CAREER ENDEAVOUR
Best Institute for IIT-JAM, NET & GATE

CSIR-UGC-NET/JRF | GATE - PHYSICS

UNIT TEST - 2

[QUANTUM MECHANICS]

Time : 00: 50 Hour **Date : 06-10-2017**

M.M. : 30

INSTRUCTION:

(I) Question Paper contains 15 objective type questions, each question carry 2 marks.

(II) There is negative marking, 0.5 mark will be deducted for each wrong answer.

(III) Attempt all the questions, use of calculator is not allowed.

1. The average force on a particle in the first excited energy eigenstate, moving in time independent potential $V(x)$ that satisfies the condition $V(-x)=V(x)$,
(a) is always zero
(b) is always non-zero
(c) is either zero or non-zero
(d) depends on the variation of potential with respect to position

2. The number of energy states for a cubical box of side a in energy range, $0<E<15\pi^2\hbar^2/2ma^2$ is
(a) 6 (b) 11 (c) 15 (d) 17

3. A quantum particle of mass m moves in a two dimensional anisotropic harmonic oscillator potential $V(x,y)=\frac{1}{2}m\omega^2x^2+8m\omega^2y^2$. The energy eigenvalues are (n = 0, 1, 2, 3, 4, ...) given by
(a) $(n+1)\hbar\omega$ (b) $\left(2n+\frac{3}{2}\right)\hbar\omega$ (c) $\left(n+\frac{3}{2}\right)\hbar\omega$ (d) $\left(n+\frac{5}{2}\right)\hbar\omega$

4. Consider an electron in Hydrogen atom in a state whose unnormalized wave function is given by

$R(r)=Ar\left(1-\frac{r}{6a_0}\right)e^{-\frac{r}{3a_0}}$, where a_0 is first Bohr radius.

The expectation value of square of orbital angular momentum is
(a) $6\hbar^2$ (b) $3\hbar^2$ (c) $2\hbar^2$ (d) 0

5. A particle of mass constrained to move in one dimensional potential

$$V(x)=\begin{cases}0, & if\ -\frac{L}{2}<x<+\frac{L}{2}\\ \infty, & otherwise\end{cases}$$

The period of time evolution of the state $|\psi(0)\rangle=\alpha|n\rangle+\beta|n+1\rangle$, where α,β are constant, is
(a) $\frac{4mL^2}{3\hbar}$ (b) $\frac{8mL^2}{3h}$ (c) $\frac{8mL^2}{(2n+1)h}$ (d) $\frac{8mL^2}{(2n+1)\hbar}$

6. Let us consider a system whose Hamiltonian is

$$\hat{H} = k\frac{L_+L_-}{\hbar}$$

where, $L_{\pm}$ are the raising and lowering operators for the z component of the orbital angular momentum and k is constant of suitable dimension. At time $t = 0$ the system is described by the following wave- function

$$\psi(\theta, \phi) = A\sin\theta\sin\phi$$

where, θ, ϕ are usual spherical polar co-ordinates. The expectation value of energy of the system at $t = 0$, is

(a) $2\hbar k$ (b) $3\hbar k$ (c) $\hbar k$ (d) $-\hbar k$

7. An electron is in the spin state $|\psi\rangle = A\begin{pmatrix}1-2i\\2\end{pmatrix}$, where A is normalization constant. If its spin is measured in this state then the probabilities of finding its spin along positive z-axis and positive x-axis respectively, are

(a) $\frac{5}{9}, \frac{5}{9}$ (b) $\frac{5}{9}, \frac{5}{18}$ (c) $\frac{4}{9}, \frac{13}{18}$ (d) $\frac{5}{9}, \frac{13}{18}$

8. A harmonic oscillator with mass m and angular frequency ω at time $t = 0$ is in the state

$$|\psi(t=0)\rangle = (2a^\dagger + 1)|0\rangle,$$

where $|0\rangle$ is the ground state and $a^\dagger$ is raising operator for linear harmonic oscillator. The expectation vlaue of energy of the oscillator at time t is given by

(a) $\frac{13}{2}\hbar\omega$ (b) $\frac{13}{10}\hbar\omega$ (c) $\hbar\omega$ (d) $\frac{\hbar\omega}{2}$

9. A particle which is rotating with uniform velocity in the x-y plane around a fixed point is prepared in the state

$$\psi(\phi) = \frac{2}{\sqrt{2\pi}}\cos^2\phi$$

where, ϕ is the angle measured in x-y plane. The standard deviation in the z-component of orbital angular momentum of the particle in this state is

(a) $\frac{4}{3}\hbar$ (b) $\frac{2}{3}\hbar$ (c) $\sqrt{\frac{2}{3}}\hbar$ (d) $\frac{2\hbar}{\sqrt{3}}$

10. If $\vec{\sigma}$ is Pauli's spin operator for a spin-½ particle and $\hat{n}$ is unit vector, then the commutator $[\vec{\sigma}\cdot\hat{n}, \vec{\sigma}]$ is

(a) $2i\vec{\sigma}\cdot(\vec{\sigma}\times\hat{n})$ (b) $i(2\hat{n}(\vec{\sigma}\cdot\hat{n}) - \vec{\sigma})$ (c) $2i(\vec{\sigma}\times\hat{n})$ (d) $i\vec{\sigma}\cdot(\vec{\sigma}\times\hat{n}) - i\vec{\sigma}$

11. A polar representation of the creation and annihilation operators for a simple harmonic oscillator can be introduced as $a = \sqrt{N+1}e^{i\phi}$ and $a^\dagger = e^{-i\phi}\sqrt{N+1}$

The operators N and ϕ are assumed to be Hermitian. Given $[a, a^\dagger] = 1$, the value of $[\cos\phi, N]$ is

(a) $\cos\phi$ (b) $-e^{-i\phi}$ (c) $e^{i\phi}$ (d) $i\sin\phi$

12. Consider an electron bound in a hydrogen atom under the influence of a homogeneous magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. Ignore the electron spin. The Hamiltonian of the system is $H = H_0 - \omega L_z$, with $\omega = eB/2mc$. The eigenstates $|nlm\rangle$ and eigenvalues as known. Assume that initially (at t =0) the system is in state

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|21-1\rangle - |211\rangle\right)$$

The expectation value of the magnetic dipole moment associated with the orbital angular momentum at time t, is

(a) $\frac{e\hbar}{2mc}$ (b) $\frac{e\hbar}{mc}$ (c) $\frac{2e\hbar}{mc}$ (d) 0

13. A free particle is initially (at t=0) in a state corresponding to the wave function

$$\psi(r) = \left(\frac{\gamma}{\pi}\right)^{3/4} e^{-\gamma r^2/2}$$

(a) The wave function is an eigen function of Hamiltonian only.

(b) The wave function is an eigen function of L^2 and L_z only.

(c) The wave function is an eigen function of H, L^2 and L_z at time $t = 0$ but not at all time.

(d) The wave function is an eigen function of H, L^2 and L_z at time $t = 0$ and also at all time t.

14. A particle of mass m is moving in the potential $V(x) = \frac{1}{2}kx^2$, where k is spring constant. The particle is in the ground state. If spring constant is suddenly quadrupled, the probability of finding the energy of particle $\hbar\omega$ is

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2\sqrt{2}}{3}$

15. The average kinetic energy of the electron in the first excited state of Li^{++}, will be approximately

(a) 30.6 eV (b) 61.2 eV (c) 13.6 eV (d) 122.4 eV

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[ANSWERS]

1. (a)	2. (d)	3. (d)	4. (c)	5. (c)	6. (c)	7. (d)
8. (b)	9. (d)	10. (c)	11. (d)	12. (d)	13. (d)	14. (d)
15. (a)						