P_n^2(x)\, dx$ will be

(a) $\frac{2(n+1)^2}{(2n+1)^2(2n-3)} + \frac{2n^2}{(2n+1)^2(2n-1)}$ (b) $\frac{2(n+1)^2}{(2n+1)^2(2n+3)} + \frac{2n^2}{(2n+1)^2(2n-1)}$

(c) $\frac{2n^2}{(2n+1)^2(2n+3)} + \frac{2(n+1)^2}{(2n+1)^2(2n-1)}$ (d) $\frac{2n^2}{(2n+1)^2(2n-3)} + \frac{2(n+1)^2}{(2n+1)^2(2n-1)}$

17. The inverse Laplace transform of the function $f(s) = \frac{1}{s^3\left(s^2+1\right)}$ is

(a) $\frac{t^2}{2} - \cos t - 1$ (b) $\frac{t^2}{2} + \cos t - 1$ (c) $\frac{t}{2} + \cos t$ (d) $\frac{t}{2} - \cos t$

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PHYSICAL SCIENCES (NET-JRF/GATE)

Unit Test II: Mathematical Physics

Answer Key (Mathematical Physics)

1. (b)	**2. (d)**	**3. (d)**	**4. (c)**	**5. (d)**	**6. (a)**	**7. (b)**
8. (b)	**9. (b)**	**10. (a)**	**11. (b)**	**12. (a)**	**13. (a)**	**14. (b)**
15. (c)	**16. (b)**	**17. (b)**				