PART - B

21. Consider two radioactive atoms, each of which has a decay rate of 1 per year. The probability that at least one of them decays in the first two years is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $1-e^{-4}$ (d) $(1-e^{-2})^2$

22. The Fourier transform $\int_{-\infty}^{\infty} dx\, f(x) e^{ikx}$ of the function $f(x)=\frac{1}{x^2+2}$ is

(a) $\sqrt{2}\pi e^{-\sqrt{2}|k|}$ (b) $\sqrt{2}\pi e^{-\sqrt{2}k}$ (c) $\frac{\pi}{\sqrt{2}} e^{-\sqrt{2}k}$ (d) $\frac{\pi}{\sqrt{2}} e^{-\sqrt{2}|k|}$

23. A ball of mass m is dropped from a tall building with zero initial velocity. In addition to gravity, the ball experiences a damping force of the form $-\gamma v$, where v is its instantaneous velocity and γ is a constant. Given the values $m=10$ kg, $\gamma=10$ kg/s, and $g \approx 10$ m/s^2, the distance travelled (in metres) in time t in seconds, is

(a) $10(t+1-e^{-t})$ (b) $10(t-1+e^{-t})$ (c) $5t^2-(1-e^{t})$ (d) $5t^2$

24. The matrix $M=\begin{pmatrix} 1 & 3 & 2 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ satisfies the equation

(a) $M^3-M^2-10M+12I=0$ (b) $M^3+M^2-12M+10I=0$

(c) $M^3-M^2-10M+10I=0$ (d) $M^3+M^2-10M+10I=0$

25. The Laplace transform of $f(t)=\begin{cases} \frac{t}{T}, & 0<t<T \\ 1 & t>T \end{cases}$, is

(a) $-\frac{(1-e^{-sT})}{s^2T}$ (b) $\frac{(1-e^{-sT})}{s^2T}$ (c) $\frac{(1+e^{-sT})}{s^2T}$ (d) $\frac{(1-e^{sT})}{s^2T}$

26. A relativistic particle moves with a constant velocity v with respect to the laboratory frame. In time τ, measured in the rest frame of the particle, the distance that it travels in the laboratory frame is

(a) $v\tau$ (b) $\frac{c\tau}{\sqrt{1-\frac{v^2}{c^2}}}$ (c) $v\tau\sqrt{1-\frac{v^2}{c^2}}$ (d) $\frac{v\tau}{\sqrt{1-\frac{v^2}{c^2}}}$

27. A particle in two dimensions is in a potential $V(x,y)=x+2y$. Which of the following (apart from the total energy of the particle) is also a constant of motion ?

(a) p_y-2p_x (b) p_x-2p_y (c) p_x+2p_y (d) p_y+2p_x

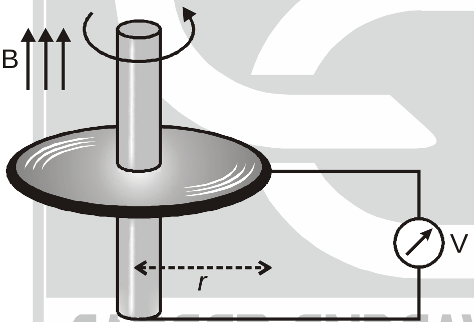
28. The dynamics of a particle governed by the Lagrangian $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2-kx\dot{x}t$ describes

(a) an undamped simple harmonic oscillator
(b) a damped harmonic oscillator with a time varying damping factor
(c) an undamped harmonic oscillator with a time dependent frequency
(d) a free particle

29. The parabolic coordinates (ξ,η) are related to the Cartesian coordinates (x,y) by $x=\xi\eta$ and $y=\frac{1}{2}(\xi^2-\eta^2)$. The Lagrangian of a two-dimensional simple harmonic oscillator of mass m and angular frequency ω is

(a) $\frac{1}{2}m\left[\dot{\xi}^2+\dot{\eta}^2-\omega^2\left(\xi^2+\eta^2\right)\right]$
(b) $\frac{1}{2}m\left(\xi^2+\eta^2\right)\left[\left(\dot{\xi}^2+\dot{\eta}^2\right)-\frac{1}{4}\omega^2\left(\xi^2+\eta^2\right)\right]$
(c) $\frac{1}{2}m\left(\xi^2+\eta^2\right)\left(\dot{\xi}^2+\dot{\eta}^2-\frac{1}{2}\omega^2\xi\eta\right)$
(d) $\frac{1}{2}m\left(\xi^2+\eta^2\right)\left(\dot{\xi}^2+\dot{\eta}^2-\frac{1}{4}\omega^2\right)$

30. A conducting circular disc of radius r and resistivity ρ rotates with an angular velocity ω in a magnetic field B perpendicular to it. A voltmeter is connected as shown in the figure below.

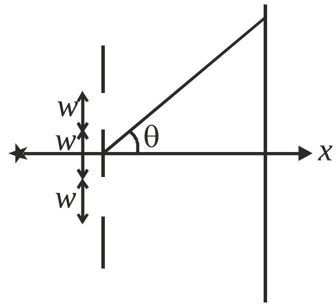


Assuming its internal resistance to be infinite, the reading on the voltmeter
(a) depends on ω, B, r and ρ
(b) depends on ω, B and r, but not on ρ
(c) is zero because the flux through the loop is not changing
(d) is zero because a current flows in the direction of B

31. The charge per unit length of a circular wire of radius a in the xy-plane, with its center at the origin, is $\lambda=\lambda_0\cos\theta$, where λ_0 is a constant and the angle θ is measured from the positive x-axis. The electric field at the center of the circle is

(a) $\vec{E}=-\frac{\lambda_0}{4\varepsilon_0 a}\hat{i}$ (b) $\vec{E}=\frac{\lambda_0}{4\varepsilon_0 a}\hat{i}$ (c) $\vec{E}=-\frac{\lambda_0}{4\varepsilon_0 a}\hat{j}$ (d) $\vec{E}=\frac{\lambda_0}{4\pi\varepsilon_0 a}\hat{k}$

32. A screen has two slits, each of width w, with their centres at a distance $2w$ apart. It is illuminated by a monochromatic plane wave travelling along the x-axis.



The intensity of the interference pattern, measured on a distant screen, at an angle $\theta = n\lambda/w$ to the x-axis is

(a) zero for $n = 1, 2, 3...$ (b) maximum for $n = 1, 2, 3...$

(c) maximum for $n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ (d) zero for $n = 0$ only

33. The electric field of an electromagnetic wave is $\vec{E}(z,t) = E_0 \cos(kz + \omega t)\hat{i} + 2E_0 \sin(kz + \omega t)\hat{j}$, where ω and k are positive constants. This represents

(a) a linearly polarised wave travelling in the positive z-direction
(b) a circularly polarised wave travelling in the negative z-direction
(c) an elliptically polarised wave travelling in the negative z-direction
(d) an unpolarised wave travelling in the positive z-direction

34. Consider the two lowest normalized energy eigenfunctions $\psi_0(x)$ and $\psi_1(x)$ of a one dimensional system. They satisfy $\psi_0(x) = \psi_0^*(x)$ and $\psi_1(x) = \alpha \frac{d\psi_0}{dx}$, where α is a real constant. The expectation value of the momentum operator in the state ψ_1 is

(a) $-\frac{\hbar}{\alpha^2}$ (b) 0 (c) $\frac{\hbar}{\alpha^2}$ (d) $\frac{2\hbar}{\alpha^2}$

35. Consider the operator $a = x + \frac{d}{dx}$ acting on smooth functions of x. The commutator $[a, \cos x]$ is

(a) $-\sin x$ (b) $\cos x$ (c) $-\cos x$ (d) 0

36. Let $a = \frac{1}{\sqrt{2}}(x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2}}(x - ip)$ be the lowering and raising operators of a simple harmonic oscillator in units where the mass, angular frequency and $\hbar$ have been set to unity. If $|0\rangle$ is the ground state of the oscillator and λ is a complex constant, the expectation value of $\langle\psi|x|\psi\rangle$ in the state $|\psi\rangle = \exp(\lambda a^\dagger - \lambda^* a)|0\rangle$, is

(a) $|\lambda|$ (b) $\sqrt{|\lambda|^2 + \frac{1}{|\lambda|^2}}$ (c) $\frac{1}{\sqrt{2}i}(\lambda - \lambda^*)$ (d) $\frac{1}{\sqrt{2}}(\lambda + \lambda^*)$

37. Consider the operator $\vec{\pi} = \vec{p} - q\vec{A}$, where $\vec{p}$ is the momentum operator, $\vec{A} = (A_x, A_y, A_z)$ is the vector potential and q denotes the electric charge. If $\vec{B} = (B_x, B_y, B_z)$ denotes the magnetic field, the z-component of the vector operator $\vec{\pi} \times \vec{\pi}$ is

(a) $iq\hbar B_z + q(A_x p_y - A_y p_x)$
(b) $-iq\hbar B_z - q(A_x p_y - A_y p_x)$
(c) $-iq\hbar B_z$
(d) $iq\hbar B_z$

38. Consider a gas of N classical particles in a two-dimensional square box of side L. If the total energy of the gas is E, the entropy (apart from an additive constant) is

(a) $Nk_B \ln\left(\frac{L^2 E}{N}\right)$
(b) $Nk_B \ln\left(\frac{LE}{N}\right)$
(c) $2Nk_B \ln\left(\frac{L\sqrt{E}}{N}\right)$
(d) $L^2 k_B \ln\left(\frac{E}{N}\right)$

39. Consider a continuous time random walk. If a step has taken place at time $t = 0$, the probability that the next step takes place between t and $t + dt$ is given by $bt\,dt$, where b is a constant. What is the average time between successive steps ?

(a) $\sqrt{\frac{2\pi}{b}}$
(b) $\sqrt{\frac{\pi}{b}}$
(c) $\frac{1}{2}\sqrt{\frac{\pi}{b}}$
(d) $\sqrt{\frac{\pi}{2b}}$

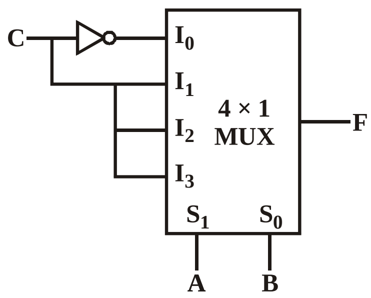
40. The partition function of a two-level system governed by the Hamiltonian $H = \begin{bmatrix} \gamma & -\delta \\ -\delta & -\gamma \end{bmatrix}$ is

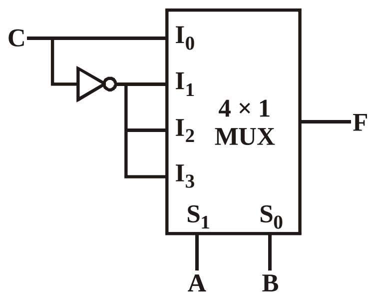
(a) $2\sinh\left(\beta\sqrt{\gamma^2 + \delta^2}\right)$
(b) $2\cosh\left(\beta\sqrt{\gamma^2 + \delta^2}\right)$
(c) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2 + \delta^2}\right) + \sinh\left(\beta\sqrt{\gamma^2 + \delta^2}\right)\right]$
(d) $\frac{1}{2}\left[\cosh\left(\beta\sqrt{\gamma^2 + \delta^2}\right) - \sinh\left(\beta\sqrt{\gamma^2 + \delta^2}\right)\right]$

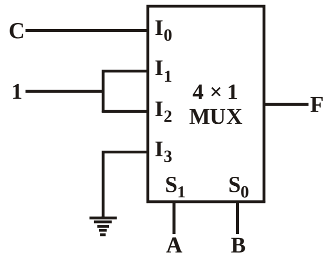
41. A silica particle of radius 0.1 μm is put in a container of water at $T = 300$ K. The densities of silica and water are 2000 kg/m^3 and 1000 kg/m^3, respectively. Due to thermal fluctuations, the particle is not always at the bottom of the container. The average height of the particle above the base of the container is approximately

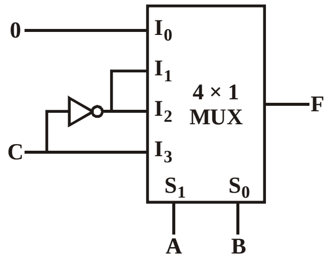
(a) 10^{-3} m
(b) 3×10^{-4} m
(c) 10^{-4} m
(d) 5×10^{-5} m

42. Which of the following circuits implements the Boolean function $F(A, B, C) = \Sigma(1, 2, 4, 6)$?

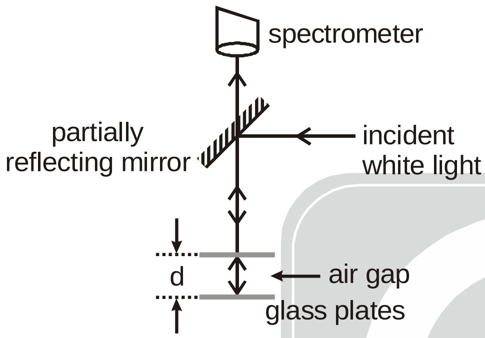
(a)


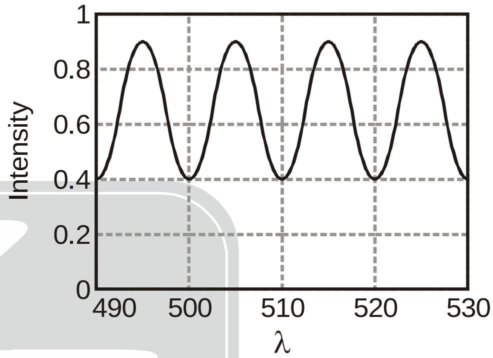
(b)


(c) 

(d) 

43. A pair of parallel glass plates separated by a distance d is illuminated by white light as shown in the figure below. Also shown is the graph of the intensity of the reflected light light I as a function of the wavelength λ recorded by a spectrometer.

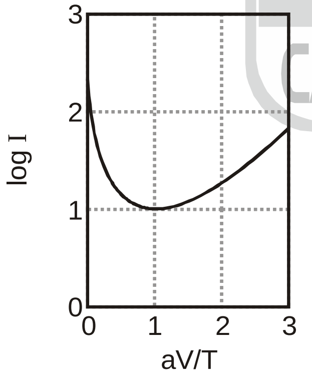


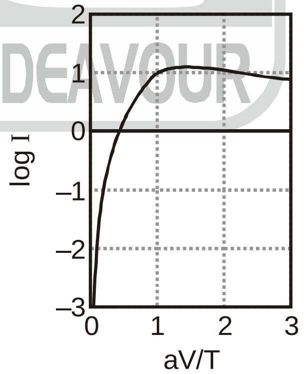


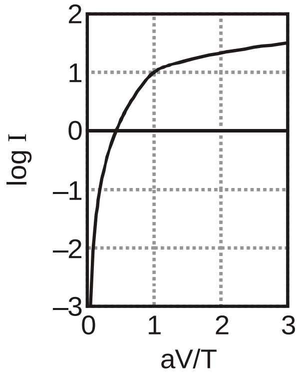
Assuming that the interference takes place only between light reflected by the bottom surface of the top plate and the top surface of bottom plate, the distance d is closest to

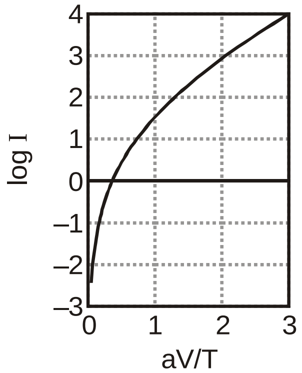
(a) 12 μm (b) 24 μm (c) 60 μm (d) 120 μm

44. The I-V characteristics of a device is $I = I_s\left[\exp\left(\frac{aV}{T}\right)-1\right]$, where T is the temperature and a and I_s are constants independent of T and V. Which one of the following plots is correct for a fixed applied voltage V ?

(a) 

(b) 

(c) 

(d) 

45. The active medium in a blue LED (Light Emitting Diode) is a $\mathrm{Ga}_x\mathrm{In}_{1-x}\mathrm{N}$ alloy. The band gaps of GaN and InN are 3.5 eV and 1.5 eV respectively. If the band gap of $\mathrm{Ga}_x\mathrm{In}_{1-x}\mathrm{N}$ varies approximately linearly with x, the value of x required for the emission of blue light of wavelength 400 nm is (take $hc \approx 1200$ eV-nm)

(a) 0.95 (b) 0.75 (c) 0.50 (d) 0.33

PART - C

46. A stable asymptotic solution of the equation $x_{n+1} = 1 + \frac{3}{1+x_n}$ is $x = 2$. If we take $x_n = 2 + \varepsilon_n$ and $x_{n+1} = 2 + \varepsilon_{n+1}$, where ε_n and ε_{n+1} are both small, the ratio $\varepsilon_{n+1}/\varepsilon_n$ is approximately

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

47. The 2 × 2 identity matrix I and the Pauli matrices $\sigma^x, \sigma^y, \sigma^z$ do not form a group under matrix multiplication. The minimum number of 2 × 2 matrices, which includes these four matrices, and form a group (under matrix multiplication) is

(a) 20 (b) 8 (c) 12 (d) 16

48. Given the values sin 45º = 0.7071, sin 50º = 0.7660, sin 55º = 0.8192 and sin 60º = 0.8660, the approximate value of sin 52º, computed by Newton's forward difference method, is

(a) 0.804 (b) 0.776 (c) 0.788 (d) 0.798

49. Let $f(x,t)$ be a solution of the heat equation $\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$ in one dimension. The initial condition at $t = 0$ is $f(x,0) = e^{-x^2}$ for $-\infty < x < \infty$. Then for all $t > 0$, $f(x,t)$ is given by

[Useful integral : $\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$].

(a) $\frac{1}{\sqrt{1+Dt}}e^{-\frac{x^2}{1+Dt}}$ (b) $\frac{1}{\sqrt{1+2Dt}}e^{-\frac{x^2}{1+2Dt}}$ (c) $\frac{1}{\sqrt{1+4Dt}}e^{-\frac{x^2}{1+4Dt}}$ (d) $e^{-\frac{x^2}{1+Dt}}$

50. After a perfectly elastic collision of two identical balls, one of which was initially at rest, the velocities of both the balls are non-zero. The angle θ between the final velocities (in the lab frame) is

(a) $\theta = \frac{\pi}{2}$ (b) $\theta = \pi$ (c) $0 < \theta < \frac{\pi}{2}$ (d) $\frac{\pi}{2} < \theta \le \pi$

51. Consider circular orbits in a central force potential $V(r) = -\frac{k}{r^n}$, where $k > 0$ and $0 < n < 2$. If the time period of a circular orbit of radius R is T_1 and that of radius $2R$ is T_2, then T_2/T_1 is

(a) $2^{\frac{n}{2}}$ (b) $2^{\frac{2}{3}n}$ (c) $2^{\frac{n}{2}+1}$ (d) 2^n

52. Consider a radioactive nucleus that is travelling at a speed $c/2$ with respect to the lab frame. It emits γ-rays of frequency ν_0 in its rest frame. There is a stationary detector (which is not on the path of the nucleus) in the lab. If a γ-ray photon is emitted when the nucleus is closest to the detector, its observed frequency at the detector is

(a) $\frac{\sqrt{3}}{2}\nu_0$ (b) $\frac{1}{\sqrt{3}}\nu_0$ (c) $\frac{1}{\sqrt{2}}\nu_0$ (d) $\sqrt{\frac{2}{3}}\nu_0$

53. Suppose that free charges are present in a material of dielectric constant $\varepsilon = 10$ and resistivity $\rho = 10^{11}\,\Omega\text{-m}$. Using Ohm's law and the equation of continuity for charge, the time required for the charge density inside the material to decay by $1/e$ is closest to

(a) 10^{-6} s (b) 10^{6} s (c) 10^{12} s (d) 10 s

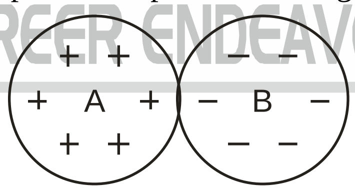
54. A particle with charge $-q$ moves with a uniform angular velocity ω in a circular orbit of radius a in the xy-plane, around a fixed charge $+q$, which is at the centre of the orbit at (0, 0, 0). Let the intensity of radiation at the point $(0, 0, R)$ be I_1 and at $(2R, 0, 0)$ be I_2. The ratio I_2/I_1, for $R \gg a$, is

(a) 4 (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) 8

55. A parallel plate capacitor is formed by two circular conducting plates of radius a separated by a distance d, where $d \ll a$. It is being slowly charged by a current that is nearly constant. At an instant when the current is I, the magnetic induction between the plates at a distance $a/2$ from the centre of the plate, is

(a) $\frac{\mu_0 I}{\pi a}$ (b) $\frac{\mu_0 I}{2\pi a}$ (c) $\frac{\mu_0 I}{a}$ (d) $\frac{\mu_0 I}{4\pi a}$

56. Two uniformly charged insulating solid spheres A and B, both of radius a, carry total charges $+Q$ and $-Q$, respectively. The spheres are placed touching each other as shown in the figure.



If the potential at the center of the sphere A is V_A and that at the center of B is V_B, then the difference $V_A - V_B$ is

(a) $\frac{Q}{4\pi\varepsilon_0 a}$ (b) $\frac{-Q}{2\pi\varepsilon_0 a}$ (c) $\frac{Q}{2\pi\varepsilon_0 a}$ (d) $\frac{-Q}{4\pi\varepsilon_0 a}$

57. A particle is scattered by a central potential $V(r) = V_0\, r e^{-\mu r}$, where V_0 and μ are positive constants. If the momentum transfer $\vec{q}$ is such that $q = |\vec{q}| \gg \mu$, the scattering cross-section in the Born approximation, as $q \to \infty$, depends on q as

[You may use $\int x^n e^{ax}\, dx = \frac{d^n}{da^n}\int e^{ax}\, dx$]

(a) q^{-8} (b) q^{-2} (c) q^{2} (d) q^{6}

58. A particle in one dimension is in a potential $V(x) = A\delta(x-a)$. Its wavefunction $\psi(x)$ is continuous everywhere. The discontinuity in $d\psi/dx$ at $x = a$ is

(a) $\frac{2m}{\hbar^2}A\psi(a)$ (b) $A(\psi(a)-\psi(-a))$ (c) $\frac{\hbar^2}{2m}A$ (d) 0

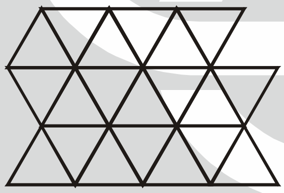
59. The dynamics of a free relativistic particle of mass m is governed by the Dirac Hamiltonian $H = c\vec{\alpha}.\vec{p} + \beta mc^2$, where $\vec{p}$ is the momentum operator and $\vec{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ and β are four 4×4 Dirac matrices. The acceleration operator can be expressed as

(a) $\frac{2ic}{\hbar}(c\vec{p} - \vec{\alpha}H)$ (b) $2ic^2\vec{\alpha}\beta$ (c) $\frac{ic}{\hbar}H\vec{\alpha}$ (d) $-\frac{2ic}{\hbar}(c\vec{p} + \vec{\alpha}H)$

60. A particle of charge q in one dimension is in a simple harmonic potential with angular frequency ω. It is subjected to a time dependent electric field $E(t) = Ae^{-(t/\tau)^2}$, where A and τ are positive constants and $\omega\tau \gg 1$. If in the distant past $t \to -\infty$ the particle was in its ground state, the probability that it will be in the first excited state as $t \to +\infty$ is proportional to

(a) $e^{-\frac{1}{2}(\omega\tau)^2}$ (b) $e^{\frac{1}{2}(\omega\tau)^2}$ (c) 0 (d) $\frac{1}{(\omega\tau)^2}$

61. Consider a random walk on an infinite two-dimensional triangular lattice, a part of which is shown in the figure below.



If the probabilities of moving to any of the nearest neighbour sites are equal, what is the probability that the walker returns to the starting position at the end of exactly three steps ?

(a) $\frac{1}{36}$ (b) $\frac{1}{216}$ (c) $\frac{1}{18}$ (d) $\frac{1}{12}$

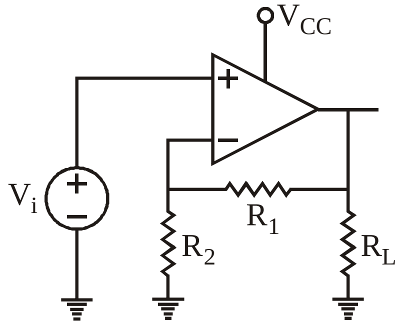
62. An atom has a non-degenerate ground state and a doubly-degenerate excited state. The energy difference between the two states is ε. The specific heat at very low temperatures $(\beta\varepsilon \gg 1)$ is given by

(a) $k_B(\beta\varepsilon)$ (b) $k_B\,e^{-\beta\varepsilon}$ (c) $2k_B(\beta\varepsilon)^2 e^{-\beta\varepsilon}$ (d) k_B

63. The electrons in graphene can be thought of as a two-dimensional gas with a linear energy-momentum relation $E = |\vec{p}|v$, where $\vec{p} = (p_x, p_y)$ and v is a constant. If ρ is the number of electrons per unit area, the energy per unit area is proportional to

(a) $\rho^{3/2}$ (b) ρ (c) $\rho^{1/3}$ (d) ρ^2

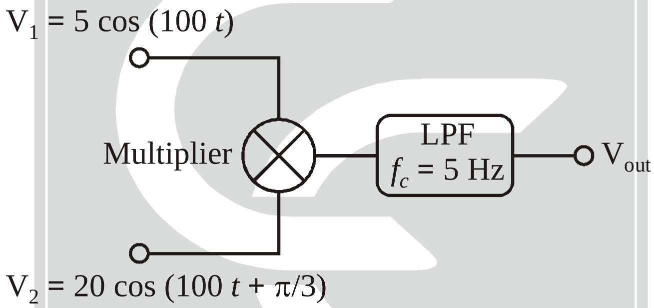
64. In the circuit below, the input voltage V_i is 2 V, $V_{cc} = 16$ V, $R_2 = 2$ kΩ and $R_L = 10$ kΩ.



The value of R_1 required to deliver 10 mW of power across R_L is

(a) 12 kΩ (b) 4 kΩ (c) 8 kΩ (d) 14 kΩ

65. Two sinusoidal signals are sent to an analog multiplier of scale factor 1 V^{-1} followed by a low pass filter (LPF).



If the roll-off frequency of the LPF is $f_c = 5$ Hz, the output voltage V_{out} is

(a) 5 V (b) 25 V (c) 100 V (d) 50 V

66. The resistance of a sample is measured as a function of temperature, and the data are shown below.

T(ºC)	2	4	6	8
$R(\Omega)$	90	105	110	115

The slope of R vs T graphs, using a linear least-squares fit to the data, will be

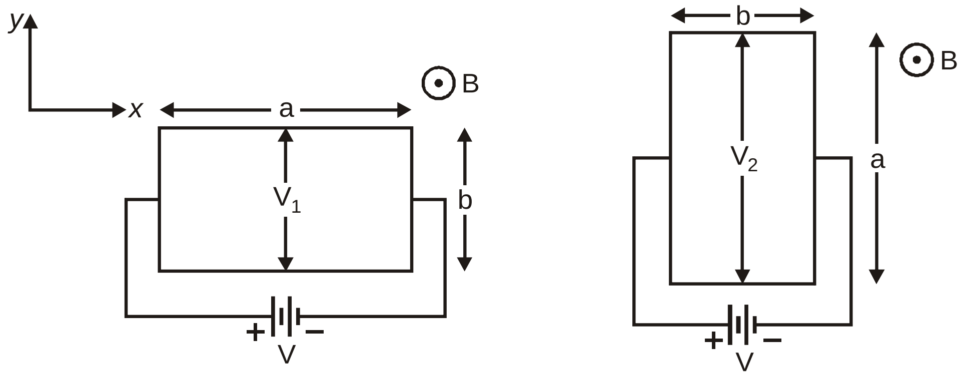
(a) 6Ω/ºC (b) 4Ω/ºC (c) 2Ω/ºC (d) 8Ω/ºC

67. Consider a one-dimensional chain of atoms with lattice constant a. The energy of an electron with wave-vector k is $\varepsilon(k) = \mu - \gamma\cos(ka)$, where μ and γ are constants. If an electric field E is applied in the positive x-direction, the time dependent velocity of an electron is
(in the following B is the constant)

(a) proportional to $\cos\left(B - \frac{eE}{\hbar}at\right)$ (b) proportional to E

(c) independent of E (d) proportional to sin $\left(B - \frac{eE}{\hbar}at\right)$

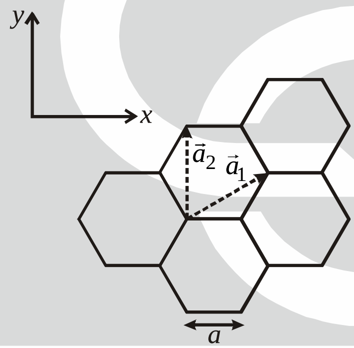
68. A thin rectangular conducting plate of length *a* and width *b* is placed in the *xy*-plane in two different orientations, as shown in the figures below. In both cases a magnetic field *B* is applied in the *z*-direction and a current flows in the *x*-direction due to the applied voltage *V*.



If the Hall voltage across the *y*-direction in the two cases satisfy $V_2 = 2V_1$, the ratio $a:b$ must be

(a) 1 : 2 (b) $1:\sqrt{2}$ (c) 2 : 1 (d) $\sqrt{2}:1$

69. Consider a hexagonal lattice with basis vectors as shown in the figure below.



If the lattice spacing is $a = 1$, the reciprocal lattice vectors are

(a) $\left(\frac{4\pi}{3}, 0\right), \left(-\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right)$ (b) $\left(\frac{4\pi}{3}, 0\right), \left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right)$

(c) $\left(0, \frac{4\pi}{\sqrt{3}}\right), \left(\pi, \frac{2\pi}{\sqrt{3}}\right)$ (d) $\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right), \left(-2\pi, \frac{2\pi}{\sqrt{3}}\right)$

70. In the L-S coupling scheme, the terms arising from two non-equivalent *p*-electrons are

(a) $^3S, ^1P, ^3P, ^1D, ^3D$ (b) $^1S, ^3S, ^1P, ^1D$

(c) $^1S, ^3S, ^3P, ^3D$ (d) $^1S, ^3S, ^1P, ^3P, ^1D, ^3D$

71. The total spin of a hydrogen atom is due to the contribution of the spins of the electron and the proton. In the high temperature limit, the ratio of the number of atoms in the spin-1 state to the number in the spin-0 state is

(a) 2 (b) 3 (c) 1/2 (d) 1/3

72. A two level system in a thermal (black body) environment can decay from the excited state by both spontaneous and thermally stimulated emission. At room temperature (300 K), the frequency below which thermal emission dominates over spontaneous emission is nearest to

(a) 10^{13} Hz (b) 10^{8} Hz (c) 10^{5} Hz (d) 10^{11} Hz

73. What should be the minimum energy of a photon for it to split an α-particle at rest into a tritium and a proton?

(The masses of ${}_{2}^{4}\text{He}$, ${}_{1}^{3}\text{H}$ and ${}_{1}^{1}\text{H}$ are 4.0026 amu, 3.0161 amu and 1.0073 amu, respectively, and 1 amu $\approx$ 938 MeV).

(a) 32.2 MeV (b) 3 MeV (c) 19.3 MeV (d) 931.5 MeV

74. Which of the following reaction(s) is/are allowed by the conservation laws ?

(i) $\pi^+ + n \to \Lambda^0 + K^+$

(ii) $\pi^- + p \to \Lambda^0 + K^0$

(a) Both (i) and (ii) (b) Only (i) (c) Only (ii) (d) Neither (i) nor (ii)

75. A particle, which is a composite state of three quarks u, d and s, has electric charge, spin and strangeness respectively, equal to

(a) $1, \frac{1}{2}, -1$ (b) $0, 0, -1$ (c) $0, \frac{1}{2}, -1$ (d) $-1, -\frac{1}{2}, +1$

PART - B

21. Radioactive decay process follows Poisson distribution with number of decaying atoms as random variable '*r*'

Here, $\langle r \rangle$ = average number of radioactive atoms decaying in two years = 2

Probability that first atom does not decay in first 2 years = $P_1 = \dfrac{e^{-\langle r \rangle} \cdot \langle r \rangle^0}{0!} = e^{-2}$

Probability that second atom does not decay in first 2 years = $P_2 = e^{-2}$

Therefore, probability that atleast one of them decays in first two years

=1–(None of the atoms decay in first two years) = $1-(P_1 \times P_2) = 1 - e^{-4}$

Correct option is (c)

22. Fourier Transform of $f(x) = \dfrac{1}{x^2+2}$ is

$$g(k) = \int_{-\infty}^{\infty} \frac{e^{ikx}}{x^2+2} dx$$

To solve the integral, we can use complex integration technique

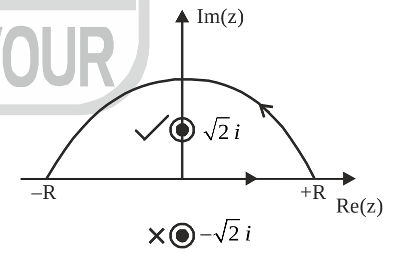
$$f(z) = \frac{e^{ikz}}{z^2+2}$$

For positive *k*, we will choose the contour to be semicircle (counter clockwise) in the upper half

Singular points : $z = +\sqrt{2}i, -\sqrt{2}i$

$$\int_{-\infty}^{\infty} \frac{e^{ikz}}{z^2+2} dz = 2\pi i \left[\text{Res.} f\left(z = \sqrt{2}i\right) \right]$$

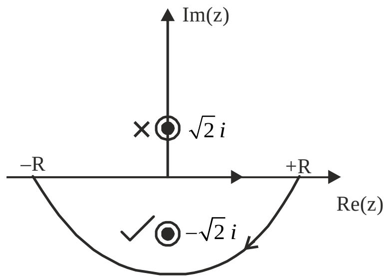
$$= 2\pi i \left. \frac{e^{ikz}}{2z} \right|_{z=\sqrt{2}i} = \frac{\pi}{\sqrt{2}} e^{-\sqrt{2}k}$$



For negative 'k', we will choose the contour to be semicircle (clockwise) in the lower half.

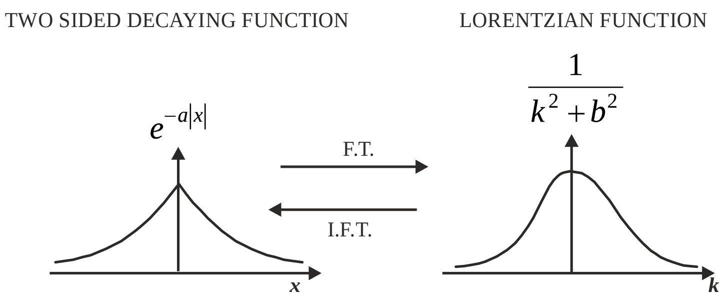
$$\int_{-\infty}^{\infty} \frac{e^{ikz}}{z^2+2} dz = -2\pi i \left[\text{Res.} f\left(z = -\sqrt{2}\, i\right) \right]$$

$$= \frac{\pi}{\sqrt{2}} e^{\sqrt{2}\, k}$$



So, $g(k) = \dfrac{\pi}{\sqrt{2}} e^{-\sqrt{2}\,|k|}$

Shortcut method:



Correct option is (d)

23. Equation of motion of the ball:

$$m\frac{dv}{dt} = mg - \gamma v \Rightarrow \frac{dv}{dt} = g - \frac{\gamma}{m}v$$

$$\Rightarrow \quad \frac{dv}{dt} + \frac{\gamma}{m}v = g \quad \left(1^{st} \text{ order linear D.E.}\right) \quad \text{... (1)}$$

Integrating factor for equation (1), $e^{\int \frac{\gamma}{m}dt} = e^{\frac{\gamma}{m}t}$

Solution of equation (1) will be

$$v \cdot e^{\frac{\gamma}{m}t} = \int g \cdot e^{\frac{\gamma}{m}t} dt = \frac{mg}{\gamma} e^{\frac{\gamma}{m}t} + C$$

$$\Rightarrow \quad \boxed{v = \frac{mg}{\gamma} + C\, e^{-\frac{\gamma}{m}t}} \quad \text{... (2)}$$

At $t = 0, v = 0 \Rightarrow C = -\frac{mg}{\gamma}$

So, $\boxed{v(t) = \frac{mg}{\gamma}\left(1 - e^{-\frac{\gamma}{m}t}\right)}$ ... (3)

$$\Rightarrow \quad x(t) = \frac{mg}{\gamma}\left(t + \frac{m}{\gamma}e^{-\frac{\gamma}{m}t}\right) + C_2$$

At $t = 0, x = 0 \Rightarrow C_2 = -\frac{m^2 g}{\gamma^2}$

So, $x(t) = \frac{mg}{\gamma}\left(t + \frac{m}{\gamma}e^{-\frac{\gamma}{m}\cdot t}\right) - \frac{m^2}{\gamma^2}g$

Putting, $m = 10\ kg,\ g = 10\ m/s^2,\ \gamma = 10\ kg/s$

We get, $\boxed{x(t) = 10\left(t - 1 + e^{-t}\right)}$

Correct option is (b)

24. Eigenvalue equation of matrix M: $|M-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 & 2 \\ 3 & -1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}=0$$

$$\Rightarrow (1-\lambda)\left[(\lambda+1)(\lambda-1)-9\right]=0$$

$$\Rightarrow (1-\lambda)\left[\lambda^2-10\right]=0$$

$$\Rightarrow -\lambda^3+10\lambda+\lambda^2-10=0$$

$$\Rightarrow \lambda^3-\lambda^2-10\lambda+10=0$$

According to Cayley Hamilton theorem,

$$M^3-M^2-10M+10I=0$$

Correct option is (c)

25. $L\left[f(t)\right]=\int_0^\infty e^{-st}\cdot f(t)\,dt$

$$=\int_0^T \frac{t}{T}e^{-st}dt+\int_T^\infty 1\cdot e^{-st}dt=\frac{1}{T}\left[t\frac{e^{-st}}{-s}\Big|_0^T-\int_0^T 1\cdot\frac{e^{-st}}{s}dt\right]+\frac{e^{-st}}{-s}\Big|_T^\infty$$

$$=\frac{1}{T}\left[-\frac{T}{s}e^{-sT}-\left\{\frac{e^{-sT}-1}{s^2}\right\}\right]+\frac{e^{-sT}}{s}=\frac{1-e^{-sT}}{s^2T}$$

Correct option is (b)

26. Time measured in lab frame is $t=\frac{\tau}{\sqrt{1-\upsilon^2/c^2}}$

Therefore, distance in lab frame $=\upsilon t=\frac{\upsilon\tau}{\sqrt{1-\upsilon^2/c^2}}$

Correct option is (d)

27. $V(x,y)=x+2y$

$$\therefore\ L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-\left(x+2y\right)$$

This Lagrangian is invariant under transformation $x\to x\pm 2\in,\ y\to y\mp\in$

Therefore $\pm 2p_x\mp p_y$ is conserved i.e., conserved quantity is $2p_x-p_y$ or $-2p_x+p_y$

Correct option is (a)

28. $$L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2-kx\dot{x}t=\frac{1}{2}m\dot{x}^2-\frac{d}{dt}\left(\frac{kx^2t}{2}\right)$$

$$L=L'-\frac{dF}{dt}(x,t)\qquad L'=\frac{1}{2}m\dot{x}^2,\ F=\frac{1}{2}kx^2$$

This is guage transformation of Lagrangian. Therefore L and L' will have same equation of motion. Since L' represents a free particle, therefore, L will also represent a free particle.

Correct option is (d)

29. Lagrangian for two dimensional harmonic oscillator is

$$L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-\frac{1}{2}m\omega^2\left(x^2+y^2\right)\qquad \text{... (1)}$$

Given: $x=\xi\eta,\quad y=\frac{1}{2}\left(\xi^2-\eta^2\right)\quad\Rightarrow\ \dot{x}=\dot{\xi}\eta+\xi\dot{\eta}$ and $\dot{y}=\xi\dot{\xi}-\eta\dot{\eta}$

Thus, $x^2+y^2=\frac{1}{4}\left(\xi^2+\eta^2\right)^2,\quad \left(\dot{x}^2+\dot{y}^2\right)=\left(\xi^2+\eta^2\right)\left(\dot{\xi}^2+\dot{\eta}^2\right)$

$$\Rightarrow\quad L=\frac{1}{2}m\left(\xi^2+\eta^2\right)\left[\left(\dot{\xi}^2+\dot{\eta}^2\right)-\frac{1}{4}m\omega^2\left(\xi^2+\eta^2\right)^1\right]$$

Correct option is (b)

30. At equilibrium, electric and magnetic force on electrons are equal therefore,

$qE=qvB\ \Rightarrow E=vB$

The induced emf is

$$V=\int\vec{E}\cdot d\vec{r}=B\int\vec{v}\cdot d\vec{r}=B\omega\int\vec{r}\cdot d\vec{r}\int=\frac{B\omega r^2}{2}$$

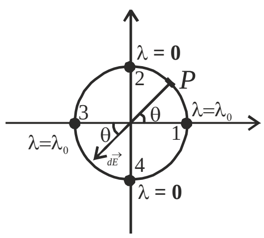
Clearly, the induced emf would depend on ω, B and r but not on ρ

Correct option is (b)

31. Electric field due to a charged element at P is

$-\left(dE\cos\theta\right)\hat{i}-\left(dE\sin\theta\right)\hat{j}$

So, the total electric field at the centre is



$$\vec{E}=-\hat{i}\int dE\cos\theta-\hat{j}\int dE\sin\theta$$

$$\vec{E}=-i\int\frac{\lambda d\ell}{4\pi\varepsilon_0a^2}\cos\theta-j\int\frac{\lambda d\ell}{4\pi\varepsilon_0a^2}\sin\theta$$

$$=-\frac{\hat{i}\lambda_0}{4\pi\varepsilon_0a^2}a\int_0^{2\pi}\cos^2\theta d\theta-\frac{\hat{j}\lambda_0}{4\pi\varepsilon_0a^2}a\int_0^{2\pi}\cos\theta\sin\theta d\theta\qquad\left[\because\ d\ell=ad\theta\right]$$

$$=-\frac{\hat{i}\lambda_0}{4\pi\varepsilon_0a}\int_0^{2\pi}\cos^2\theta d\theta=-\hat{i}\frac{\lambda_0}{4\pi\varepsilon_0a}\pi=-\frac{\lambda_0}{4\varepsilon_0a}i$$

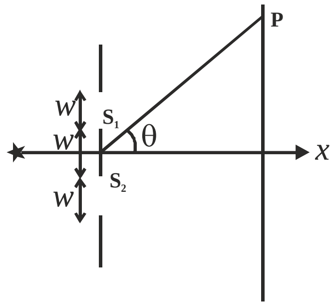
Correct option is (a)

32. For bright fringes, path difference $=\lambda, 2\lambda, \cdots$

Path difference between two rays S_1P and S_2P is

$\Delta x = d\sin\theta$

Here, d = distance between slit.



$\Rightarrow \quad 2w\sin\theta \qquad (d = 2w: \text{ given})$

For small angle $\sin\theta \sim \theta$

$\Delta x = 2w\theta$

At angle $\theta = \dfrac{n\lambda}{w}$, path difference would be

$$\Delta x = 2w\cdot\frac{n\lambda}{w} = 2n\lambda = \text{condition of maxima}$$

Correct option is (b).

33. Given : $\vec{E}(z,t) = E_0\cos(kz+\omega t)\hat{i} + 2E_0\sin(kz+\omega t)\hat{j}$

The amplitude along $\hat{i}$ and $\hat{j}$ are different i.e. $\vec{E}_0$ and $2\vec{E}_0$ respectively and phase difference is 90º So, the resultant electric field is elliptically polarised.

Correct option is (a)

34. Given : $\psi_0(x) = \psi_0^*(x)$ and $\psi_1(x) = \alpha\dfrac{d\psi_0}{dx}$, where α is a real constant.

Now, $$\langle p_x\rangle = \int_{-\infty}^{+\infty}\psi_1^*\left(\frac{-i\hbar d}{dx}\right)\psi_1 dx = \int_{-\infty}^{+\infty}\left(\alpha^*\right)\left(-i\hbar\frac{d\psi_0^*}{dx}\right)\left(\alpha\frac{d\psi_0^*}{dx}\right)dx$$

$$= |\alpha|^2(-i\hbar)\int_{-\infty}^{+\infty}\left(\frac{d\psi_0}{dx}\right)\frac{d}{dx}\left(\frac{d\psi_0}{dx}\right)dx$$

$$= -i\hbar|\alpha|^2\int_{-\infty}^{+\infty}\left(\frac{d\psi_0}{dx}\right)\left(\frac{d^2\psi_0}{dx^2}\right)dx \qquad \ldots(1)$$

Integrating by parts,

$$\langle p_x\rangle = -i\hbar|\alpha|^2\left(\left[\frac{d\psi_0}{dx}\cdot\frac{d\psi_0}{dx}\right]_{-\infty}^{\infty} - \int_{-\infty}^{+\infty}\frac{d^2\psi_0}{dx}\cdot\frac{d\psi_0}{dx}dx\right)$$

$$= -i\hbar|\alpha|^2\int_{-\infty}^{+\infty}\frac{d^2\psi_0}{dx}\cdot\frac{d\psi_0}{dx}dx = -\langle p_x\rangle \qquad [\because \text{ using }(1)]$$

$$\Rightarrow \quad \langle p_x \rangle = 0, \text{ where } \frac{d\psi_0}{dx} \to 0 \text{ as } x \to \pm\infty$$

Note : The expectation value of momentum for real wavefunction is zero.

Correct option is (b)

35. Given : $a = x + \frac{d}{dx}$

So, $[a, \cos x]\psi(x) = a\cos x\psi(x) - (\cos x)a\psi(x)$

$$= \left(x + \frac{d}{dx}\right)(\cos x\psi) - \cos x\left(x + \frac{d}{dx}\right)\psi$$

$$= x\cos x\psi + \frac{d\psi}{dx}\cos x - \sin x\,\psi - \cos x(x\psi) - \cos x\frac{d\psi}{dx}$$

$$= -\sin x\psi$$

$$\Rightarrow \quad [a, \cos x] = -\sin x$$

36. $x = \frac{1}{\sqrt{2}}\left(a + a^\dagger\right)$

$$|\psi\rangle = e^{\lambda a^\dagger - \lambda^* a}|0\rangle \quad \Rightarrow \langle\psi| = \langle 0| e^{\lambda^* a - \lambda a^\dagger}$$

$$\langle\psi|x|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left\langle 0\left|e^{\lambda^* a - \lambda a^\dagger} a\, e^{\lambda a^\dagger - \lambda^* a}\right|0\right\rangle + \left\langle 0\left|e^{\lambda^* a - \lambda a^\dagger} a^\dagger e^{\lambda a^\dagger - \lambda a^* a}\right|0\right\rangle\right) \quad \text{... (1)}$$

$$\langle\psi|x|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left\langle 0\left|e^{\lambda^* a}\; e^{-\lambda a^\dagger} a\, e^{\lambda a^\dagger} e^{-\lambda^* a}\right|0\right\rangle + \left\langle 0\left|e^{\lambda^* a}\; e^{-\lambda a^\dagger} a^\dagger e^{\lambda a^\dagger} e^{-\lambda^* a}\right|0\right\rangle\right) \quad \text{... (2)}$$

$$\boxed{\begin{aligned}
\left[a, e^{\lambda a^\dagger}\right] &= [a,1] + \lambda\left[a, a^\dagger\right] + \frac{\lambda^2}{2!}\left[a, a^{\dagger 2}\right] + \ldots\ldots.. \\
&= \lambda + \lambda a^\dagger + \frac{\lambda^3}{2!}a^{\dagger 2} + \ldots\ldots\ldots\ldots. \\
&= \lambda\left(1 + \lambda a^\dagger + \frac{\lambda^2 a^\dagger}{2!} + \ldots\ldots\right) = \lambda a^{\lambda a^\dagger} \\
\left[e^{\lambda^* a}, a^\dagger\right] &= \lambda^*\left[a, a^\dagger\right] + \frac{\lambda^{*2}}{2!}\left[a^2, a^\dagger\right] + \ldots\ldots\ldots.. \\
&= \lambda^*\left(1 + \lambda^* a + \frac{\lambda^{*2} a^2}{2!} + \ldots..\right) = \lambda^* e^{\lambda^* a}
\end{aligned}}$$

$$\langle\psi|x|\psi\rangle = \frac{1}{\sqrt{2}}\left(\left\langle 0\left|e^{\lambda^* a}\, e^{-\lambda a^\dagger}\left(\lambda e^{\lambda a\dagger} + e^{\lambda a^\dagger} a\right)e^{-\lambda^* a}\right|0\right\rangle + \left\langle 0\left|e^{\lambda^* a} a^\dagger e^{-\lambda^* a}\right|0\right\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\left\langle 0\left|e^{\lambda^* a}\lambda\, e^{-\lambda^* a}\right|0\right\rangle + \left\langle 0\left|e^{\lambda^* a} a\, e^{-\lambda^* a}\right|0\right\rangle + \left\langle 0\left|\left(\lambda^* e^{\lambda^* a} + a^\dagger e^{\lambda^* a}\right)e^{-\lambda^* a}\right|0\right\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\lambda\langle 0|0\rangle + \langle 0|a|0\rangle + \lambda^*\langle 0|0\rangle + \langle 0|a^\dagger|0\rangle\right) = \frac{1}{\sqrt{2}}\left(\lambda + 0 + \lambda^* + 0\right) = \frac{1}{\sqrt{2}}\left(\lambda + \lambda^*\right)$$

Correct option is (d)

37. **Given :** $\vec{\pi} = \vec{p} - q\vec{A}$

Now, $\left(\vec{\pi}\times\vec{\pi}\right)_z\psi = \left(\vec{p}-q\vec{A}\right)_x\left(\vec{p}-q\vec{A}\right)_y\psi - \left(\vec{p}-q\vec{A}\right)_y\left(\vec{p}-q\vec{A}\right)_x\psi$

$$= \left(-i\hbar\frac{d}{dx} - qA_x\right)\left(-i\hbar\frac{d}{dy} - qA_y\right)\psi - \left(-i\hbar\frac{d}{dy} - qA_y\right)\left(-i\hbar\frac{d}{dx} - qA_x\right)\psi$$

$$= \left(i\hbar q\frac{d}{dx}A_y\psi + i\hbar qA_x\frac{d\psi}{dy}\right) - \left(i\hbar q\frac{d}{dy}A_x\psi + i\hbar qA_y\frac{d\psi}{dx}\right) \qquad \left[\because\ \frac{d^2\psi}{dxdy} = \frac{d^2\psi}{dydx}\right]$$

$$= i\hbar q\left(\frac{dA_y}{dx} - \frac{dA_x}{dy}\right)\psi = i\hbar q\left(\vec{\nabla}\times\vec{A}\right)_z\psi = i\hbar qB_z\psi \Rightarrow \quad \left(\vec{\pi}\times\vec{\pi}\right)_z \equiv i\hbar qB_z$$

Correct option is (d)

38. The partition function for a particle in a 2-D box is

$$Z = \frac{1}{\hbar^2}\int_0^L\int_0^L dxdy\int_{-\infty}^{+\infty} e^{-p_x^2/2k_BT}dp_x\int_{-\infty}^{+\infty} e^{-p_y^2/k_BT}dp_y \;= \frac{L^2}{\hbar^2}\left(\sqrt{2mk_BT}\right)^2 = 2mk_BT\,\frac{L^2}{\hbar^2}$$

So, the partition function for N independent particle is

$$Z = \left[2\pi mk_BT\,\frac{L^2}{\hbar^2}\right]^N \qquad \therefore \qquad Z = \left(2\pi m\frac{EL^2}{N\hbar^2}\right)^N \text{ where, } E = Nk_BT$$

The free energy is, $F = -Nk_BT\ln z = -Nk_BT\ln\left(\frac{2\pi mEL^2}{N\hbar^2}\right)$

$$S = -\left(\frac{\partial F}{\partial T}\right) = Nk_B\ln\left(\frac{2\pi m}{N}\frac{EL^2}{\hbar^2}\right) = Nk_B\ln\left(\frac{L^2E}{N}\right) + Nk_B\ln\left(\frac{2\pi m}{\hbar^2}\right) \;= Nk_B\ln\left(\frac{L^2E}{N}\right) + \text{constant}$$

Correct option is (a)

39. **Correct option is (d)**

40. The possible values of energy are the eigen values of the Hamiltonian.

Thus $\left|H-\varepsilon I\right|=0$

$$\Rightarrow \quad \begin{vmatrix} \gamma-\varepsilon & -\delta \\ -\delta & -(\gamma+\varepsilon) \end{vmatrix}=0$$

$$\Rightarrow \quad -\left(\gamma^2-\varepsilon^2\right)-\delta^2=0 \Rightarrow \varepsilon=\pm\sqrt{\delta^2+\gamma^2}$$

So, $z=e^{\beta\sqrt{\delta^2+\gamma^2}}+e^{-\beta\sqrt{\delta^2+\gamma^2}}=2\cosh\left(\beta\sqrt{\delta^2+\gamma^2}\right)$

Correct option is (b)

41. Average thermal kinetic energy in thermal equilibrium at temperature T in 3 dimensional motion $=\frac{3}{2}kT$

Average thermal kinetic energy = Average potential energy at height 'h'

$$\frac{3}{2}kT=mgh=\rho\times\frac{4}{3}\pi r^3\times g\times h$$

$$h=\frac{3\times 3kT}{2\times 4\times\pi r^3\times g\times\rho}=\frac{9\times 1.38\times 10^{-23}\times 300}{8\times 3.14\times\left(0.1\times 10^{-6}\right)^3\times 9.8\times 1000}$$

$$=0.015\times 10^{-2}=1.5\times 10^{-4}\ m\simeq 10^{-4}\ m$$

Correct option is (c)

42. $$f(\underset{\substack{\uparrow\\ msb}}{A},\ B,\ \underset{\substack{\uparrow\\ \ell sb}}{C})=\sum m(1,2,4,6)$$

$1=\bar{A}\bar{B}C$

$2=\bar{A}B\bar{C}$

$4=A\bar{B}\bar{C}$

$6=AB\bar{C}$

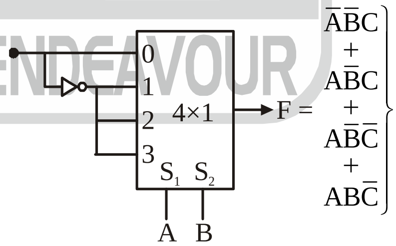


Fig. 4×1 MUX

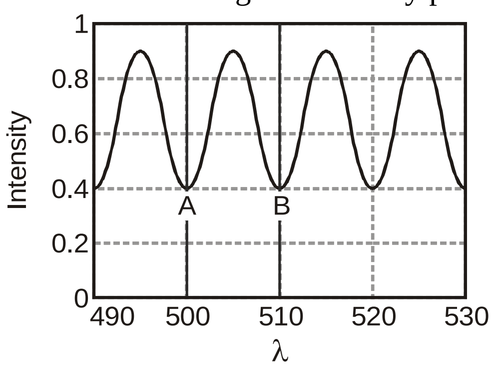
Correct option is (b)

43. Condition of minima in case of reflected light from thin film is given by

$$2\mu d=n\lambda \qquad \ldots (1)$$

here, $\mu = 1$, refractive index of air

Now, consider successive position of minima in given intensity pattern (Position A and B)



For the position A : $2d = n\lambda_1$

For the position B : $2d = (n+1)\lambda_2$

where $\lambda_1 > \lambda_2$ and λ_1 and λ_2 are corresponding to adjacent minima.

$$\Rightarrow n+1 = \frac{2d}{\lambda_1}+1, \text{ and } n+1 = \frac{2d}{\lambda_2}$$

$$\Rightarrow \frac{2d}{\lambda_1}+1 = \frac{2d}{\lambda_2} \Rightarrow 2d\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)=1 \Rightarrow d = \left(\frac{\lambda_1\lambda_2}{\lambda_1-\lambda_2}\right)\times\frac{1}{2} = \frac{1}{2}\times\frac{510\times 500}{10}$$

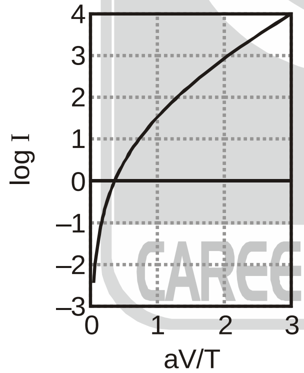
$$= 12.750 \text{ nm} \approx 12\,\mu\text{m}$$

Hence, correct option is (a).

44. **Given :** $I = I_s\left[\exp\left(\frac{aV}{T}\right)-1\right]$

For $\frac{aV}{T} >> 1,\ I = I_s \exp\left(\frac{aV}{T}\right) \Rightarrow \ln I = \frac{aV}{T} + \ln I_s$

So, the graph between ln I and $\frac{aV}{T}$ will be a straight line for large $\frac{aV}{T}$. So, the graph would be



Correct option is (d)

45. For $x = 1, GaN, E_g = 3.5\ eV$

For $x = 0, InN, E_g = 1.5 eV$

For $\lambda = 400\ nm, E_g = \frac{hc}{\lambda} = \frac{1200}{400} = 3\ eV$

If $E_g = 3.5\ eV$ then $x = 1$

Then for $E_g = 3.0\ eV, x$ required is 0.75

Correct option is (b)

PART - C

46. **Given :** $x_{n+1}=1+\dfrac{3}{1+x_n}\Rightarrow 2+\varepsilon_{n+1}=1+\dfrac{3}{1+(2+\varepsilon_n)}\Rightarrow 2+\varepsilon_{n+1}=1+\dfrac{3}{3}\left(1+\dfrac{\varepsilon_n}{3}\right)^{-1}$

$\Rightarrow \quad 2+\varepsilon_{n+1}=1+\left(1-\dfrac{\varepsilon_n}{3}\right)$ (Since, ε_n is small)

$\Rightarrow \quad \dfrac{\varepsilon_{n+1}}{\varepsilon_n}=-\dfrac{1}{3}$

Correct option is (c)

47. The following relations are very much well known for Pauli-Spin matrices

$\sigma_x\sigma_y=i\sigma_z,\sigma_y\sigma_x=-i\sigma_z,\sigma_y\sigma_z=i\sigma_x,\sigma_z\sigma_y=-i\sigma_x$

$\sigma_z\sigma_x=i\sigma_y,\sigma_x\sigma_z=-i\sigma_y,\sigma_x^2=\sigma_y^2=\sigma_z^2=I,(i\sigma_x)^2=(i\sigma_y)^2=(i\sigma_z)^2=-I$

Using above relations,

$(i\sigma_x)(i\sigma_y)=-i\sigma_z,(i\sigma_y)(i\sigma_x)=i\sigma_z,(i\sigma_y)(i\sigma_z)=-i\sigma_x,$

$(i\sigma_z)(i\sigma_y)=i\sigma_x,(i\sigma_z)(i\sigma_x)=-i\sigma_y,(i\sigma_x)(i\sigma_z)=i\sigma_y$

Since, $(-i\sigma_x)^2=(-i\sigma_y)^2=(-i\sigma_z)^2=-I$

$\Rightarrow(\sigma_x)(i\sigma_x)=(\sigma_y)(i\sigma_y)=(\sigma_z)(i\sigma_z)=iI$

$(\sigma_x)(-i\sigma_x)=(\sigma_y)(-i\sigma_y)=(\sigma_z)(-i\sigma_z)=-iI$

$G=\{\sigma_x,\sigma_y,\sigma_z,I,-\sigma_x,-\sigma_y,-\sigma_z,-I,i\sigma_x,i\sigma_y,i\sigma_z,iI,-i\sigma_x,-i\sigma_y,-i\sigma_z,-iI\}\Rightarrow$ 16 elements

You can also check all the properties those have to be satisfied by the elements of the group.

Correct option is (d)

48. Newton's forward difference method:

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
$x_0=45^0$	$y_0=0.7071$	$\Delta y_0=0.0589$	$\Delta^2 y_0=-0.0057$	-0.0007
$x_1=50^0$	$y_1=0.7660$	$\Delta y_1=0.0532$	$\Delta^2 y_1=-0.0064$	
$x_2=55^0$	$y_2=0.8192$	$\Delta y_2=0.0464$		
$x_3=60^0$	$y_3=0.8660$			

$$\sin 52^0=y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

Here, $u=\dfrac{52^0-x_0}{x_1-x_0}=\dfrac{7}{5}$

So, $\sin 52^0=(0.7071+0.08246-0.0016+0.00008)=0.7881\,(\text{approx})$

Correct option is (c)

49. Only option (c) satisfies the given partial differential equation,

$$f(x,t)=\frac{1}{\sqrt{1+4Dt}}\;e^{-\frac{x^2}{1+4Dt}}$$

$$\frac{\partial f}{\partial t}=-\frac{1}{2}\frac{4D}{(1+4Dt)^{3/2}}\;e^{-\frac{x^2}{1+4Dt}}+\frac{1}{(1+4Dt)^{1/2}}e^{-\frac{x^2}{1+4Dt}}\cdot\frac{4Dx^2}{(1+4Dt)^2}$$

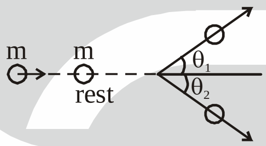
$$\frac{\partial f}{\partial x}=\frac{1}{(1+4Dt)^{3/2}}e^{-\frac{x^2}{1+4Dt}}(-2x)$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{1}{(1+4Dt)^{5/2}}e^{-\frac{x^2}{1+4Dt}}\left(4x^2\right)+\frac{1(-2)}{(1+4Dt)^{3/2}}e^{-\frac{x^2}{(1+4Dt)}}$$

Correct option is (c)

50. If θ_c be angle of scattering in center of mass frame then

$$\tan\theta_1=\frac{\sin\theta_c}{\cos\theta_c+\frac{m_1}{m_2}}=\frac{\sin\theta_c}{\cos\theta_c+\frac{m}{m}}$$



$$=\frac{2\cdot\sin\frac{\theta_c}{2}\cdot\cos\frac{\theta_c}{2}}{2\cos^2\frac{\theta_c}{2}}=\tan\frac{\theta_c}{2}$$

$$\therefore\quad \theta_1=\frac{\theta_c}{2}\quad\text{and}\quad \tan\theta_2=\frac{\sin\theta_c}{1-\cos\theta_c}=\frac{2\cdot\sin\frac{\theta_c}{2}\cdot\cos\frac{\theta_c}{2}}{2\sin^2\frac{\theta_c}{2}}=\cot\frac{\theta_c}{2}\Rightarrow\theta_2=\frac{\pi}{2}-\frac{\theta_c}{2}$$

$$\therefore\quad \theta_1+\theta_2=\pi/2$$

Correct answer is (a)

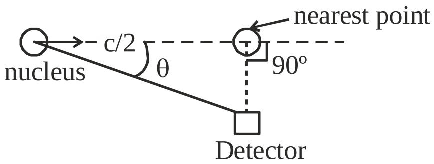
51. According to Keppler's law for potential energy $V(r)=-\frac{k}{r}$,

Time period $T\propto r^{3/2}$, where r is major axis

$$\left(\frac{T_1}{T_2}\right)=\left(\frac{r_1}{r_2}\right)^{3/2}=\left(\frac{R}{2R}\right)^{3/2}=2^{-3/2}$$

Therefore, for $n=+1$ $\left(\frac{T_1}{T_2}\right)=2^{-3/2}$ *or* $\left(\frac{T_2}{T_1}\right)=2^{3/2}$

Correct option is (c).

52. 

We know that observed frequency in case of Doppler effect is given as

$$\nu = \frac{\nu_0\sqrt{1-\mathrm{v}^2/c^2}}{1-\frac{\mathrm{v}}{c}\cos\theta}$$

At nearest point, $\theta = 90^\circ$

$$\therefore \quad \nu = \nu_0\sqrt{1-\mathrm{v}^2/c^2} = \nu_0\sqrt{1-\frac{1}{4}} = \frac{\nu_0\sqrt{3}}{2}$$

Correct answer is (a)

53. Given : $\varepsilon = 10$, $\rho = 10^{11}\,\Omega$-m

Let the volume charge density be d. Then

Ohm's law $\Rightarrow \vec{E} = \rho\vec{J}$ ... (1)

From continuity, equation $\vec{\nabla}\cdot\vec{J} + \frac{\partial d}{\partial t} = 0$ ... (2)

Also, $\vec{\nabla}\cdot\vec{E} = \frac{d}{\varepsilon} = \frac{d}{10\varepsilon_0}$ ... (3)

Putting equation (1) in equation (3), $\rho\vec{\nabla}\cdot\vec{J} = \frac{d}{10\varepsilon_0}$ ... (4)

Using equation (4) in equation (2),

$$\frac{d}{10\rho\varepsilon_0} + \frac{\partial d}{\partial t} = 0 \Rightarrow \frac{\partial d}{d} = -\frac{\partial t}{10\rho\varepsilon_0} \Rightarrow \ln d = -\frac{t}{10\rho\varepsilon_0} + c$$

$\Rightarrow \quad \boxed{d = d_0 e^{-t/10\rho\varepsilon_0}}$ where, d_0 = initial charge density.

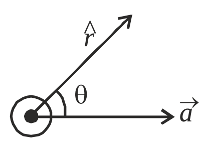
Now, $d \to \frac{d_0}{e} \;\therefore\; \frac{1}{e} = e^{-t/10\rho\varepsilon_0} \Rightarrow t = 10\rho\varepsilon_0 = 10\times10^{11}\times8.85\times10^{-12}$

$\Rightarrow \quad \boxed{t = 8.85\,\text{sec}}$

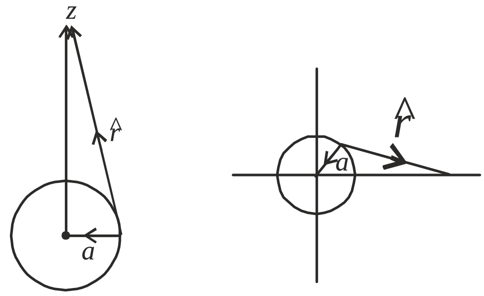
Correct option is (d)

54. Poynting vector due to an accelerated charge particle is given as (in direction $\hat{r}$)

$$\vec{S} = \frac{k\sin^2\theta}{r^2}\hat{r}$$ where θ is angle between acceleration and $\hat{r}$.



When a particle moves in circle with constant speed or under central force, its acceleration is towards center of circle.



θ is variable (between 0 to π)

$$\theta = 90 \qquad \sin\theta \simeq 1 \qquad \langle \sin^2\theta \rangle = \frac{1}{2}$$

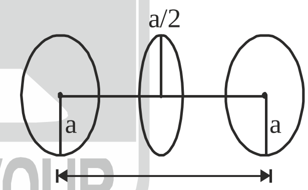
$$I = \langle |\vec{S}| \rangle \qquad \therefore \ \frac{I_2}{I_1} = \frac{\langle \sin^2\theta_2 \rangle}{\langle \sin^2\theta_1 \rangle} \cdot \left(\frac{r_1}{r_2}\right)^2 = \frac{\left(\frac{1}{2}\right)}{1} \cdot \frac{R^2}{4R^2} = \frac{1}{8}$$

Correct option is (c)

55. In between capacitor plates, displacement current will be equal to conduction current.

Displacement current density $J_d = \dfrac{I}{\pi a^2}$

From modified ampere's law,



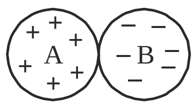
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \int_s \vec{J}_d \cdot d\vec{s} \ \Rightarrow B \cdot 2\pi\left(\frac{a}{2}\right) = \mu_0 \frac{I}{\pi a^2} \pi \left(\frac{a}{2}\right)^2$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi a^2} \times \frac{a}{2} = \frac{\mu_0 I}{4\pi a}$$

Correct option is (d)

56. Potential at centre of uniformly charged sphere

$V_{centre} = \dfrac{3}{2}\left(\dfrac{q}{4\pi\varepsilon_0 a}\right)$ and at outside point is $\dfrac{q}{4\pi\varepsilon_0 r}$



$$V_A = \frac{3q}{8\pi\varepsilon_0 a} - \frac{q}{4\pi\varepsilon_0 (2a)}$$

$$V_B = \frac{-3q}{8\pi\varepsilon_0 a} + \frac{q}{4\pi\varepsilon_0 (2a)}$$

$$\therefore \qquad V_A - V_B = \frac{q}{2\pi\varepsilon_0 a}$$

Correct optino is (c)

57. The $f(\theta,\phi)$ for high energy when, $q \to \infty$ is

$$f(\theta,\phi) = -\frac{2m}{\hbar^2 q}\int_0^\infty rV(r)\sin qr\,dr$$

$$= -\frac{2m}{\hbar^2 q}\int_0^\infty r^2 V_0 e^{-\mu r}\sin qr\,dr = \frac{-2m}{\hbar^2 q}V_0\int_0^\infty r^2 e^{-\mu r}\left(\frac{e^{iqr}-e^{-iqr}}{2i}\right)dr$$

$$= -\frac{2m}{2i\hbar^2 q}V_0\int_0^\infty \left(r^2 e^{-(\mu - iq)r} - r^2 e^{-(\mu+iq)r}\right)dr \qquad \ldots (1)$$

$$\therefore \quad f(\theta,\phi) = \frac{imV_0}{\hbar^2 q}\left[\frac{2}{(\mu - iq)^3} - \frac{2}{(\mu+iq)^3}\right] = \frac{2imV_0}{\hbar^2 q}\left[\frac{(\mu+iq)^3 - (\mu - iq)^3}{(\mu^2+q^2)^3}\right]$$

$$\left[\because \int_0^\infty x^2 e^{-ax}dx = \frac{2}{a^3}\right]$$

$$= \frac{2imV_0}{\hbar^2 q}\frac{2iq\left(3\mu^2 - q^2\right)}{\left(\mu^2+q^2\right)^3} \quad \Rightarrow \quad f(\theta,\phi) \propto \frac{q^2}{q^6} = q^{-4}$$

So, cross-section, $\sigma(\theta) \propto \left|f(\theta,\phi)\right|^2$

$$\Rightarrow \quad \boxed{\sigma(\theta) \propto q^{-8}}$$

Correct option is (a)

58. Given : $V(x) = A\delta(x-a)$

The Schrodinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + A\delta(x-a)\psi = E\psi$$

Integrating from $a-\varepsilon$ to $a+\varepsilon$,

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\int_{a-\varepsilon}^{a+\varepsilon}\frac{d^2\psi}{dx^2}dx + A\int_{a-\varepsilon}^{a+\varepsilon}\delta(x-a)\psi\,dx = E\int_{a-\varepsilon}^{a+\varepsilon}\psi\,dx$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\left[\frac{d\psi}{dx}\right]_{a-\varepsilon}^{a+\varepsilon} + A\psi(a) = 0$$

$$\Rightarrow \quad \left.\frac{d\psi}{dx}\right|_{a+\varepsilon} - \left.\frac{d\psi}{dx}\right|_{a-\varepsilon} = \frac{2mA}{\hbar^2}\psi(a)$$

Correct option is (a)

59. Dirac matrices

$$\sigma_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}_{4\times 4}, \ \sigma_x = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \ \beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}_{4\times 4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\alpha_i\alpha_j = -\alpha_j\alpha_i, \ \alpha^2 = \beta^2 = 1$$

$$\alpha_i\beta = -\beta\alpha_i, \ [\vec{\gamma}, \vec{\alpha}] = [\vec{\gamma}, \beta] = 0$$

$$H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{\partial\vec{r}}{\partial t} + \frac{1}{i\hbar}\left[\vec{r}, \vec{H}\right] = \frac{1}{i\hbar}[\vec{r}, H] \quad \text{as } \frac{\partial\vec{r}}{\partial t} = 0$$

$$= \frac{1}{i\hbar}\left[\vec{r}, c\vec{\alpha}\cdot\vec{p} + \beta m_0c^2\right] = \frac{c}{i\hbar}[\vec{r}, \vec{\alpha}\cdot\vec{p}] + \frac{m\alpha^2}{i\hbar}\left[\vec{r}, \vec{\beta}\right]$$

$$= \frac{c}{i\hbar}[\vec{r}, \vec{\alpha}\cdot\vec{p}] = \frac{c}{i\hbar}\left([x, \alpha_x p_x]\hat{i} + \left[y, \alpha_y p_y\right]\hat{j} + [z, \alpha_z p_z]\hat{k}\right)$$

$$= \frac{c}{i\hbar}\times i\hbar\left(\alpha_x\hat{i} + \alpha_y\hat{j} + \alpha_z\hat{k}\right) = c\vec{\alpha} \qquad \text{... (1)}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial t} + \frac{1}{i\hbar}[\hat{v}, H] = \frac{1}{i\hbar}[\vec{v}, H] \text{ as } \frac{\partial\vec{v}}{\partial t} = 0$$

$$\vec{a} = \frac{1}{i\hbar}[\vec{v}, c\vec{\alpha}\cdot\vec{p}] + \frac{1}{i\hbar}\left[\vec{v}, \beta mc^2\right]$$

$$= \frac{c^2}{i\hbar}[\vec{\alpha}, \vec{\alpha}\cdot\vec{p}] + \frac{c\times mc^2}{i\hbar}[\vec{\alpha}, \beta] \qquad \text{... (2)}$$

$$[\vec{\alpha}, \beta] = [\alpha_x, \beta]\hat{i} + \left[\alpha_y, \beta\right]\hat{j} + [\alpha_z, \beta]\hat{k}$$

$$= (\alpha_x\beta - \beta\alpha_x)\hat{i} + \left(\alpha_y\beta - \beta\alpha_y\right)\hat{j} + (\alpha_z\beta - \beta\alpha_z)\hat{k}$$

$$= 2\alpha_x\beta\hat{i} + 2\alpha_y\beta\hat{j} + 2\alpha_z\beta\hat{k} = 2\vec{\alpha}\beta \qquad \text{... (3)}$$

$$\left[\vec{\alpha}, \vec{\alpha}\cdot\vec{\beta}\right] = \left[\alpha_x\hat{i} + \alpha_y\hat{j} + \alpha_z\hat{k}, \alpha_x p_x + \alpha_y p_y + \alpha_z p_z\right] \qquad \text{... (4)}$$

$$\left[\alpha_x, \alpha_x p_x + \alpha_y p_y + \alpha_z p_z\right] = [\alpha_x, \alpha_x p_x] + \left[\alpha_x, \alpha_y x\ p_y\right] + [\alpha_x, \alpha_z p_z]$$

$$= \alpha_y\left[\alpha_x, p_y\right] + \left[\alpha_x, \alpha_y\right]p_y + \alpha_z[\alpha_x, p_z] + [\alpha_x, p_z]$$

$$= 2\alpha_x\alpha_y p_y + 2\alpha_x\alpha_z p_z = 2\alpha_x\left(\alpha_y p_y + \alpha_z p_z\right)$$

$$= 2\alpha_x(\vec{\alpha}\cdot\vec{p} - \alpha_x p_x) = 2\alpha_x\vec{\alpha}\cdot\vec{p} - 2p_x$$

Similarly, $\left[\alpha_y, \alpha_x p_x + \alpha_y p_y + \alpha_z p_z\right] = 2\alpha_y\vec{\alpha}\cdot\vec{p} - 2p_y$

$$\left[\alpha_z, \alpha_x p_x + \alpha_y p_y + \alpha_z p_z\right] = 2\alpha_z\vec{\alpha}\cdot\vec{p} - 2p_z$$

$$\left[\vec{\alpha},\vec{\alpha}\cdot\vec{p}\right]=2\alpha_x\hat{i}\vec{\alpha}\cdot\vec{p}-2p_x\hat{i}+2\alpha_y\hat{j}\vec{\alpha}\cdot\vec{p}-2p_y\hat{j}+2\alpha_z\hat{k}\vec{\alpha}\cdot\vec{p}-2p_z\hat{k}$$

$$=2\vec{\alpha}\vec{\alpha}\cdot\vec{p}-2\vec{p} \qquad \ldots (5)$$

From equation (2),

$$\vec{a}=\frac{c^2}{i\hbar}\left(2\vec{\alpha}\vec{\alpha}\cdot\vec{p}-2\vec{p}\right)+\frac{c\times mc^2}{i\hbar}2\vec{\alpha}\beta$$

$$=\frac{2c}{i\hbar}\left(c\vec{\alpha}\vec{\alpha}\cdot\vec{p}-c\vec{p}+\vec{\alpha}\beta mc^2\right)=\frac{2ic}{\hbar}\left(c\vec{p}-\vec{\alpha}H\right)$$

Correct option is (a)

60. Given : $E(t)=Ae^{-\left(\frac{t}{\tau}\right)}=H'_{fi}$

$$\omega_{fi}=\frac{E_1-E_0}{\hbar}=\frac{\frac{3}{2}\hbar\omega-\frac{\hbar\omega}{2}}{\hbar}=\omega$$

Now, $C_f(t)=\frac{1}{i\hbar}\int_0^t H'_{fi}\,e^{i\omega_{fi}t}dt=\frac{1}{i\hbar}\int_0^\infty Ae^{\frac{-t^2}{\tau}}e^{i\omega t}dt=\frac{A}{i\hbar}\sqrt{\pi\tau^2}\exp\left\{\frac{-\omega^2\tau^2}{4}\right\}$

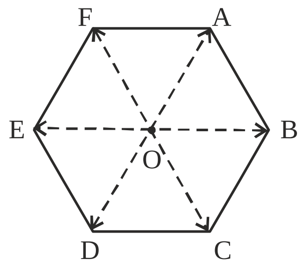
$$\left[\because \int_0^\infty e^{-\alpha x^2+\beta x}dx=\sqrt{\frac{\pi}{\alpha}}e^{\beta^2/4\alpha}\right]$$

The probability of being in the first excited state is $P_f(t)=\left|C_f(t)\right|^2\frac{A^2}{\hbar^2}\pi\tau^2\exp\left(\frac{-\omega^2\tau^2}{2}\right)$

So, $P_f(t)\propto e^{\frac{-\omega^2\tau^2}{2}}$

Correct option is (a)

61. Assume the starting point for the walker to be point ‘O’.
First step : Walker can move in either of the six directions and the same is true for **second** and **third** step.
Total number of different paths the walker can choose, in total 3 steps = (6×6×6) 216 paths.
Favourable paths, in which the walker will return to starting point ‘O’ are following:



$$\left.\begin{array}{l}\text{OABO, OBAO, OBCO, OCBO, OCDO, ODCO,}\\ \text{ODEO, OEDO, OEFO, OFEO, OFAO, OAFO}\end{array}\right]\rightarrow 12 \text{ paths}$$

Required probabiltiy = $\frac{12}{216}=\frac{1}{18}$

Correct option is (c)

62. Energy at ground state is 0 and at excited state is ε.
The partition function is,

$$z=1\,e^{0.\beta}+2\,e^{-\beta\varepsilon}=1+2e^{-\beta\varepsilon}$$

The energy is ,

$$u = -\frac{\partial}{\partial \beta}(\ell nz) = -\frac{\partial}{\partial \beta}\left[\ln\left(1+2e^{-\beta\varepsilon}\right)\right] = \frac{2\varepsilon e^{-\beta\varepsilon}}{1+2e^{-\beta\varepsilon}}$$

So, $$C_V = \left(\frac{\partial u}{\partial T}\right)_V = -\frac{1}{k_B T^2}\left(\frac{\partial u}{\partial \beta}\right)_V = -k_B\beta^2\left(\frac{\partial u}{\partial \beta}\right)_V \qquad \left[\because\ \beta = \frac{1}{kT}\right]$$

$$= -k_B\beta^2 \frac{\left(1+2e^{-\beta\varepsilon}\right)\left(-2\varepsilon^2 e^{-\beta\varepsilon}\right) - \left(2\varepsilon e^{-\beta\varepsilon}\right)\left(-2\varepsilon e^{-\beta\varepsilon}\right)}{\left(1+2e^{-\beta\varepsilon}\right)^2} = -k_B\beta^2 \frac{-2\varepsilon^2 e^{-\beta\varepsilon}}{\left(1+2e^{-\beta\varepsilon}\right)^2}$$

$$= 2k_B\beta^2 \frac{\varepsilon^2 e^{-\beta\varepsilon}}{\left(1+2e^{-\beta\varepsilon}\right)^2} = 2k_B(\beta\varepsilon)^2 e^{-\beta\varepsilon}$$

(when $\beta\varepsilon >> 1$]

Correct option is (c)

63. Fermi wave vector of 2-D electron gas is

$$k_F = \left(\frac{2\pi N}{A}\right)^{1/2} \qquad \text{where, } \frac{N}{A} = \rho(\text{given})$$

Given : fermi energy is $E_F = |\vec{p}|v = \hbar k_F v \qquad [\because \vec{p} = \hbar k_F]$

$$\boxed{E_F = hv(2\pi)^{1/2}\rho^{1/2}}$$

Zero point energy per electron is

$$\varepsilon_0 = \frac{1}{2}E_F = \frac{\hbar v(2\pi)^{1/2}}{2}\rho^{1/2}$$

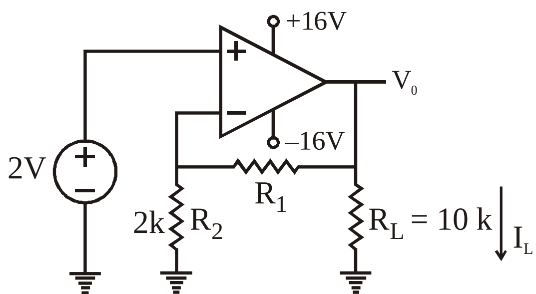
Total zero point energy per unit area

$$\frac{E_0}{A} = \frac{N}{2A}E_F = \frac{hv(2\pi)^{1/2}}{2}\rho^{3/2}$$

Hence, $\boxed{\frac{E_0}{A} \propto \rho^{3/2}}$

Correct option is (a)

64.



$$P_{R_L} = 10mW = I_L{}^2 \cdot R_L$$

$$10\,mW = \frac{V_0{}^2}{R_L}$$

$$\boxed{V_0 = 10\,V}$$

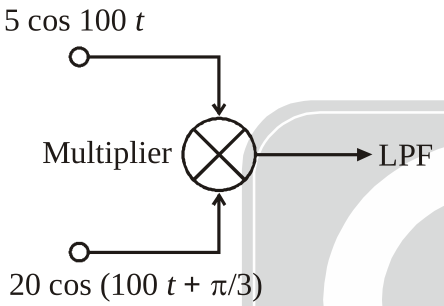
Virtual-ground concept at (–) input

$$\frac{2}{2} + \frac{2-V_0}{R_1} = 0 \qquad 1 = \frac{V_0 - 2}{R_1} \qquad (\because\ V_0 = 10)$$

$$\boxed{R_1 = 10 - 2 = 8k\Omega}$$

Correct option is (c)

65.



Multiplied-signal $= 20\cos\left(100t + \frac{\pi}{3}\right) \cdot 5\cos 100t$

$$\Rightarrow \quad 50\left[2\cos\left(100t + \frac{\pi}{3}\right) \cdot \cos(100t)\right] \quad \Rightarrow \quad 50\left[\cos\left(200t + \frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right)\right]$$

$$\Rightarrow \quad \left[50\cos\left(200t + \frac{\pi}{3}\right) + 50 \cdot \cos\frac{\pi}{3}\right] \quad \Rightarrow \quad \underbrace{50\cos\frac{\pi}{3}}_{25\,volt} \text{ is output voltage}$$

$$V_0 = 25\,V$$

Correct option (b)

66. The line to be fitted will be

$R = aT + b$ ... (1)

The normal equations are

$\Sigma R = a\Sigma T + nb$ ... (2)

and $\Sigma TR = a\Sigma T^2 + b\Sigma T$ ... (3)

Where, n = number of data points = 4 [Given]

Now,

$T(^\circ C)$	$R(\Omega)$	T^2	TR
2	90	4	180
4	105	16	420
6	110	36	660
8	115	164	920
$\Sigma T = 20$	$\Sigma R = 420$	$\Sigma T^2 = 120$	$\Sigma TR = 2180$

Using all these in equation (2) and equation (3),

420 = 20a + 4b ... (4)

And 2180 = 120a + 20b ... (5)

Solving equation (4) and (5),

a = 4

Thus, slope of equation (1) is $4\Omega/^{o}C$

Correct option is (b)

67. From Newton's law, force due to electric field is

$$\vec{F} = -e\vec{E} = \frac{d\vec{p}}{dt} = \frac{\hbar d\vec{k}}{dt}$$

$$\Rightarrow \quad \frac{d\vec{k}}{dt} + \frac{e}{\hbar}\vec{E} = 0$$

$$\Rightarrow \quad \boxed{k(t) = k(0) - \frac{eE}{\hbar}t} \quad \text{... (1)}$$

The group velocity is

$$V_g = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{d\varepsilon}{dk} \quad \text{... (2)} \quad [\text{since, energy } \varepsilon = \hbar\omega]$$

Now, given energy $\varepsilon = \mu - \gamma\cos(ka)$ ... (3)

Using equation (3) in equation (2),

$$V_g = \frac{\gamma q}{\hbar}\sin(ka) \quad \text{... (4)}$$

The time dependent velocity is

$$V_g = \frac{\gamma q}{\hbar}\sin\left[\left(k(0) - \frac{eE}{\hbar}t\right)a\right] = \frac{\gamma q}{\hbar}\sin\left[B - \frac{eEat}{\hbar}\right] \quad (\text{Letting } B = k(0)a = \text{constant})$$

$$\Rightarrow \quad V_g \propto \sin\left(B - \frac{eEat}{\hbar}\right)$$

Correct option is (d)

68. Hall coefficient $\frac{1}{ne} = \frac{E_H}{J_x B_z}$

So, $\frac{E_1}{J_1 B} = \frac{E_2}{J_2 B} \quad \Rightarrow \frac{E_1}{J_1} = \frac{E_2}{J_2}$

$$\Rightarrow \frac{\frac{V_1}{b}}{\sigma\frac{V_2}{a}} = \frac{\frac{V_2}{a}}{\sigma\frac{V_1}{b}} \quad \Rightarrow \frac{a^2}{b^2} = \frac{V_2}{V_1} \quad \Rightarrow \frac{a}{b} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{2}{1}} \qquad \left[E = \frac{V}{d} \text{ and } J = \sigma E\right]$$

Correct option is (d)

69. From figure, we have

$$\vec{a}_1 = \frac{\sqrt{3}}{2}a\left(\sqrt{3}\hat{x}+\hat{y}\right);\quad \vec{a}_2 = \sqrt{3}\,a\hat{y};\quad \vec{a}_3 = c\hat{z}\left(\text{assuming}\right)$$

Now, $V = \vec{a}_1 \cdot \vec{a}_2 \times \vec{a}_3 = \frac{3\sqrt{3}}{2}a^2 c$

Now, $\vec{b}_1 = 2\pi\frac{\vec{a}_2 \times \vec{a}_3}{V} = \frac{4\pi}{3}\hat{x} + 0\hat{y}$ and $\vec{b}_2 = 2\pi\frac{\vec{a}_3 \times \vec{a}_1}{V} = -\frac{2\pi}{3}\hat{x} + \frac{2\pi}{\sqrt{3}}\hat{y}$

So, $\vec{b}_1 = \left(\frac{4\pi}{3}, 0\right)$ and $\vec{b}_2 = \left(-\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right)$

Correct option is (a)

70. We have two non-equivalent p-electrons

So, $\left.\ell_1 = 1, \ell_2 = 2\right\} \Rightarrow L = 0,1,2$

$$\left.s_1 = \frac{1}{2}, s_2 = \frac{1}{2}\right\} \Rightarrow S = 0,1$$

Thus, $L = 0, S = 0 \Rightarrow {}^1S$

$L = 0, S = 1 \Rightarrow {}^3S$

$L = 1, S = 0 \Rightarrow {}^1P$

$L = 1, S = 1 \Rightarrow {}^3P$

$L = 2, S = 0 \Rightarrow {}^1D$

$L = 2, S = 1 \Rightarrow {}^3D$

Correct option is (d)

71. ${}^2S_{1/2}$ —
- F = 1 (when spins are parallel) Spin-1 state
- F = 0 (when spins are anti-parallel) Spin-0 state

where, $F = |J-S|$ to $|J+S|$

The degeneracy of quantum level is 2F + 1

Thus, $\frac{2F(=1)+1}{2F(=0)+1} = \frac{3}{2} = 3$

Correct option is (b)

72. $$\frac{R_{stimulated}}{R_{spon\tan eous}} = \frac{B_{21}u(v)}{A} = \frac{1}{\left(e^{\frac{hv}{k_BT}} - 1\right)} > 1 \ (\text{given})$$

$$\Rightarrow \quad 1 > \left(e^{\frac{hv}{k_BT}} - 1\right) \Rightarrow e^{\frac{hv}{k_BT}} < 2$$

$$\Rightarrow \quad \frac{hv}{k_BT} < \ln(2) \Rightarrow v < \frac{k_BT}{h}\ln(2) \Rightarrow v < \frac{1.38\times10^{-23}\times300\times10^{13}}{6\times10^{-34}}\times0.693 \Rightarrow \boxed{v < 0.48\times10^{13}}$$

Correct option is (a)

73. The energy of photon will be minimum only if the tritium and proton are created at rest.

Thus, $E_\gamma = \left[m\left({}_1H^1\right) + m\left({}_1H^3\right) - m\left({}_2He^4\right)\right]\times 931MeV$

$$= [1.0073 + 3.0161 - 4.0026]\times 931\text{MeV}$$

$$= 0.0208\times931\,\text{MeV} = 19.36\,\text{MeV}$$

Hence, correct option is (c).

74. **(i)** $\pi^+ + n \longrightarrow \Lambda^0 + k^+$

	π^+	n	Λ^0	k^+	
Q	1	0	0	1	$\Rightarrow \Delta Q = 0$
L	0	0	0	0	$\Rightarrow \Delta L = 0$
B	0	1	1	0	$\Rightarrow \Delta B = 0$
S	0	0	-1	1	$\Rightarrow \Delta S = 0$

(ii) $\pi^- + p \longrightarrow \Lambda^0 + k^+$

	π^-	p	Λ^0	k^+	
Q	-1	1	0	0	$\Rightarrow \Delta Q = 0$
L	0	0	0	0	$\Rightarrow \Delta L = 0$
B	0	1	1	0	$\Rightarrow \Delta B = 0$
S	0	0	-1	1	$\Rightarrow \Delta S = 0$

Since, physical quantities like charge, lepton number, Baryon number, strangeness are all conserved. So, both reactions are allowed.

Correct option is (a).

75. Charge on u is 2/3, on d is –1/3 and on s is –1/3. Thus, total $Q = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$. Since, the particle is formed by three quarks so we know this is quark content of baryons. And baryons are fermions. So, possible spin = $\frac{1}{2}$.

For strangeness of u = 0, of d = 0 and for s = –1. So, total strangeness S = 0 + 0 + 1 = –1

Hence, correct option is (c).