CSIR-UGC–NET/JRF– JUNE - 2012
PHYSICAL SCIENCES

Part-B

21. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is

(a) $\hat{i}+\hat{j}$ (b) $\hat{j}+\hat{k}$ (c) $\hat{i}+\hat{j}+\hat{k}$ (d) $2\hat{i}+3\hat{j}+5\hat{k}$

22. The eigenvalues of the matrix, $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ are

(a) (1, 4, 9) (b) (0, 7, 7) (c) (0, 1, 13) (d) (0, 0, 14)

23. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \le |z| \le 2$, and around $z = 1$ is

(a) $\frac{1}{2}\left[1+z+z^2+z^3+......\right]\left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+.....\right]$

(b) $\frac{1}{1-z}+z-(1-z)^2+(1-z)^3+........$

(c) $\frac{1}{z^2}\left[1+\frac{1}{z}+\frac{1}{z^2}+......\right]\left[1+\frac{2}{z}+\frac{4}{z^2}+.....\right]$

(d) $2(z-1)+5(z-1)^2+7(z-1)^3+......$

24. The radioactive decay of a certain material satisfies Poisson statistics with a mean rate of λ per second, what should be the minimum duration of counting (in seconds) so that the relative error is less than 1% ?

(a) $100/\lambda$ (b) $10^4/\lambda^2$ (c) $10^4/\lambda$ (d) $1/\lambda$

25. Let $u(x,y) = x + \frac{1}{2}(x^2 - y^2)$ be the real part of an analytic function f(z) of the complex variable $z = x + iy$. The imaginary part of f(z) is

(a) $y + xy$ (b) xy (c) y (d) $y^2 - x^2$

26. Let $y(x)$ be a continuous real function in the range 0 and 2π, satisfying the inhomogeneous differential equaltion:

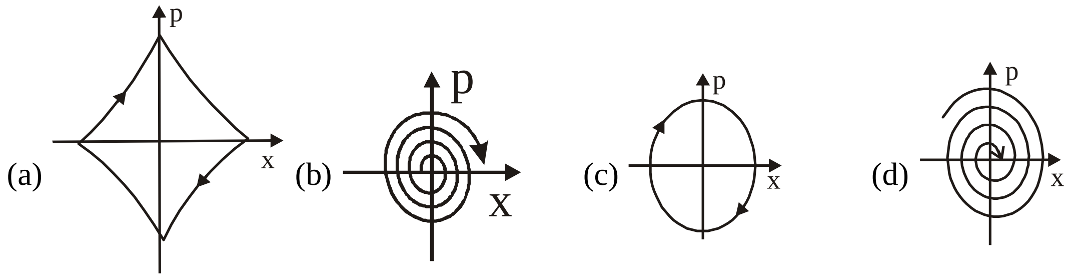
$\sin x\frac{d^2y}{dx^2}+\cos x\frac{dy}{dx}=\delta\left(x-\frac{\pi}{2}\right)$. The value of dy/dx at the point $x = \pi/2$

(a) is continuous (b) has a discontinuity of 3
(c) has a discontinuity of 1/3 (d) has a discontinuity of 1

27. A ball is picked at random from one of two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white?

(a) 34/70 (b) 41/70 (c) 36/70 (d) 29/70

28. The bob of a simple pendulum,which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum?



29. Two events, separated by a (spatial) distance 9×10^{9} m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed 0.8c (where the speed of light $c = 3\times10^{8}$ m/s) is:

(a) 60 s (b) 40 s (c) 20 s (d) 0 s

30. If the Lagrangian of a particle moving in one dimension is given by $L = \frac{\dot{x}^2}{2x} - V(x)$, the Hamiltonian is:

(a) $\frac{1}{2}xp^2 + V(x)$ (b) $\frac{\dot{x}^2}{2x} + V(x)$ (c) $\frac{1}{2}\dot{x}^2 + V(x)$ (d) $\frac{p^2}{2x} + V(x)$

31. A horizontal circular platform rotates with a constant angular velocity Ω directed vertically upwards. A person seated at the centre shoots a bullet of mass ‘m’ horizontally with speed ‘v’. The acceleration of the bullet, in the reference frame of the shooter, is

(a) $2v\Omega$ to his right (b) $2v\Omega$ to his left (c) $v\Omega$ to his right (d) $v\Omega$ to his left

32. The magnetic field corresponding to the vector potential, $\vec{A} = \frac{1}{2}\vec{F}\times\vec{r} + \frac{10}{r^3}\vec{r}$

where $\vec{F}$ is a constant vector, is

(a) $\vec{F}$ (b) $-\vec{F}$ (c) $\vec{F} + \frac{30}{r^4}\vec{r}$ (d) $\vec{F} - \frac{30}{r^4}\vec{r}$

33. An electromagnetic wave is incident on a water-air interface. The phase of the perpendicular component of the electric field, $E_{\perp}$, of the reflected wave into the water is found to remain the same for all angles of incidence. The phase of the magnetic field *H*.

(a) does not change (b) changes by $3\pi/2$ (c) changes by $\pi/2$ (d) changes by π

34. The magnetic field at a distance R from a long straight wire carrying a steady current I is proportional to

(a) *IR* (b) I/R^2 (c) I^2/R^2 (d) *I/R*

35. The component along an arbitrary direction $\hat{n}$, with direction cosines (n_x, n_y, n_z), of the spin of a spin-½ particle is measured. The result is:

(a) 0 (b) $\pm\frac{\hbar}{2}n_z$ (c) $\pm\frac{\hbar}{2}(n_x, n_y, n_z)$ (d) $\pm\frac{\hbar}{2}$

36. A particle of mass m is in a cubic box of size a. The potential inside the box $\left(0 \le x < a, 0 \le y < a, 0 \le z < a\right)$ is zero and infinte outside. If the particle is in an eigenstate of energy $E = \frac{14\pi^2 h^2}{2ma^2}$, its wave function is:

(a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{3\pi x}{a} \sin\frac{5\pi y}{a} \sin\frac{6\pi z}{a}$

(b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{7\pi x}{a} \sin\frac{4\pi y}{a} \sin\frac{3\pi z}{a}$

(c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{4\pi x}{a} \sin\frac{8\pi y}{a} \sin\frac{2\pi z}{a}$

(d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a} \sin\frac{2\pi y}{a} \sin\frac{3\pi z}{a}$

37. Let $\Psi_{n\ell m}$ denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The wavefunction $\Psi = \frac{1}{4}\left[\Psi_{210} + \sqrt{5}\Psi_{21-1} + \sqrt{10}\Psi_{213}\right]$ is an eigenfunction only of

(a) H, L^2 and L_z (b) H and L_z (c) H and L^2 (d) L^2 and L_z

38. The commutator $\left[x^2, p^2\right]$ is

(a) $2 i\hbar\, xp$ (b) $2i\hbar\left(xp + px\right)$ (c) $2i\hbar px$ (d) $2i\hbar\left(xp - px\right)$

39. Consider a system of non-interacting particles in d dimensions obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy, k is the wavevector, 's' is an integer and A a constant. The density of states, $N(\varepsilon)$, is proportional to

(a) $\varepsilon^{\frac{1}{d}-1}$ (b) $\varepsilon^{\frac{d}{s}+1}$ (c) $\varepsilon^{\frac{d}{s}-1}$ (d) $\varepsilon^{\frac{s}{d}-s}$

40. The number of ways in which N identical bosons can be distributed in two energy levels, is

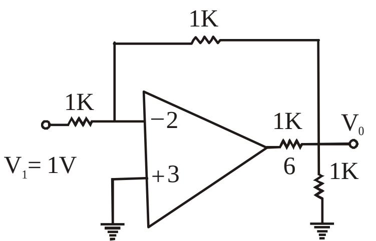
(a) $N + 1$ (b) $N(N-1)/2$ (c) $N(N+1)/2$ (d) N

41. The free energy of a gas N particled in a volume V and at a temperature T is $F = NK_BT \ln\left[a_0 V \left(k_B T\right)^{5/2} / N\right]$, where a_0 is a constant and k_B denotes the Boltzmann constant. The internal energy of gas is

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{5}{2}Nk_BT$

(c) $Nk_BT \ln\left[a_0 V \left(k_BT\right)^{5/2}/N\right] - \frac{3}{2}Nk_BT$ (d) $Nk_BT \ln\left[a_0V/\left(k_BT\right)^{5/2}\right]$

42. In the op-amp circuit shown in the figure below, the input voltage V_i is $1V$. The value of the output V_0 is

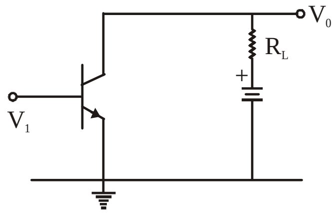


(a) $-0.33\ V$ (b) $-0.50\ V$ (c) $-1.00\ V$ (d) $-0.25\ V$

43. An LED operates at 1.5 V and 5 mA in forward bias. Assuming an 80 % external efficiency of the LED, how many photons are emitted per second?

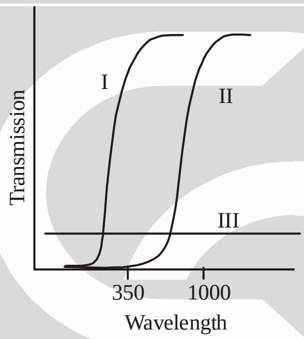
(a) 5.0×10^{16} (b) 1.5×10^{16} (c) 0.8×10^{16} (d) 2.5×10^{16}

44. The transistor in the given circuit has $h_{fe} = 35\Omega$ and $h_{ie} = 1000\Omega$. If the load resistance $R_L = 1000\Omega$, the voltage and current gain are, respectively.



(a) –35, + 35 (b) 35, – 35 (c) 35, – 0.97 (d) 0.98, –35

45. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond respectively, to



(a) Insulator, semiconductor and metal (b) Semiconductor, metal and insulator
(c) Metal, semiconductor and insulator (d) Insulator, metal and semiconductor

46. The eigenvalues of the antisymmetric matrix, $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where n_1, n_2 and n_3 are the components of a unit vector, are

(a) 0, i, – i (b) 0, 1, – 1 (c) 0, 1 + i, –1 – i (d) 0, 0, 0

47. Which of the following limits exists?

(a) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m}+\ln N\right)$ (b) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{m}-\ln N\right)$

(c) $\lim_{N\to\infty}\left(\sum_{m=1}^{N}\frac{1}{\sqrt{m}}-\ln N\right)$ (d) $\lim_{N\to\infty}\sum_{m=1}^{N}\frac{1}{m}$

48. A bag contains many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to

(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$

49. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{t^2 - R^2} \cos\left(\frac{\pi t}{2R}\right) dt$ is

(a) $-2\pi / R$ (b) $-\pi / R$ (c) π / R (d) $2\pi / R$

50. The Poisson bracket $\{|r|, |p|\}$ has the value

(a) $|r||p|$ (b) $\hat{r}.\hat{p}$ (c) 3 (d) 1

51. Consider the motion of a classical particle in a one dimensional double-well potential $V(x) = \frac{1}{4}(x^2 - 2)^2$. If the particle is displaced infinitesimally from the minimum the positive x-axis (and friction is neglected), then

(a) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = \sqrt{2}$

(b) the particle will execute simple harmonic motion in the right well with an angular frequency $\omega = 2$

(c) the particle will switch between the right and left wells

(d) the particle will approach the bottom of the right well and settle there

52. What is the proper time interval between the occurence of two events if in one inertial frame the events are separated by 7.5×10^5 m and occur 6.5 s apart?

(a) 6.50 s (b) 6.00 s (c) 5.75 s (d) 5.00 s

53. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential $V(r) = \begin{cases} V_0 & if \ r \le R \\ 0 & if \ r > R \end{cases}$ If V_0 is changes to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the Born approximation,

(a) increases to four times the original value (b) increases to twice the original value

(c) decreases to half the original value (d) decreases to one fourth the original value

54. A variational calculation is done with the normalized trial wavefunction $\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}(a^2 - x^2)$ for the one dimensional potential well

$$V(x) = \begin{cases} 0 & if \ |x| \le a \\ \infty & if \ |x| > a \end{cases}$$

The ground state energy is estimated to be

(a) $\frac{5h^2}{3ma^2}$ (b) $\frac{3h^2}{2ma^2}$ (c) $\frac{3h^2}{5ma^2}$ (d) $\frac{5h^2}{4ma^2}$

55. A particle in one-dimension is in the potental

$$V(x) = \begin{cases} \infty & if \ x < 0 \\ -V_0 & if \ 0 \le x \le \ell \\ 0 & if \ x > \ell \end{cases}$$

If there is at least one bound state, the minimum depth of the potential is

(a) $\frac{h^2\pi^2}{8m\ell^2}$ (b) $\frac{h^2\pi^2}{2m\ell^2}$ (c) $\frac{2h^2\pi^2}{m\ell^2}$ (d) $\frac{h^2\pi^2}{m\ell^2}$

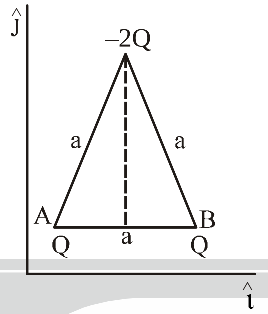
56. Which of the following is a self-adjoint operator in the spherical polar coordinate system (r,θ,φ)?

(a) $-\frac{i\hbar}{\sin^2\theta}\frac{\partial}{\partial\theta}$ (b) $-i\hbar\frac{\partial}{\partial\theta}$ (c) $-\frac{i\hbar}{\sin\theta}\frac{\partial}{\partial\theta}$ (d) $-i\hbar\sin\theta\frac{\partial}{\partial\theta}$

57. Which of the following quantities is Lorentz invariant?

(a) $|E\times B|^2$ (b) $|E|^2-|B|^2$ (c) $|E|^2+|B|^2$ (d) $|E|^2|B|^2$

58. Charges Q, Q and $-2Q$ are placed on the vertices of an equilateral triangle ABC of sides of length a, as shown in the figure



The dipole moment of this configuration of charges, irrespective of the choice of origin, is

(a) $+2aQ\,\hat{i}$ (b) $+\sqrt{3}aQ\,\hat{j}$ (c) $-\sqrt{3}aQ\,\hat{j}$ (d) 0

59. The vector potential $\vec{A}$ due to a magnetic moment 'm' at a point 'r' is given by $\vec{A}=\frac{\vec{m}\times\vec{r}}{r^3}$. If $\vec{m}$ is directed along the positive z-axis, the x-component of the magnetic field, at the point r, is

(a) $\frac{3myz}{r^5}$ (b) $-\frac{3mxy}{r^5}$ (c) $\frac{3mxz}{r^5}$ (d) $\frac{3m(z^2-xy)}{r^5}$

60. A system has two normal modes of vibration, with frequencies ω_1 and $\omega_2=2\omega_1$. What is the probability that at temperature T, the system has an energy less than $4\hbar\omega_1$? [In the following $x=e^{-\beta\hbar\omega}$ and Z is the partition function]

(a) $x^{3/2}(x+2x^2)/Z$ (b) $x^{3/2}(1+x+x^2)/Z$

(c) $x^{3/2}(1+2x^2)/Z$ (d) $x^{3/2}(1+x+2x^2)/Z$

61. The magnetization M of a ferromagnet, as a function of the temperature T and the magnetic field H, is described by the equaltion $M=\tanh\left(\frac{T_c}{T}M+\frac{H}{T}\right)$. In these units, the zero-field magnetic susceptibility in terms of $M(0)=M(H=0)$ is given by

(a) $\frac{1-M^2(0)}{T-T_c(1-M^2(0))}$ (b) $\frac{1-M^2(0)}{T-T_c}$ (c) $\frac{1-M^2(0)}{T+T_c}$ (d) $\frac{1-M^2(0)}{T}$

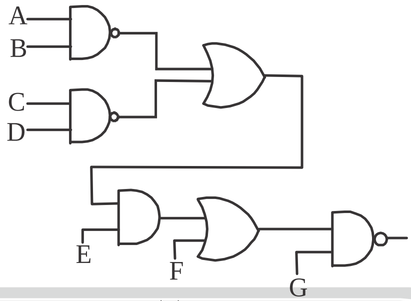
62. Bose condensation occurs in liquid He^4 kept at ambient pressure at 2.17 K. At which temperature will Bose condensation occur in He^4 in gaseous state, the density of which is 1000 times smaller than that of liquid He^4? (Assume that it is a perfect Bose gas)

(a) 2.17 mK (b) 21.7 mK (c) 21.7 μK (d) 2.17 μK

63. Consider black body radiation contained in a cavity whose walls are at temperature T. The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to $2T$ and the radiation is allowed to come to equilibrium at the new temperature, the entropy of the radiation increases by a factor of

(a) 2 (b) 4 (c) 8 (d) 16

64. The output 0, of the given circuit in cases I and II, where
Case I : A, B = 1; C, D = 0; E, F = 1 and G = 0
Case II : A,B = 0; C, D = 0; E, F = 0 and G = 1 are respectively



(a) 1, 0 (b) 0, 1 (c) 0, 0 (d) 1, 1

65. A resistance strain gauge is fastened to a steel fixture and subjected to a stress of 1000 kg/m^2. If the gauge factor is 3 and the modulus of elasticity of steel is $2\times10^{10}\ \text{kg/m}^2$, then the fractional change in resistance of the strain gauge due to the applied stress is:
(Note: The gauge factor is defined as the ratio of the fractional change in resistance to the fractional change in length.)

(a) 1.5×10^{-7} (b) 3.0×10^{-7} (c) 0.16×10^{-10} (d) 0.5×10^{-7}

66. Consider a sinusoidal waveform of amplitude 1V and frequency f_0. Starting from an arbitrary intial time, the waveform is sampled at intervals of $1/(2f_0)$. If the corresponding Fourier spectrum peaks at a frequency $\overline{f}$ and an amplitude $\overline{A}$, then

(a) $\overline{f}=2f_0$ and $\overline{A}=1V$ (b) $\overline{f}=f_0$ and $0\le\overline{A}\le 1V$

(c) $\overline{f}=0$ and $\overline{A}=1V$ (d) $\overline{f}=\frac{f_0}{2}$ and $\overline{A}=\frac{1}{\sqrt{2}}V$

67. The first absorption spectrum of $^{12}C^{16}O$ is at $3.842\,\text{cm}^{-1}$ while that of $^{13}C^{16}O$ is at $3.673\,\text{cm}^{-1}$. The ratio of their moments of inertia is

(a) 1.851 (b) 1.286 (c) 1.046 (d) 1.038

68. The spin-orbit interaction in an atom is given by $H = a\,\vec{L}.\vec{S}$, where L and S denote the orbital and spin angular moment, respectively of electron. The splitting between the levels $^2P_{3/2}$ and $^2P_{1/2}$ is:

(a) $\frac{3}{2}a\hbar^2$ (b) $\frac{1}{2}a\hbar^2$ (c) $3a\hbar^2$ (d) $\frac{5}{2}a\hbar^2$

69. The spectral line corresponding to an atomic transition from $J = 1$ to $J = 0$ states splits in a magnetic field of 1 KG into three components seperated by 1.6×10^{-3} Å. If the zero field spectral line corresponding to 1849Å, what is the g-factor corresponding to the $J = 1$ state? (You may use $\frac{hc}{\mu_0}\approx 2\times10^4$ cm.)

(a) 2 (b) 3/2 (c) 1 (d) 1/2

70. The energy required to create a lattice vacancy in a crystal is equal to 1 eV. The ratio of the number densities of vacancies $n(1200\text{ K})/n(300\text{ K})$, when the crystal is at equilibrium at 1200 K and 300 K, respectively is approximately
(a) exp(–30) (b) exp(–15) (c) exp(15) (d) exp(30)

71. The dispersion relation of phonons in a solid is given by
$$\omega^2(k) = \omega_0^2\left(3 - \cos k_x a - \cos k_y a - \cos k_z a\right)$$
The velocity of the phonons at large wavelength is
(a) $\omega_0 a/\sqrt{3}$ (b) $\omega_0 a$ (c) $\sqrt{3}\omega_0 a$ (d) $\omega_0 a/\sqrt{2}$

72. Consider an electron in a box of lenght L with periodic boundary condition $\psi(x) = \psi(x+L)$. If the electron is in the $\psi_k(x) = \frac{1}{\sqrt{L}} e^{jkx}$ with energy $\varepsilon_k = \frac{\hbar^2 k^2}{2m}$, what is the correction to its energy, to second order of perturbation theory, when it is subjected to weak periodic potential $V(x) = V_0 \cos gx$, where g is an integral multiple of the $2\pi/L$?
(a) $V_0^2\, \varepsilon_g/\varepsilon_k^2$ (b) $-\frac{mV_0^2}{2\hbar^2}\left(\frac{1}{g^2+2kg} + \frac{1}{g^2-2kg}\right)$
(c) $V_0^2\left(\varepsilon_k - \varepsilon_g\right)/\varepsilon_g^2$ (d) $V_0^2/\left(\varepsilon_k + \varepsilon_g\right)$

73. The ground state of ${}^{207}_{82}\text{Pb}$ nuclcus has spin-parity $J^P = \frac{1^-}{2}$, while the first excited state has $J^P = \frac{5^-}{2}$. The electromagnetic radiation emitted when the nucleus makes a transition from the first excited state to the ground state are
(a) E2 and E3 (b) M2 and E3 (c) E2 and M3 (d) M2 and M3

74. The dominant interactions underlying the following processes
A. $K^- + p \to \Sigma^- + \pi^+$, B. $\mu^- + \mu^+ \to K^- + K'$, C. $\Sigma^+ \to p + \pi^0$ are
(a) A: strong, B: electromagnetic and C: weak (b) A: strong, B: weak and C: weak
(c) A: weak, B: electromagnetic and C: strong (d) A: weak, B: electromagnetic and C: weak

75. If a Higgs boson of mass m_H with a speed $\beta = \frac{v}{c}$ decays into a pair photons, then the invariant mass of the photon pair is
[**Note:** The invariant mass of a system of two particles, with four-momenta p_1 and p_2 is $(p_1 + p_2)^2$]
(a) βm_H (b) m_H (c) $m_H/\sqrt{1-\beta^2}$ (d) $\beta m_H/\sqrt{1-\beta^2}$