



CSIR-UGC-NET/JRF | GATE - PHYSICS

UNIT TEST - 2

[QUANTUM MECHANICS]

Time : 00: 50 Hour

Date : 06-10-2017

M.M. : 30

INSTRUCTION:

- (I) Question Paper contains 15 objective type questions, each question carry 2 marks.
 (II) There is negative marking, 0.5 mark will be deducted for each wrong answer.
 (III) Attempt all the questions, use of calculator is not allowed.

- The average force on a particle in the first excited energy eigenstate, moving in time independent potential $V(x)$ that satisfies the condition $V(-x) = V(x)$,
 - is always zero
 - is always non-zero
 - is either zero or non-zero
 - depends on the variation of potential with respect to position
- The number of energy states for a cubical box of side a in energy range, $0 < E < 15\pi^2\hbar^2/2ma^2$ is
 - 6
 - 11
 - 15
 - 17
- A quantum particle of mass m moves in a two dimensional anisotropic harmonic oscillator potential $V(x, y) = \frac{1}{2}m\omega^2x^2 + 8m\omega^2y^2$. The energy eigenvalues are ($n = 0, 1, 2, 3, 4, \dots$) given by
 - $(n+1)\hbar\omega$
 - $\left(2n + \frac{3}{2}\right)\hbar\omega$
 - $\left(n + \frac{3}{2}\right)\hbar\omega$
 - $\left(n + \frac{5}{2}\right)\hbar\omega$
- Consider an electron in Hydrogen atom in a state whose unnormalized wave function is given by $R(r) = Ar\left(1 - \frac{r}{6a_0}\right)e^{-\frac{r}{3a_0}}$, where a_0 is first Bohr radius. The expectation value of square of orbital angular momentum is
 - $6\hbar^2$
 - $3\hbar^2$
 - $2\hbar^2$
 - 0
- A particle of mass constrained to move in one dimensional potential

$$V(x) = \begin{cases} 0, & \text{if } -\frac{L}{2} < x < +\frac{L}{2} \\ \infty, & \text{otherwise} \end{cases}$$

The period of time evolution of the state $|\psi(0)\rangle = \alpha|n\rangle + \beta|n+1\rangle$, where α, β are constant, is

 - $\frac{4mL^2}{3\hbar}$
 - $\frac{8mL^2}{3\hbar}$
 - $\frac{8mL^2}{(2n+1)\hbar}$
 - $\frac{8mL^2}{(2n+1)\hbar}$



6. Let us consider a system whose Hamiltonian is

$$\hat{H} = k \frac{L_+ L_-}{\hbar}$$

where, L_{\pm} are the raising and lowering operators for the z component of the orbital angular momentum and k is constant of suitable dimension. At time $t = 0$ the system is described by the following wave- function

$$\psi(\theta, \phi) = A \sin \theta \sin \phi$$

where, θ, ϕ are usual spherical polar co-ordinates. The expectation value of energy of the system at $t = 0$, is

- (a) $2\hbar k$ (b) $3\hbar k$ (c) $\hbar k$ (d) $-\hbar k$

7. An electron is in the spin state $|\psi\rangle = A \begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$, where A is normalization constant. If its spin is measured in this state then the probabilities of finding its spin along positive z -axis and positive x -axis respectively, are

- (a) $\frac{5}{9}, \frac{5}{9}$ (b) $\frac{5}{9}, \frac{5}{18}$ (c) $\frac{4}{9}, \frac{13}{18}$ (d) $\frac{5}{9}, \frac{13}{18}$

8. A harmonic oscillator with mass m and angular frequency ω at time $t = 0$ is in the state

$$|\psi(t=0)\rangle = (2a^\dagger + 1)|0\rangle,$$

where $|0\rangle$ is the ground state and a^\dagger is raising operator for linear harmonic oscillator. The expectation value of energy of the oscillator at time t is given by

- (a) $\frac{13}{2}\hbar\omega$ (b) $\frac{13}{10}\hbar\omega$ (c) $\hbar\omega$ (d) $\frac{\hbar\omega}{2}$

9. A particle which is rotating with uniform velocity in the x - y plane around a fixed point is prepared in the state

$$\psi(\phi) = \frac{2}{\sqrt{2\pi}} \cos^2 \phi$$

where, ϕ is the angle measured in x - y plane. The standard deviation in the z -component of orbital angular momentum of the particle in this state is

- (a) $\frac{4}{3}\hbar$ (b) $\frac{2}{3}\hbar$ (c) $\sqrt{\frac{2}{3}}\hbar$ (d) $\frac{2\hbar}{\sqrt{3}}$

10. If $\vec{\sigma}$ is Pauli's spin operator for a spin- $1/2$ particle and \hat{n} is unit vector, then the commutator $[\vec{\sigma} \cdot \hat{n}, \vec{\sigma}]$ is

- (a) $2i\vec{\sigma} \cdot (\vec{\sigma} \times \hat{n})$ (b) $i(2\hat{n}(\vec{\sigma} \cdot \hat{n}) - \vec{\sigma})$ (c) $2i(\vec{\sigma} \times \hat{n})$ (d) $i\vec{\sigma} \cdot (\vec{\sigma} \times \hat{n}) - i\vec{\sigma}$

11. A polar representation of the creation and annihilation operators for a simple harmonic oscillator can be introduced as $a = \sqrt{N+1}e^{i\phi}$ and $a^\dagger = e^{-i\phi}\sqrt{N+1}$

The operators N and ϕ are assumed to be Hermitian. Given $[a, a^\dagger] = 1$, the value of $[\cos \phi, N]$ is

- (a) $\cos \phi$ (b) $-e^{-i\phi}$ (c) $e^{i\phi}$ (d) $i \sin \phi$

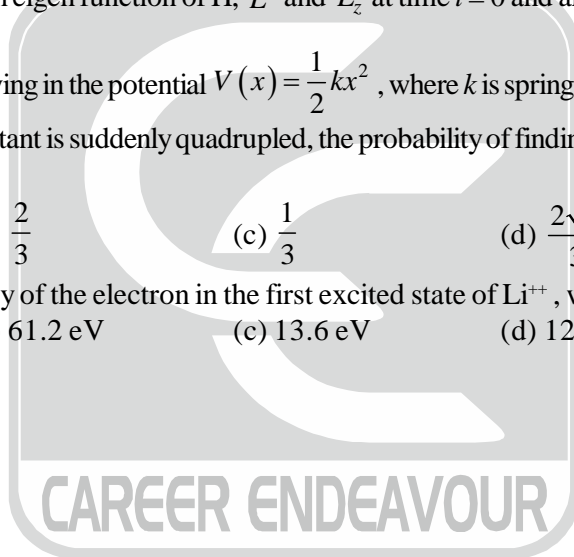


12. Consider an electron bound in a hydrogen atom under the influence of a homogeneous magnetic field $\mathbf{B} = B\hat{z}$. Ignore the electron spin. The Hamiltonian of the system is $H = H_0 - \omega L_z$, with $\omega = eB/2mc$. The eigenstates $|nlm\rangle$ and eigenvalues as known. Assume that initially (at $t=0$) the system is in state

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|21-1\rangle - |211\rangle)$$

The expectation value of the magnetic dipole moment associated with the orbital angular momentum at time t , is

- (a) $\frac{e\hbar}{2mc}$ (b) $\frac{e\hbar}{mc}$ (c) $\frac{2e\hbar}{mc}$ (d) 0
13. A free particle is initially (at $t=0$) in a state corresponding to the wave function
- $$\psi(r) = \left(\frac{r}{\pi}\right)^{3/4} e^{-r^2/2}$$
- (a) The wave function is an eigen function of Hamiltonian only.
 (b) The wave function is an eigen function of L^2 and L_z only.
 (c) The wave function is an eigen function of H , L^2 and L_z at time $t=0$ but not at all time.
 (d) The wave function is an eigen function of H , L^2 and L_z at time $t=0$ and also at all time t .
14. A particle of mass m is moving in the potential $V(x) = \frac{1}{2}kx^2$, where k is spring constant. The particle is in the ground state. If spring constant is suddenly quadrupled, the probability of finding the energy of particle $\hbar\omega$ is
- (a) 0 (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2\sqrt{2}}{3}$
15. The average kinetic energy of the electron in the first excited state of Li^{++} , will be approximately
- (a) 30.6 eV (b) 61.2 eV (c) 13.6 eV (d) 122.4 eV





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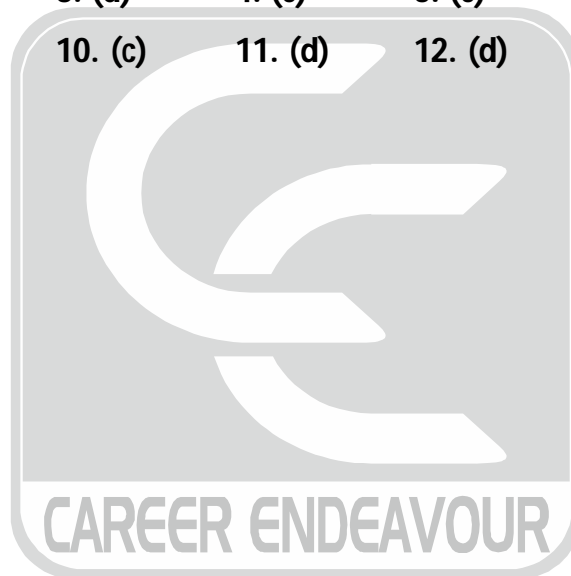
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[ANSWERS]

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|---------|--------|---------|---------|---------|---------|---------|
| 1. (a) | 2. (d) | 3. (d) | 4. (c) | 5. (c) | 6. (c) | 7. (d) |
| 8. (b) | 9. (d) | 10. (c) | 11. (d) | 12. (d) | 13. (d) | 14. (d) |
| 15. (a) | | | | | | |



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