PART - B

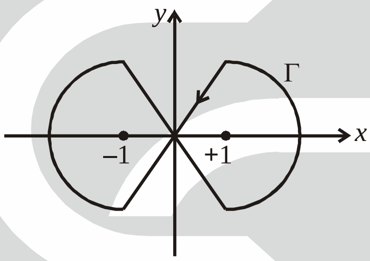
21. Which of the following cannot be eigen values of a real 3 × 3 matrix

(a) $2i, 0, -2i$ (b) 1, 1, 1 (c) $e^{i\theta}, e^{-i\theta}, 1$ (d) $i, 1, 0$

22. Let $u(x, y) = e^{ax}\cos(by)$ be the real part of a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where a, b are real constants and $a \neq 0$. The function $f(z)$ is complex analytic everywhere in the complex plane if and only if

(a) $b = 0$ (b) $b = \pm a$ (c) $b = \pm 2\pi a$ (d) $b = a \pm 2\pi$

23. The integral $\oint_\Gamma \frac{z\, e^{i\pi z/2}}{z^2 - 1} dz$ along the closed contour Γ shown in the figure is



(a) 0 (b) 2π (c) -2π (d) $4\pi i$

24. The function $y(x)$ satisfies the differential equation $x\frac{dy}{dx} + 2y = \frac{\cos \pi x}{x}$. If $y(1) = 1$, the value of $y(2)$ is

(a) π (b) 1 (c) 1/2 (d) 1/4

25. The random variable $x(-\infty < x < \infty)$ is distributed according to the normal distribution $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$. The probability density of the random variable $y = x^2$ is

(a) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$ (b) $\frac{1}{2\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$

(c) $\frac{1}{\sqrt{2\sigma^2}} e^{-\frac{y}{2\sigma^2}}, 0 \le y < \infty$ (d) $\frac{1}{\sqrt{2\pi\sigma^2 y}} e^{-\frac{y}{\sigma^2}}, 0 \le y < \infty$

26. The Hamiltonian for a system described by the generalized coordinate x and generalized momentum p is

$$H = \alpha x^2 p + \frac{p^2}{2(1 + 2\beta x)} + \frac{1}{2}\omega^2 x^2$$

where α, β and ω are constant. The corresponding Lagrangian is

(a) $\frac{1}{2}(\dot{x}-\alpha x^2)^2(1+2\beta x)-\frac{1}{2}\omega^2 x^2$ (b) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2 x^2-\alpha x^2\dot{x}$

(c) $\frac{1}{2}(\dot{x}^2-\alpha^2 x)^2(1+2\beta x)-\frac{1}{2}\omega^2 x^2$ (d) $\frac{1}{2(1+2\beta x)}\dot{x}^2-\frac{1}{2}\omega^2 x^2+\alpha x^2\dot{x}$

27. An inertial observer sees two events E_1 and E_2 happening at the same location but 6 μs apart in time. Another observer moving with a constant velocity v (with respect to the first one) sees the same events to be 9 μs apart. The spatial distance between the events, as measured by the second observer, is approximately

(a) 300 m (b) 1000 m (c) 2000 m (d) 2700 m

28. A ball weighing 100 gm, released from a height of 5 m, bounces perfectly elastically off a plate. The collision time between the ball and the plate is 0.5 s. The average force on the plate is approximately

(a) 3 N (b) 2 N (c) 5 N (d) 4 N

29. A solid vertical rod, of length L, and cross-sectional area A, is made of a material of Young's modulus Y. The rod is loaded with a mass M, and as a result, extends by a small amount ΔL in the equilibrium condition. The mass is then suddenly reduced to $M/2$. As a result the rod will undergo longitudinal oscillation with an angular frequency

(a) $\sqrt{\frac{2YA}{ML}}$ (b) $\sqrt{\frac{YA}{ML}}$ (c) $\sqrt{\frac{2YA}{M\Delta L}}$ (d) $\sqrt{\frac{YA}{M\Delta L}}$

30. If the root-mean-squared momentum of a particle in the ground state of a one-dimensional simple harmonic potential is p_0, then its root-mean-squared momentum in the first excited state is

(a) $p_0\sqrt{2}$ (b) $p_0\sqrt{3}$ (c) $p_0\sqrt{2/3}$ (d) $p_0\sqrt{3/2}$

31. Consider a potential barrier A of height V_0 and width b, and another potential barrier B of height $2V_0$ and the same width b. The ratio T_A/T_B of tunnelling probabilities T_A and T_B, through barriers A and B respectively, for a particle of energy $V_0/100$, is best approximated by

(a) $\exp\left[\left(\sqrt{1.99}-\sqrt{0.99}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$ (b) $\exp\left[\left(\sqrt{1.98}-\sqrt{0.98}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$

(c) $\exp\left[\left(\sqrt{2.99}-\sqrt{0.99}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$ (d) $\exp\left[\left(\sqrt{2.98}-\sqrt{0.98}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$

32. A constant perturbation H' is applied to a system for time Δt (where $H'\Delta t \ll \hbar$) leading to a transition from a state with energy E_i to another with energy E_f. If the time of application is doubled, the probability of transition will be

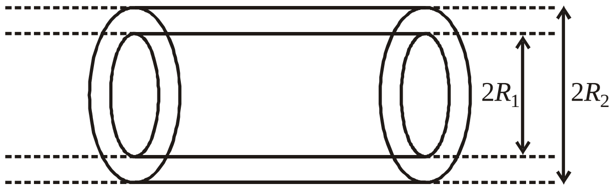
(a) unchanged (b) doubled (c) quadrupled (d) halved

33. The two vectors $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} b \\ c \end{pmatrix}$ are orthogonal if

(a) $a=\pm 1,\ b=\pm 1/\sqrt{2},\ c=\pm 1/\sqrt{2}$ (b) $a=\pm 1,\ b=\pm 1,\ c=0$

(c) $a=\pm 1,\ b=0,\ c=\pm 1$ (d) $a=\pm 1,\ b=\pm 1/2,\ c=1/2$

34. Two long hollow co-axial conducting cylinders of radii R_1 and R_2 $(R_1 < R_2)$ are placed in vacuum as shown in the figure below.



The inner cylinder carries a charge $+\lambda$ per unit length and the outer cylinder carries a charge $-\lambda$ per unit length. The electrostatic energy per unit length of this system is

(a) $\frac{\lambda^2}{\pi\varepsilon_0}\ln\left(\frac{R_1}{R_2}\right)$ (b) $\frac{\lambda^2}{4\pi\varepsilon_0}\left(\frac{R_2^2}{R_1^2}\right)$ (c) $\frac{\lambda^2}{4\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$ (d) $\frac{\lambda^2}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$

35. A set N concentric circular loops of wire, each carrying a steady current I in the same direction, is arranged in a plane. The radius of the first loop is $r_1 = a$ and the radius of the n^{th} loop is given by $r_n = n\,r_{n-1}$. The magnitude B of the magnetic field at the centre of the circles in the limit $N \to \infty$, is

(a) $\frac{\mu_0 I(e^2-1)}{4\pi a}$ (b) $\frac{\mu_0 I(e-1)}{\pi a}$ (c) $\frac{\mu_0 I(e^2-1)}{8a}$ (d) $\frac{\mu_0 I(e-1)}{2a}$

36. An electromagnetic wave (of wavelength λ_0 in free space) travels through an absorbing medium with dielectric permittivity given by $\varepsilon = \varepsilon_R + i\varepsilon_I$, where $\frac{\varepsilon_I}{\varepsilon_R} = \sqrt{3}$. If the skin depth is $\frac{\lambda_0}{4\pi}$, the ratio of the amplitude of electric field E to that of the magnetic field B, in the medium (in ohms) is

(a) 120π (b) 377 (c) $30\sqrt{2}\,\pi$ (d) 30π

37. The vector potential $\vec{A} = ke^{-at}r\hat{r}$, (where a and k are constants) corresponding to an electromagnetic field is changed to $\vec{A}' = -ke^{-at}r\hat{r}$. This will be a gauge transformation if the corresponding change $\phi' - \phi$ in the scalar potential is

(a) akr^2e^{-at} (b) $2akr^2e^{-at}$ (c) $-akr^2e^{-at}$ (d) $-2akr^2e^{-at}$

38. A thermodynamic function $G(T, P, N) = U - TS + PV$ is given in terms of the internal energy U, temperature T, entropy S, pressure P, volume V and the number of particles N. Which of the following relations is true ? (In the following μ is the chemical potential).

(a) $S = -\left.\frac{\partial G}{\partial T}\right|_{N,P}$ (b) $S = \left.\frac{\partial G}{\partial T}\right|_{N,P}$ (c) $V = -\left.\frac{\partial G}{\partial P}\right|_{N,T}$ (d) $\mu = -\left.\frac{\partial G}{\partial N}\right|_{P,T}$

39. A box, separated by a movable wall, has two compartment filled by a monoatomic gas of $\frac{C_P}{C_V} = \gamma$.

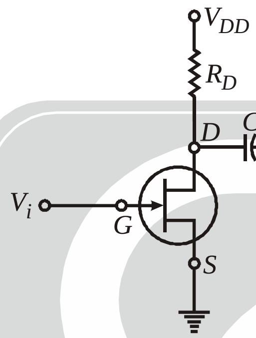
Initially the volumes of the two compartments are equal, but the pressure are $3P_0$ and P_0, respectively. When the wall is allowed to move, the final pressure in the two compartments become equal. The final pressure is

(a) $\left(\frac{2}{3}\right)^{\gamma} P_0$ (b) $3\left(\frac{2}{3}\right)^{\gamma} P_0$ (c) $\frac{1}{2}(1+3^{1/\gamma})^{\gamma} P_0$ (d) $\left(\frac{3^{1/\gamma}}{1+3^{1/\gamma}}\right)^{\gamma} P_0$

40. A gas of photons inside a cavity of volume V is in equilibrium at temperature T. If the temperature of the cavity is changed to $2T$, the radiation pressure will change by a factor of
(a) 2 (b) 16 (c) 8 (d) 4

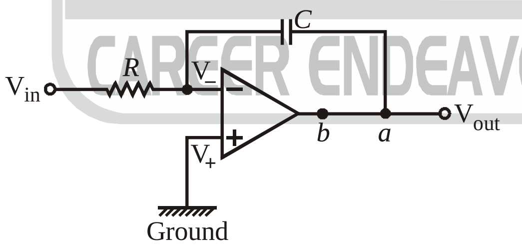
41. In a thermodynamic system in equilibrium, each molecule can exist in three possible states with probabilities $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively. The entropy per molecule is
(a) $k_B \ln 3$
(b) $\frac{1}{2}k_B \ln 2+\frac{2}{3}k_B \ln 3$
(c) $\frac{2}{3}k_B \ln 2+\frac{1}{2}k_B \ln 3$
(d) $\frac{1}{2}k_B \ln 2+\frac{1}{6}k_B \ln 3$

42. In the n-channel JFET shown in figure below, $V_i=-2\text{V}$, $C=10\,\text{pF}$, $V_{DD}=+16\,\text{V}$ and $R_D=2\,\text{k}\Omega$.



If the drain D-source S saturation current I_{DSS} is 10 mA and the pinch-off voltage V_P is –8V, then the voltage across points D and S is
(a) 11.125 V (b) 10.375 V (c) 5.75 V (d) 4.75 V

43. The gain of the circuit given below is $-\frac{1}{\omega RC}$.



The modification in the circuit required to introduce a dc feedback is to add a resistor
(a) between a and b
(b) between positive terminal of the op-amp and ground
(c) in series with C
(d) parallel to C

44. A 2 × 4 decoder with an enable input can function as a
(a) 4 × 1 multiplexer
(b) 1 × 4 demultiplexer
(c) 4 × 2 encoder
(d) 4 × 2 priority encoder

45. The experimentally measured values of the variables x and y are 2.00 ± 0.05 and 3.00 ± 0.02, respectively. What is the error in the calculated value of $z=3y-2x$ from the measurements ?
(a) 0.12 (b) 0.05 (c) 0.03 (d) 0.07

PART - C

46. The Green's function satisfying $\frac{d^2}{dx^2}g(x, x_0) = \delta(x - x_0)$, with the boundary conditions $g(-L, x_0) = 0 = g(L, x_0)$, is

(a) $\begin{cases} \frac{1}{2L}(x_0 - L)(x + L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0 + L)(x - L), & x_0 \le x \le L \end{cases}$

(b) $\begin{cases} \frac{1}{2L}(x_0 + L)(x + L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0 - L)(x - L), & x_0 \le x \le L \end{cases}$

(c) $\begin{cases} \frac{1}{2L}(L - x_0)(x + L), & -L \le x < x_0 \\ \frac{1}{2L}(x_0 + L)(L - x), & x_0 \le x \le L \end{cases}$

(d) $\frac{1}{2L}(x - L)(x + L), \quad -L \le x \le L$

47. Let $\sigma_x, \sigma_y, \sigma_z$ be the Pauli matrices and

$$x'\sigma_x + y'\sigma_y + z'\sigma_z = \exp\left(\frac{i\theta\sigma_z}{2}\right) \times \left[x\sigma_x + y\sigma_y + z\sigma_z\right] \exp\left(-\frac{i\theta\sigma_z}{2}\right)$$

Then the coordinates are related as follows

(a) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(b) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(c) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

(d) $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} & 0 \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

48. The interval [0, 1] is divided into $2n$ parts of equal length to calculate the integral $\int_0^1 e^{i2\pi x}dx$ using Simpson's $\frac{1}{3}$-rule. What is the minimum value of n for the result to be exact ?

(a) ∞ (b) 2 (c) 3 (d) 4

49. Which of the following sets of 3 × 3 matrices (in which a and b are real numbers) form a group under matrix multiplication ?

(a) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ b & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

(b) $\left\{\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

(c) $\left\{\begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

(d) $\left\{\begin{pmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : a, b \in \mathbb{R}\right\}$

50. The Lagrangian of a free relativistic particle (in one-dimension) of mass m is given by $L = -m\sqrt{1-\dot{x}^2}$, where $\dot{x} = dx/dt$. If such a particle is acted upon by a constant force in the direction of its motion, the phase space trajectories obtained from the corresponding Hamiltonian are

(a) ellipses (b) cycloids (c) hyperbolas (d) parabolas

51. A Hamiltonian system is described by the canonical coordinate q and canonical momentum p. A new coordinate Q is defined as $Q(t) = q(t+\tau) + p(t+\tau)$, where t is the time and τ is a constant, that is, the new coordinate is a combination of the old coordinate and momentum at a shifted time. The new canonical momentum $P(t)$ can be expressed as

(a) $p(t+\tau) - q(t+\tau)$

(b) $p(t+\tau) - q(t-\tau)$

(c) $\frac{1}{2}\left[p(t-\tau) - q(t+\tau)\right]$

(d) $\frac{1}{2}\left[p(t+\tau) - q(t+\tau)\right]$

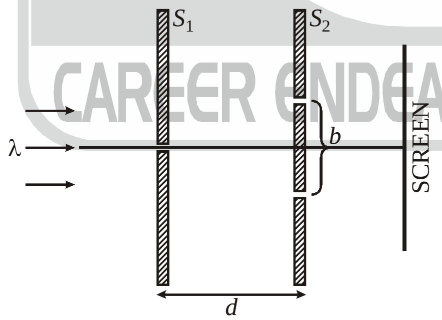
52. The energy of a one-dimensional system, governmed by the Lagrangian $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^{2n}$, where k and n are two positive constants, is E_0. The time period of oscillation τ satisfies

(a) $\tau \propto k^{-1/n}$ (b) $\tau \propto k^{-1/2n} E_0^{\frac{1-n}{2n}}$ (c) $\tau \propto k^{-1/2n} E_0^{\frac{n-2}{2n}}$ (d) $\tau \propto k^{-1/n} E_0^{\frac{1+n}{2n}}$

53. An electron is decelerated at a constant rate starting from an initial velocity u (where $u \ll c$) to $u/2$ during which it travels a distance s. The amount of energy lost to radiation is

(a) $\frac{\mu_0 e^2 u^2}{3\pi mc^2 s}$ (b) $\frac{\mu_0 e^2 u^2}{6\pi mc^2 s}$ (c) $\frac{\mu_0 e^2 u}{8\pi mcs}$ (d) $\frac{\mu_0 e^2 u}{16\pi mcs}$

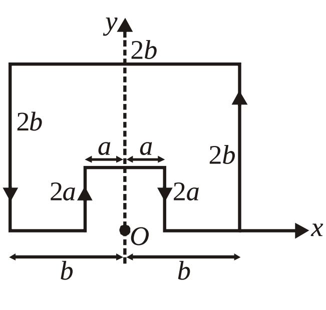
54. The figure below describes the arrangement of slits and screens in a Young's double slit experiment. The width of the slit in S_1 is a and the slits in S_2 are of negligible width.



If the wavelength of the light is λ, the value of d for which the screen would be dark is

(a) $b\sqrt{\left(\frac{a}{\lambda}\right)^2 - 1}$ (b) $\frac{b}{2}\sqrt{\left(\frac{a}{\lambda}\right)^2 - 1}$ (c) $\frac{a}{2}\left(\frac{b}{\lambda}\right)^2$ (d) $\frac{ab}{\lambda}$

55. A constant current I is flowing in a piece of wire that is bent into a loop as shown in the figure.



The magnitude of the magnetic field at the point O is

(a) $\frac{\mu_0 I}{4\pi\sqrt{5}}\ln\left(\frac{a}{b}\right)$ (b) $\frac{\mu_0 I}{4\pi\sqrt{5}}\ln\left(\frac{1}{a}-\frac{1}{b}\right)$ (c) $\frac{\mu_0 I}{4\pi\sqrt{5}}\left(\frac{1}{a}\right)$ (d) $\frac{\mu_0 I}{4\pi\sqrt{5}}\left(\frac{1}{b}\right)$

56. Consider the potential $V(\vec{r})=\sum_i V_0 a^3 \delta^{(3)}(\vec{r}-\vec{r}_i)$, where $\vec{r}_i$ are the position vectors of the vertices of a cube of length a centered at the origin and V_0 is a constant. If $V_0 a^2 \ll \frac{\hbar^2}{m}$, the total scattering cross-section, in the low energy limit, is

(a) $16a^2\left(\frac{mV_0a^2}{\hbar^2}\right)$ (b) $\frac{16a^2}{\pi^2}\left(\frac{mV_0a^2}{\hbar^2}\right)^2$ (c) $\frac{64a^2}{\pi}\left(\frac{mV_0a^2}{\hbar^2}\right)^2$ (d) $\frac{64a^2}{\pi^2}\left(\frac{mV_0a^2}{\hbar^2}\right)$

57. The Coulomb potential $V(r)=-e^2/r$ of a hydrogen atom is perturbed by adding $H'=bx^2$ (where b is a constant) to the Hamiltonian. The first order correction to the ground state energy is (The ground state wavefunction is $\psi_0=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$).

(a) $2ba_0^2$ (b) ba_0^2 (c) $ba_0^2/2$ (d) $\sqrt{2}ba_0^2$

58. Using the trial function $\psi(x)=\begin{cases}A(a^2-x^2) & ; -a<x<a \\ 0 & ; \text{otherwise}\end{cases}$, the ground state energy of a one-dimensional harmonic oscillattor is

(a) $\hbar\omega$ (b) $\sqrt{\frac{5}{14}}\hbar\omega$ (c) $\frac{1}{2}\hbar\omega$ (d) $\sqrt{\frac{5}{7}}\hbar\omega$

59. In the usual notation $|n\,l\,m\rangle$ for the states of a hydrogen like atom, consider the spontaneous transitions $|210\rangle\to|100\rangle$ and $|310\rangle\to|100\rangle$. If t_1 and t_2 are the lifetimes of the first and the second decaying states respectively, then the ratio t_1/t_2 is proportional to

(a) $\left(\frac{32}{27}\right)^3$ (b) $\left(\frac{27}{32}\right)^3$ (c) $\left(\frac{2}{3}\right)^3$ (d) $\left(\frac{3}{2}\right)^3$

60. A random variable n obeys Poisson statistics. The probability of finding $n=0$ is 10^{-6}. The expectation value of n is nearest to

(a) 14 (b) 10^6 (c) e (d) 10^2

61. The single particle energy levels of a non-interacting three-dimensional isotropic system, labelled by momentum k, are proportional to k^3. The ratio $\vec{P}/\varepsilon$ of the average pressure $\vec{P}$ to the energy density ε at a fixed temperature, is

(a) 1/3 (b) 2/3 (c) 1 (d) 3

62. The Hamiltonian for three Ising spins S_0, S_1 and S_2, taking values ± 1, is $H = -jS_0(S_1 + S_2)$. If the system is in equilibrium at temperature *T*, the average energy of the system, in terms of $\beta = (k_B T)^{-1}$, is

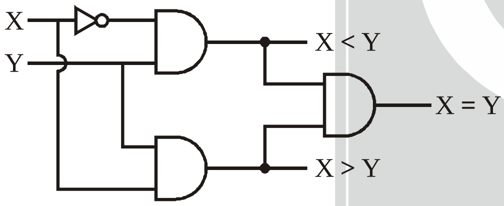
(a) $-\dfrac{1+\cosh(2\beta j)}{2\beta \sinh(2\beta j)}$ (b) $-2j[1+\cosh(2\beta j)]$

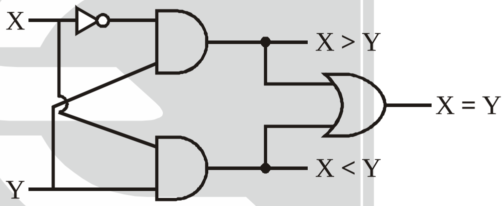
(c) $-2/\beta$ (d) $-2j\dfrac{\sinh(2\beta j)}{1+\cosh(2\beta j)}$

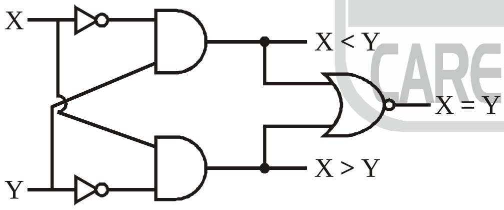
63. Let I_0 be the saturation current, η the ideality factor and v_F and v_R the forward and reverse potentials, respectively, for a diode. The ratio R_R/R_F of its reverse and forward resistances R_R and R_F respectively, varies as (In the following k_B is the Boltzmann constant, *T* is the absolute temperature and *q* is the charge).

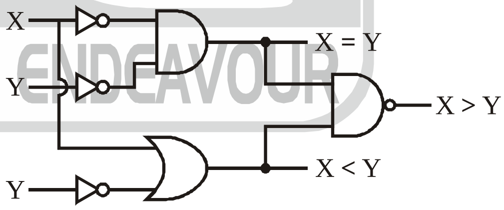
(a) $\dfrac{v_R}{v_F}\exp\left(\dfrac{qv_F}{\eta k_B T}\right)$ (b) $\dfrac{v_F}{v_R}\exp\left(\dfrac{qv_F}{\eta k_B T}\right)$ (c) $\dfrac{v_R}{v_F}\exp\left(-\dfrac{qv_F}{\eta k_B T}\right)$ (d) $\dfrac{v_F}{v_R}\exp\left(-\dfrac{qv_F}{\eta k_B T}\right)$

64. In the figures below, X and Y are one bit inputs. The circuit which corresponds to a one bit comparator is

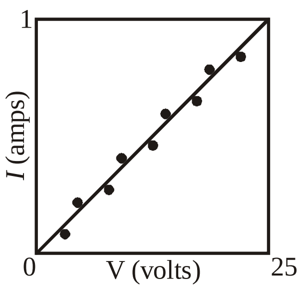
(a)


(b)


(c)


(d)


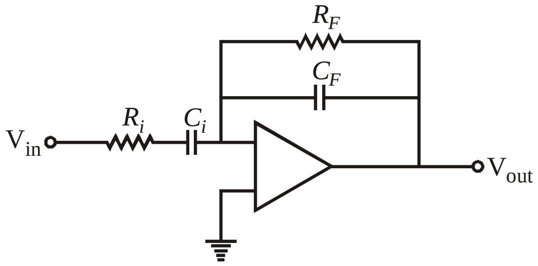
65. Both the data points and a linear fit to the current vs voltage of a resistor are shown in the graph below.
If the error in the slope is $1.255 \times 10^{-3}\ \Omega^{-1}$, then the value of resistance estimated from the graph is



(a) $(0.04 \pm 0.8)\Omega$ (b) $(25.0 \pm 0.8)\Omega$ (c) $(25 \pm 1.25)\Omega$ (d) $(25 \pm 0.0125)\Omega$

66. In the following operational amplifier circuit $C_{in} = 10\,\text{nF}$, $R_{in} = 20\,\text{k}\Omega$, $R_F = 200\,\text{k}\Omega$ and $C_F = 100\,\text{pF}$.

The magnitude of the gain at a input signal frequency of 16 kHz is



(a) 67 (b) 0.15 (c) 0.3 (d) 3.5

67. An atomic spectral line is observed to split into nine components due to Zeeman shift. If the upper state of the atom is $^3D_2$ then the lower state will be

(a) $^3F_2$ (b) $^3F_1$ (c) $^3P_1$ (d) $^3P_2$

68. If the coefficient of stimulated emission for a particular transition is 2.1×10^{19} m^3 W^{-1} s^{-3} and the emitted photon is at wavelength 3000 Å, then the lifetime of the excited state is approximately.

(a) 20 ns (b) 40 ns (c) 80 ns (d) 100 ns

69. If the bindings energies of the electron in the K and L shells of silver atom are 25.4 keV and 3.34 keV, respectively, then the kinetic energy of the Auger electron will be approximately

(a) 22 keV (b) 9.3 keV (c) 10.5 keV (d) 18.7 keV

70. The energy gap and lattice constant of an indirect band gap semiconductor are 1.875 eV ; 0.52 nm, respectively. For simplicity take the dielectric constant of the material to be unity. When it is excited by broadband radiation, an electron initially in the valence band at $k = 0$ makes a transition to the conduction band. The wavevector of the electron in the conduction band, in terms of the wavevector k_{max} at the edge of the Brillouin zone, after the transition is closest to

(a) $k_{max}/10$ (b) $k_{max}/100$ (c) $k_{max}/1000$ (d) 0

71. The electrical conductivity of copper is approximately 95% of the electrical conductivity of silver, while the electron density in silver is approximately 70% of the electron density in copper. In Drude's model, the approximate ratio τ_{Cu}/τ_{Ag} of the mean collision time in copper (τ_{Cu}) to the mean collision time in silver (τ_{Ag}) is

(a) 0.44 (b) 1.50 (c) 0.33 (d) 0.66

72. The charge distribution inside a material of conductivity σ and permittivity ε at initial tine $t = 0$ is $\rho(r, 0) = \rho_0$, constant. At subsequent times $\rho(r, t)$ is given by

(a) $\rho_0 \exp\left(-\frac{\sigma t}{\varepsilon}\right)$

(b) $\frac{1}{2}\rho_0\left[1 + \exp\left(\frac{\sigma t}{\varepsilon}\right)\right]$

(c) $\frac{\rho_0}{\left[1 - \exp\left(\frac{\sigma t}{\varepsilon}\right)\right]}$

(d) $\rho_0 \cosh \frac{\sigma t}{\varepsilon}$

73. If in a spontaneous α-decay of ${}^{232}_{92}\text{U}$ at rest, the total energy released in the reaction is Q, then the energy carried by the α-particle is

(a) $\frac{57Q}{58}$ (b) $\frac{Q}{57}$ (c) $\frac{Q}{58}$ (d) $\frac{23Q}{58}$

74. The range of the nuclear force between two nucleons due to the exchange of pions is 1.40 fm. If the mass of the pion is 140 MeV/c^2 and the mass of the rho-meson is 770 MeV/c^2, then the range of the force due to exchange of rho mesons is

(a) 1.40 fm (b) 7.70 fm (c) 0.25 fm (d) 0.18 fm

75. A baryon X decays by strong interaction as $X \rightarrow \Sigma^+ + \pi^- + \pi^0$, where Σ^+ is a member of the isotriplet $(\Sigma^+, \Sigma^0, \Sigma^-)$. The third component I_3 of the isospin of X is

(a) 0 (b) 1/2 (c) 1 (d) 3/2

PART - B

21. For a real 3×3 matrix,
Trace (A) = Sum of the eigenvalues = Real quantity
Det (A) = Product of the eigenvalues = Real quantity

	$\lambda_1+\lambda_2+\lambda_3$	$\lambda_1\lambda_2\lambda_3$
Option (a) :	0	0
Option (b) :	3	1
Option (c) :	$1+2\cos\theta$	0
Option (d) :	$\boxed{1+i}$ ↓ Not possible for a real matrix	0

Correct option is (d)

22. Real part and imaginary part of a complex analytic function, satisfies Laplace's equation i.e.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

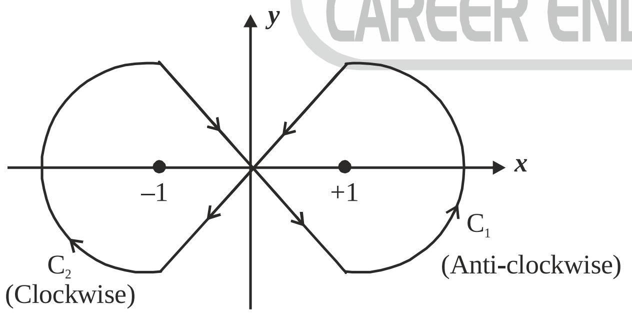
$$\Rightarrow \quad a^2\, e^{ax}\cdot\cos(by)-b^2e^{ax}\cos(by)=0$$

$$\Rightarrow \quad \left(a^2-b^2\right)\cdot e^{ax}\cdot\cos by=0$$

$$\Rightarrow \quad \left(a^2-b^2\right)=0 \Rightarrow \boxed{b=\pm a}$$

Correct option is (b)

23.



$$f(z)=\frac{z\cdot e^{i\pi z/2}}{z^2-1} \text{ has two simple poles at } z=\pm 1$$

$$\text{Res.} f(z=+1)=\lim_{z\to+1}\frac{z\cdot e^{i\pi z/2}}{\left(z^2-1\right)}(z-1)=\frac{1}{2}e^{i\pi/2}$$

$$\text{Res.} f(z=+1)=\lim_{z\to-1}\frac{z\cdot e^{i\pi z/2}}{\left(z^2-1\right)}(z+1)=\frac{1}{2}e^{-i\pi/2}$$

Required integral $= 2\pi i\left[\text{Res}\cdot f(z=+1)\right] - 2\pi i\left[\text{Res}\cdot f(z=-1)\right]$

$$= \frac{2\pi i}{2}\left[e^{i\pi/2} - e^{-i\pi/2}\right] = \pi i\left[2i\sin\frac{\pi}{2}\right] = -2\pi$$

Correct option is (c)

24. $x\frac{dy}{dx} + 2y = \frac{\cos\pi x}{x}$

$\Rightarrow \quad \frac{dy}{dx} + \frac{2}{x}y = \frac{\cos\pi x}{x^2}$ (First order linear D.E.)

$I.F. = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

Solution will be, $y \times (I.F.) = \int Q(x)\cdot(I.F.)\,dx$

$\Rightarrow \quad y\cdot x^2 = \int \frac{\cos(\pi x)}{x^2}\cdot x^2 dx$

$\Rightarrow \quad \boxed{y\cdot x^2 = \frac{\sin(\pi x)}{\pi} + C}$

Given : $y(1) = 1 \Rightarrow c = 1 \Rightarrow y\cdot x^2 = \frac{\sin(\pi x)}{\pi} + 1$

So, $y(2) = \frac{1}{4}$

Correct option is (d)

25. Probability density corresponding to any random variable should be normalized in nature i.e.

$$\int_0^\infty P(y)\,dy = 1$$

Option (a) : $\int_0^\infty P(y)\,dy = 1$

Option (b) : $\int_0^\infty P(y)\,dy = \frac{1}{2}$

Option (c) : $\int_0^\infty P(y)\,dy = \sqrt{2\,\sigma^2}$

Option (d) : $\int_0^\infty P(y)\,dy = \frac{1}{\sqrt{2}}$

Correct option is (a)

26. We have hamiltonian, $H = \alpha x^2 p + \dfrac{p^2}{2(1+2\beta x)} + \dfrac{1}{2}\omega^2 x^2$

According to Hamilton's equtaions,

$$\dot{x}. = \frac{\partial H}{\partial p} = \alpha x^2 + \frac{p}{1+2\beta x}$$

$$\Rightarrow \quad p = \left(\dot{x} - \alpha x^2\right)(1+2\beta x)$$

The Lagrangian,

$$L = p\dot{x} - H$$

$$= p\left(\alpha x^2 + \frac{p}{1+2\beta x}\right) - \alpha x^2 p - \frac{p^2}{2(1+2\beta x)} - \frac{1}{2}\omega^2 x^2$$

$$= \frac{1}{2}\frac{p^2}{(1+2\beta x)} - \frac{1}{2}\omega^2 x^2 = \frac{1}{2}\frac{\left(\dot{x}. - \alpha x^2\right)^2}{(1+2\beta x)}(1+2\beta x)^2 - \frac{1}{2}\omega^2 x^2$$

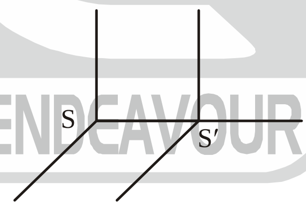
$$= \frac{1}{2}\left(\dot{x} - \alpha x^2\right)^2(1+2\beta x) - \frac{1}{2}\omega^2 x^2$$

Correct option is (a)

27. The position of two events with respect to S frame are (x_1, y_1, z_1, t_1) and (x_1, y_1, z_1, t_2)

$\therefore$ The position of two events with respect to S' frame are

$$x_1' = \frac{x_1 - vt_1}{\sqrt{1 - v^2/c^2}}$$



$$x_2' = \frac{x_1 - vt_2}{\sqrt{1 - v^2/c^2}}$$

and $\Delta t' = \dfrac{\Delta t}{\sqrt{1 - v^2/c^2}}$

$$\Rightarrow \quad 9 = \frac{6}{\sqrt{1 - v^2/c^2}} \quad \Rightarrow \quad \sqrt{1 - v^2/c^2} = \frac{6}{9} \quad \Rightarrow \quad v = \frac{\sqrt{45}}{9} \times c$$

$$\therefore \quad \Delta x' = x_1' - x_2' = \frac{v(t_2 - t_1)}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{\sqrt{45}}{9} \times c \times 6 \times \frac{9}{6} \times 10^{-6} \simeq 2000 \text{ m}$$

Correct option is (c)

28. The velocity of ball before collision with plate is

$$= \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \; m/\text{sec}$$

The change in momentum due to collision is = 10×0.1–(–10×0.1) = 2 kg-m/sec

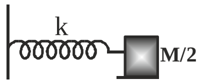
So, the average force on the plate is

$$= \text{Force due to change in momentum + force due to weight} = \frac{2}{0.5} + 10 \times 0.1 = 5N$$

Correct option is (c)

29. According to Hooke's law,

$$\frac{F/A}{-\Delta L/L} = Y$$



$$F = \frac{AY}{L}\Delta L = -k\Delta L, \text{ where force constant } k = \frac{AY}{L}$$

It is equivalent to a spring – block system with a spring constant $k = \frac{AY}{L}$ and mass $\frac{M}{2}$

$$\omega = \sqrt{\frac{k}{M/2}} = \sqrt{\frac{2AY}{ML}}$$

Correct option is (a)

30. Root-mean square momentum of nth state of one dimensional harmonic potential is given by

$$= \sqrt{\langle n | p^2 | n \rangle}$$

So, R.M.S. momentum of ground state

$$= \sqrt{\langle 0 | p^2 | 0 \rangle}$$

$$= \sqrt{\left\langle 0 \left| -\frac{\hbar m\omega}{2}\left(a - a^\dagger\right)\left(a - a^\dagger\right) \right| 0 \right\rangle} \quad \left[\because \; p = \frac{1}{i}\sqrt{\frac{\hbar m\omega}{2}}\left(a - a^\dagger\right)\right]$$

$$= \sqrt{-\frac{\hbar m\omega}{2}\left\langle 0 \left| a^2 + a^{\dagger 2} - aa^\dagger - a^\dagger a \right| 0 \right\rangle}$$

$$= \sqrt{-\frac{\hbar m\omega}{2}(-1)} \quad \left(\begin{array}{l} \because \; a|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle \\ a|0\rangle = 0, \; \langle n | m \rangle = \delta_{mn} \end{array}\right)$$

$$= \sqrt{\frac{\hbar m\omega}{2}} = p_0 \, (\text{given})$$

So, R.M.S. momentum of first excited state

$$= \sqrt{\langle 1 | p^2 | 1 \rangle} = \sqrt{-\frac{\hbar m\omega}{2}\left\langle 1 \left| a^2 + a^{\dagger 2} - aa^\dagger - a^\dagger a \right| 1 \right\rangle}$$

$$=\sqrt{-\frac{\hbar m\omega}{2}(-2-1)}=\sqrt{3\frac{\hbar m\omega}{2}}=\sqrt{3}\;p_0$$

Correct option is (b)

31. Since, the energy of the particle, $E=\frac{V_0}{100}$ very less than the potential barrier. So, tunnelling probabilities can be written as

$$T=\frac{16E(V_0-E)}{V_0^2}e^{-2\beta b} \qquad \left[\text{where, } \beta=\sqrt{\frac{2m(V_0-E)}{\hbar^2}}\right]$$

$$\therefore \quad \frac{T_A}{T_B}=\frac{\frac{16E(V_0-E)}{V_0^2}e^{-\sqrt{\frac{8mb^2}{\hbar^2}(V_0-E)}}}{\frac{16E(2V_0-E)}{V_0^2}e^{-\sqrt{\frac{8mb^2}{\hbar^2}(2V_0-E)}}}\simeq e^{-\sqrt{\frac{8mb^2}{\hbar^2}\left(V_0-\frac{V_0}{100}\right)}+\sqrt{\frac{8mb^2}{\hbar^2}\left(2V_0-\frac{V_0}{100}\right)}}$$

$$=e^{\left(\sqrt{1.99}-\sqrt{0.99}\right)\sqrt{\frac{mV_0b^2}{\hbar^2}}}$$

Correct option is (a)

32. Transition probability for $|\psi_i\rangle\to|\psi_f\rangle$ due to the presence of time-dependent perturbation $V_P(t')$, is

$$P_{if}=\left|-\frac{i}{\hbar}\int_0^t\langle\psi_f|\hat{V}_P(t')|\psi_i\rangle e^{i\omega_{fi}t'}dt'\right|^2 \quad \text{where, } \omega_{fi}=\left(E_f-E_i\right)/\hbar$$

For a constant perturbation H' applied to a system for time Δt

$$P_{if}=\frac{1}{\hbar^2}\left|\langle\psi_f|H'|\psi_i\rangle\right|^2\left|\int_0^{\Delta t}e^{i\omega_{fi}t'}dt\right|^2=\frac{1}{\hbar^2}\left|\langle\psi_f|H'|\psi_i\rangle\right|^2\left|\frac{e^{i\omega_{fi}\Delta t}-1}{i\omega_{fi}}\right|^2$$

$$=\frac{1}{\hbar^2}\left|\langle\psi_f|H'|\psi_i\rangle\right|^2\frac{\left(e^{i\omega_{fi}\Delta t}-1\right)\left(e^{-i\omega_{fi}\Delta t}-1\right)}{\omega_{fi}^2}$$

$$\Rightarrow\; P_{if}=\frac{1}{\hbar^2}\left|\langle\omega_f|H'|\psi_i\rangle\right|^2\frac{4\sin^2\left(\omega_{fi}\Delta t/2\right)}{\omega_{fi}^2}$$

For a transition from the state with energy E_i to a state with energy E_f, $\omega_{fi}=\left(E_f-E_i\right)/\hbar=\text{constant}$.

For small $\Delta t, \sin^2\left(\omega_{fi}\Delta t/2\right) \approx \omega_{fi}^2 (\Delta t)^2/4$

Therefore, transition probability,

$$P_{if} = \frac{1}{\hbar^2}\left|\left\langle \psi_f \left|\hat{H}\right|\psi_i\right\rangle\right|^2 \frac{4}{\omega_{fi}^2}\left(\frac{\omega_{fi}^2(\Delta t)^2}{4}\right) = \frac{1}{\hbar^2}\left|\left\langle \psi_f \left|\hat{H}\right|\psi_i\right\rangle\right|^2 (\Delta t)^2$$

Since, $P_{if} \propto (\Delta t)^2$, therefore, if the time of application is doubled, then the transition probability will be quadrupled i.e. 4 times.

Correct option is (c)

33. The given two vectors are $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} b \\ c \end{pmatrix}$

According to normalized condition,

$$(a \;\; 0)\begin{pmatrix} a \\ 0 \end{pmatrix} = 1 \Rightarrow a^2 = 1 \Rightarrow \; a \pm 1$$

And $(b \;\; c)\begin{pmatrix} b \\ c \end{pmatrix} = 1 \Rightarrow b^2 + c^2 = 1$

and $\begin{pmatrix} a \\ 0 \end{pmatrix}, \begin{pmatrix} b \\ c \end{pmatrix}$ are orthogonal

So, $(a \;\; 0)\begin{pmatrix} b \\ c \end{pmatrix} = 0$

$\Rightarrow \quad ab = 0 \Rightarrow b = 0 \because \; a = \pm 1$

$\therefore \quad c^2 = 1 \Rightarrow c = \pm 1$

Correct option is (c)

34. Electrostatic energy of a capacitor

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)} \qquad \therefore \; U = \frac{1}{2}\frac{\lambda_0^2}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right) = \frac{\lambda_0^2}{4\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$$

Correct option is (c)

35. Magnetic field at the center is given by

$$B = \sum \frac{\mu_0 I}{2r_n} = \frac{\mu_0 I}{2}\sum\frac{1}{r_n}$$

$$= \frac{\mu_0 I}{2}\left[\frac{1}{a} + \frac{1}{2a} + \frac{1}{3\times 2a} + \frac{1}{4\times 3\times 2a} ------\right]$$

$$= \frac{\mu_0 I}{2a}\left[1-1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}-------\right] = \frac{\mu_0 I}{2a}[e-1]$$

Correct option is (d)

36. $\varepsilon = \varepsilon_r + i\varepsilon_i$

$\Rightarrow \quad \varepsilon = \alpha\cos 60^{\circ} + i\alpha\sin 60^{\circ}$ (Given: $\frac{\varepsilon_i}{\varepsilon_r} = \sqrt{3}$)

We know that,

$$n = \sqrt{\varepsilon\mu_r} \Rightarrow n = \sqrt{(\varepsilon_r + i\varepsilon_i)} \Rightarrow n = (\alpha\cos 60 + i\alpha\sin 60)^{1/2} \Rightarrow n = \sqrt{\alpha}(\cos 30 + i\sin 30)$$

$$k = nk_0$$

$$\Rightarrow \quad k = \sqrt{\alpha}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)k_0$$

$$\therefore \quad k_i = \frac{k_0}{2}\sqrt{\alpha}$$

We know that,

$$\text{skin depth } (\delta) = \frac{1}{k_i} = \frac{2}{\sqrt{\alpha}k_0}$$

$$\frac{\lambda_0}{4\pi} = \frac{2}{\sqrt{\alpha}}\cdot\frac{\lambda_0}{2\pi} \Rightarrow \alpha = 16$$

$$\therefore \quad k = \sqrt{\alpha}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)k_0 = \frac{4}{2}\left(\sqrt{3} + i\right)k_0$$

$$\therefore \quad |k| = 4k_0$$

Again, we know that,

$$\frac{B}{E} = \frac{|k|}{\omega} = \frac{4k_0}{\omega} = \frac{4k_0}{\omega} = \frac{4k_0}{ck_0} = \frac{4}{c}$$

Therefore, ratio of electric field (E) and magnetic field in the unit of Ohm's = $\frac{E}{H} = \mu_0\frac{c}{4} = 30\,\pi$

Correct option is (d)

37. According to gauge transformation

$$A' = A + \nabla\psi \quad \text{and} \quad \varphi' = \varphi - \frac{d\psi}{dt}$$

$$\therefore \quad \nabla\psi = A' - A = -2ke^{-at}r\hat{r}$$

$$\Rightarrow \quad \frac{d\psi}{dr} = -2ke^{-at}r \quad \Rightarrow \quad \psi = -ke^{-at}r^2$$

$$\therefore \quad \varphi' - \varphi = -\frac{d\psi}{dt} = -ka\, e^{-at} r^2$$

Correct option is (c)

38. **Given :** $G(T, P, N) = U - TS + PV$

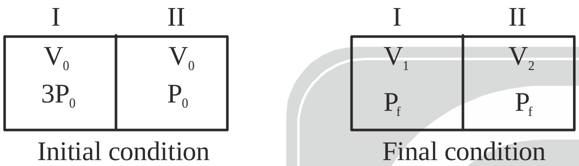
$$\Rightarrow \quad dG = dU - TdS - SdT + PdV + VdP$$

where, $dU = TdS - PdV + \mu dN$

$$\therefore \quad dG = \mu dN - SdT + VdP$$

$$\Rightarrow \quad S = -\left(\frac{\partial G}{\partial T}\right)_{N,P}$$

Correct option is (a)

39. 

Using adiabatic equation for compartment *I*, we have

$$(3P_0)V_0^\gamma = P_f V_1^\gamma$$

$$\Rightarrow \quad \left(\frac{3P_0V_0^\gamma}{P_f}\right)^{1/\gamma} = V_1 \qquad \ldots (1)$$

Similarly, for compartment II, we have

$$\left(\frac{P_0V_0^\gamma}{P_f}\right)^{1/\gamma} = V_2 \qquad \ldots (2)$$

From the above figure,

$$V_1 + V_2 = 2V_0$$

Now adding (1) and (2), we get

$$\left(\frac{3P_0V_0^\gamma}{P_f}\right)^{1/\gamma} + \left(\frac{P_0V_0^\gamma}{P_f}\right)^{1/\gamma} = 2V_0$$

$$\Rightarrow \quad \frac{(3P_0)^{1/\gamma} + (P_0)^{1/\gamma}}{2} = P_f^{1/\gamma} \Rightarrow P_f = \frac{P_0}{2^\gamma}\left(1 + 3^{1/\gamma}\right)^\gamma$$

Correct option is (c)

40. For a gas of photons inside a cavity,

$$\frac{P}{T^4} = \text{constant}$$

$$\Rightarrow \quad \frac{P_1}{T_1^4} = \frac{P_2}{T_2^4} \Rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)^4$$

$$\Rightarrow \qquad P_2 = P_1\left(\frac{2T}{T}\right)^4 = 16P_1$$

Correct option is (b)

41. The entropy per molecule is defined as

$$S = -k_B \sum p_i \ln p_i$$

$$\Rightarrow \qquad S = -k_B\left[\frac{1}{2}\ln\frac{1}{2} + \frac{1}{3}\ln\frac{1}{3} + \frac{1}{6}\ln\frac{1}{6}\right] = -k_B\left[\frac{2}{3}\ln\frac{1}{2} + \frac{1}{2}\ln\frac{1}{3}\right]$$

$$= \frac{2}{3}k_B \ln 2 + \frac{1}{2}k_B \ln 3$$

Correct option is (c)

42. $V_i = -2$ volt

$V_{DD} = 16$ volt

$R_D = 2\ \text{k}\Omega$

$I_{DSS} =$ maximum drain current = 10 mA

$V_p = -8$ (pinch off voltage)

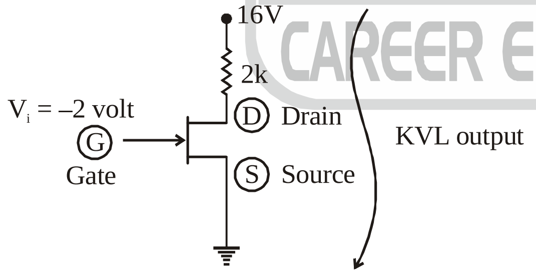
To - calculate voltage (OR) Resistance value we apply DC - Analysis.

DC - Analysis means $f = 0, \omega = 0, X_C = \infty$

$$X_C = \frac{1}{\omega C}$$

Open-circuit capacitor

Drawing circuit, making capacitor open circuit.



$V_{GS} = V_G - V_S$

$V_{GS} = -2$ volt

$\because \quad V_S = 0$

$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$ Shockeley equation $\qquad [V_P = -8V$ given]

Put value, $I_D = 10\left[1 - \frac{-2}{-8}\right]^2$

$I_D = 5.625$ mA

KVL at output, $16 = 2 \times I_D + V_{DS}$

Put I_D, $V_{DS} = 16 - 2 \times 5.625$

$V_{DS} = 4.75$ volt

Correct option is (d)

43. Gain of circuit $= \frac{-1}{\text{WRC}}$

" – " sign due to in verting op-amp.

DC - feedback "means ●—/\/\/\/—● "

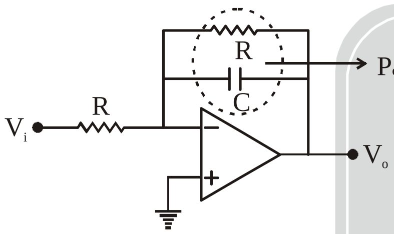
↓

negative feedback only

↓

output to input connection

Converting circuit to a practical integrator, connecting "Resistor" parallel to capacitor



Parallel "Z" $= \frac{R \times \frac{1}{CS}}{R + \frac{1}{CS}} \Rightarrow \frac{R}{1 + RCS}$

$$\frac{V_0}{V_i} = \frac{-R}{R(1+RCS)} = \frac{-Z_1}{R}$$

$$\frac{V_0}{V_i} = \frac{-1}{1+RCS}$$

$$\frac{V_0}{V_i}(s) = \frac{-1}{1+RCS} \qquad \text{Put } s = j\omega$$

$$\frac{V_0}{V_i}(j\omega) = \frac{-1}{1+j\omega RC} \qquad \text{Considering } \omega RC >> 1$$

$$\frac{V_0}{V_i}(j\omega) = \frac{-1}{j\omega RC}$$

Gain in magnitude $= \frac{1}{\omega RC}$

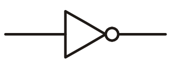
Concept : Given circuit is ideal LPF / Integrator

if we connect "R" ●—/\/\/\/—● in parallel to "Capacitor" it becomes practical integrator with gain $\frac{1}{\omega RC}$ and "−" ve sign due to inverting OPAMP.

Correct option is (d)

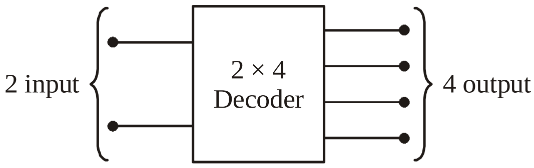
44. 2 × 4, Decoder

Enable - input means inverter/NOT gate

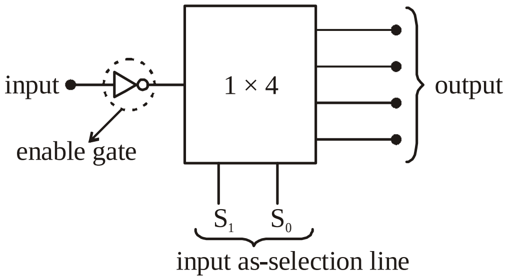


Enable input

Decoder $= n \times 2^n$



If we are considering input as selection line



Format DEMUX/demultiplexer

$1 \times 2^n \to$ n; selection line

Above - CKT is 1 × 4 Demux

Correct option is (b)

PART - C

45. $z = 3y - 2x$

$$\therefore \quad \Delta z = \sqrt{\left(\frac{\partial z}{\partial x}\Delta x\right)^2 + \left(\frac{\partial z}{\partial y}\Delta y\right)^2} = \sqrt{(-2\times 0.05)^2 + (3\times 0.02)^2} = \sqrt{0.0136} \simeq 0.12$$

Correct option is (a)

46. $\frac{d^2}{dx^2} g(x, x_0) = \delta(x - x_0)$

Since, second order derivative of the Green's function is related to Dirac Delta function, therefore first order derivative of the Green's function should be discontinuous at $x = x_0$. So, solution given in option (d) is incorrect.

Here, the strength of the Dirac-Delta function $\delta(x = x_0)$ is equal to 1. So, the amount of discontinuity of the first order derivative of Green's function will also be equal to 1. Only the solution given in option (a) satisfies this condition.

Correct option is (a)

47. Paulli spin matrices :

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$x'\sigma_x + y'\sigma_y + z'\sigma_z = \exp\left(i\frac{\theta}{2}\sigma_z\right)\left[x\sigma_x + y\sigma_y + z\sigma_z\right]\exp\left(-i\frac{\theta}{2}\sigma_z\right)$$

$$\Rightarrow \quad \begin{pmatrix} z' & x'-iy' \\ x'+iy' & -z' \end{pmatrix} = \begin{pmatrix} e^{i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{pmatrix}\begin{pmatrix} x & x-iy \\ x+iy & -z \end{pmatrix}\begin{pmatrix} e^{i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\theta/2} & 0 \\ 0 & e^{-i\theta/2} \end{pmatrix} \begin{pmatrix} e^{-i\theta/2\cdot z} & e^{i\theta/2\cdot(x-iy)} \\ e^{-i\theta/2}\cdot(x+iy) & -z\cdot e^{i\theta/2} \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} z' & x'-iy' \\ x'+iy' & -z' \end{pmatrix} = \begin{pmatrix} z & e^{i\theta}\cdot(x-iy) \\ e^{-i\theta}\cdot(x+iy) & z \end{pmatrix}$$

Comparing the matrix elements on the both sides,

$$z' = z$$

$$x' - iy' = e^{i\theta}(x-iy)$$

$$= (\cos\theta + i\sin\theta)(x-iy) = (x\cos\theta + y\sin\theta) - i(-x\sin\theta + y\cos\theta)$$

$$\Rightarrow \quad x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

So, $$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Correct option is (b)

48. **Exact Method:**

$$\int_0^1 e^{i(2\pi x)}dx = \left[\frac{e^{i2\pi x}}{i(2\pi)}\right]_0^1 = \frac{1}{2\pi i}\left(e^{i2\pi} - 1\right) = 0$$

Simpson's 1/3[rd] rule : (Taking $n = 2$, $2n = 4$)

Length of sub-intervals, $h = \frac{(b-a)}{2n} = \frac{(1-0)}{4} = \frac{1}{4}$

x:	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(x) = e^{i2\pi x}$:	$1 = y_0$	$i = y_1$	$-1 = y_2$	$-i = y_3$	$1 = y_4$

$$\int_0^1 e^{i(2\pi x)}dx = \frac{h}{3}\left[(y_0 + y_4) + 2(y_2) + 4(y_1 + y_3)\right]$$

$$= \frac{1}{12}\left[(1+1) + 2(-1) + 4(i-i)\right] = 0$$

Correct option is (b)

49. **Closure properties of a group:**

For any two elements $A, B \in G$, their composition A×B also belongs to the group G.

Take two matrices of the format given is option (c) i.e.

$$A = \begin{pmatrix} 1 & 0 & a_1 \\ 0 & 1 & b_1 \\ 0 & 0 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 0 & a_2 \\ 0 & 1 & b_2 \\ 0 & 0 & 1 \end{pmatrix}$$

Therefore, $A\times B=\begin{pmatrix}1 & 0 & a_1+a_2\\ 0 & 1 & b_1+b_2\\ 0 & 0 & 1\end{pmatrix}$

Will also be of same format i.e. $A\times B\in G$.
None of the other option satisfy closure property.
Correct option is (c)

50. Potential energy : $V=-\int F dx=-Fx$

$$L=-m\sqrt{1-\dot{x}^2}-V \Rightarrow L=-m\sqrt{1-\dot{x}^2}+Fx$$

Corresponding Hamiltonian is

$$H=\sqrt{p_x^2+m^2}-Fx$$

H is equal to energy

$$E=\sqrt{p_x^2+m^2}-Fx$$

$\therefore$ $$(E+Fx)^2=p_x^2+m^2$$

$$(E+Fx)^2-p_x^2=m^2$$

This is equation of hyperbola in x-p_x plane.
Correct option is (c)

51. In canonical transformation coordinate and momenta at one instant of time are transformed into new coordinate and momenta. Further, if $q,p\to Q,P$ be a canonical transformation then Poisson bracket $\{Q,P\}_{q,p}=1$ i.e. $\frac{\partial Q}{\partial q}\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q}=1$

This condition is satisfied only if we take

$$Q=q(t+\tau)+p(t+\tau) \text{ and } P=\frac{1}{2}\left[p(t+\tau)-q(t+\tau)\right]$$

Correct option is (d)

52. $V(x)=\frac{1}{2}kx^{2n}$

Energy , $E=\frac{1}{2}m\left(\frac{dx}{dt}\right)^2+V(x)$

$\therefore$ $$dt=\sqrt{\frac{m}{2}}\frac{dx}{\sqrt{E-V(x)}} \Rightarrow dt=\sqrt{\frac{m}{2}}E^{-1/2}\frac{dx}{\sqrt{1-\frac{V}{E}}}$$

Put $\frac{V}{E}=y$ (dimensionless variable)

$$\frac{1}{2}kx^{2n} = yE \Rightarrow x = \left(\frac{2yE}{k}\right)^{1/2n}$$

$$\therefore \quad dx = \left(\frac{2E}{k}\right)^{1/2n} \frac{1}{2n} y^{\frac{1}{2n}-1}$$

$$\therefore \quad dt \propto E^{-1/2}\left(\frac{E}{k}\right)^{1/2n} \Rightarrow dt \propto k^{\frac{-1}{2n}} E^{\frac{1-n}{2n}}$$

Correct option is (b)

53. Radiative power due to accelerated charge particle

$$P = \frac{dE}{dt} = \frac{\mu_0 q^2 a^2}{6\pi c} \qquad v_t = u - a\tau \Rightarrow \frac{u}{2} = u - a\tau \Rightarrow \tau = \frac{u}{2a}$$

$$\Rightarrow E = \frac{\mu_0 q^2 a^2}{6\pi c}\tau \qquad v_t^2 = u^2 - 2as \Rightarrow 2as = u^2 - \frac{u^2}{4} = \frac{3u^2}{4} \Rightarrow a = \frac{3u^2}{8s}$$

$$\therefore \quad E = \frac{u_0 e^2 a^2}{6\pi c} \times \frac{u}{2a} = \frac{\mu_0 e^2 ua}{12\pi c} = \frac{\mu_0 e^2 3u^3}{12\pi c \times 8s}$$

$$E = \frac{\mu_0 e^2 u^3}{32\pi cs}$$

The fraction of energy loss due to radiation

$$= \frac{\dfrac{\mu_0 e^2 u^3}{32\pi cs}}{\dfrac{1}{2}mu^2} = \frac{\mu_0 e^2 u}{16\pi csm}$$

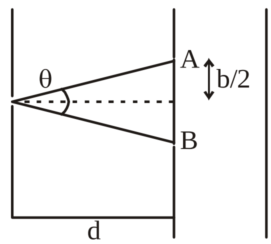
Correct option is (d)

54. The screen will be dark if both the points A and B is dark.

$\therefore \quad a\sin\theta = \lambda$ (for 1st order minimum)

$$\sin\theta = \frac{\lambda}{a}$$

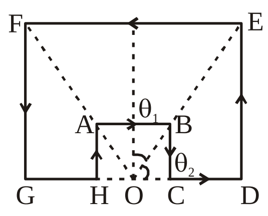
$$\Rightarrow \frac{b/2}{\sqrt{d^2 + b^2/4}} = \frac{\lambda}{a} \Rightarrow d = \frac{b}{2}\sqrt{\left(\frac{a}{\lambda}\right)^2 - 1}$$



Correct option is (b)

55. The magnetic field due to EF arm

$$B_{EF} = \frac{\mu_0 I}{2\pi r_1}\sin\theta_1 = \frac{\mu_0 i}{4\pi b}\frac{1}{\sqrt{5}}\uparrow \qquad \left[\sin\theta_1 = \frac{b}{\sqrt{b^2 + 4b^2}} = \frac{1}{\sqrt{5}}\right]$$



The magnetic field due to AB arm

$$B_{AB} = \frac{\mu_0 I}{2\pi r_2}\sin\theta_1 = \frac{\mu_0 I}{4\pi a}\frac{1}{\sqrt{5}}\downarrow$$

The magnetic field due to $\overline{BC}$

$$B_{BC} = \frac{\mu_0 I}{4\pi r_2}\sin\theta_2 = \frac{\mu_0 i}{4\pi a}\frac{2}{\sqrt{5}} = \frac{\mu_0 I}{2\pi a}\frac{1}{\sqrt{5}}\downarrow \qquad \left[\text{since}, \sin\theta_2 = \frac{2a}{\sqrt{4a^2+a^2}} = \frac{2}{\sqrt{5}}\right]$$

The magnetic field due to $\bar{H}A$

$$B_{HA} = \frac{\mu_0 I}{2\pi a}\frac{1}{\sqrt{5}}\downarrow$$

The magnetic field due to $\overline{DE}$

$$B_{DE} = \frac{\mu_0 I}{4\pi r_3}\sin\theta_2 = \frac{\mu_0 I}{2\pi b\sqrt{5}}\uparrow$$

The magnetic field due to $\overline{FG}$

$$B_{FG} = \frac{\mu_0 I}{2\pi b\sqrt{5}}\uparrow$$

So total magnetic field

$$\bar{B} = B_{DE} + B_{EF} + B_{GF} - B_{HA} - B_{AB} - B_{BC}$$

$$= \frac{\mu_0 I\sqrt{5}}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$

56. Given potential : $V(r) = V_0 a^3 \sum_i \delta^{(3)}(\bar{r} - \bar{r}_i)$

Scattering amplitude :

$$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\langle\phi|\hat{V}|\psi\rangle = -\frac{m}{2\pi\hbar^2}V_0 a^3 \int e^{i\bar{r}(\vec{k}-\vec{k}_0)}\sum_{i=1}^{8}\delta^{(3)}(\bar{r}-\bar{r}_i)\,d^3r$$

$$= -\frac{m}{2\pi\hbar^2}V_0 a^3 \int \sum_{i=1}^{8}\delta^{(3)}(\bar{r}-\bar{r}_i)\,d^3r$$

(for low energy limit $(\vec{k}-\vec{k}_0) \simeq 0$)

$$= -\frac{mV_0 a^3}{2\pi\hbar^2}\times 8$$

$$\therefore \qquad \frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2 = \frac{16a^6 m^2 V_0^2}{\pi^2\hbar^4}$$

$$\Rightarrow \qquad \sigma = \int\frac{d\sigma}{d\Omega}d\Omega = \frac{16a^6 m^2 V_0^2}{\pi^2\hbar^4}\int d\Omega = \frac{16a^6 m^2 V_0^2}{\pi^2\hbar^4}4\pi = \frac{64a^6 m^2 V_0^2}{\hbar^4\pi} = \frac{64a^2}{\pi}\left(\frac{mV_0 a^2}{\hbar^2}\right)^2$$

Correct option is (c)

57. The first order energy correction term of the ground state is given by

$$=\langle\psi_0|H'|\psi_0\rangle$$

$$=\int_0^\infty\int_0^\pi\int_0^{2\pi}\frac{1}{\pi a_0^3}e^{-\frac{2r}{a_0}}br^2\sin^2\theta\cos^2\varphi r^2\sin\theta drd\theta d\varphi$$

$$=\frac{b}{\pi a_0^3}\int_0^\infty e^{-\frac{2r}{a_0}}r^4dr\int_0^\pi\sin^3\theta d\theta\int_0^{2\pi}\cos^2\varphi d\varphi \qquad \left[\frac{2r}{a_0}=z,\ dr=dz\frac{a_0}{2}\right]$$

$$=\frac{b}{\pi a_0^3}\frac{4}{3}\times\pi\times\int_0^\infty e^{-\frac{2r}{a_0}}r^4dr=\frac{4b}{3a_0^3}\times\left(\frac{a}{2}\right)^5\int_0^\infty e^{-z}z^4dz=\frac{4}{3}\frac{b}{a_0^3}\times\frac{a^5}{2^5}\times 4!=ba_0^2$$

Correct option is (b)

58. We have trial function,

$$\psi(x)=\begin{cases}A\left(a^2-x^2\right) & \text{for } -a<x<a\\ 0 & \text{otherwise}\end{cases}$$

According to normalized condition, $\int_{-a}^{a}|\psi|^2dx=1$

$$\Rightarrow \quad |A|^2\int_{-a}^{a}\left(a^2-x^2\right)^2dx=1\Rightarrow|A|^2\int_{-a}^{a}\left(a^4+x^4-2a^2x^2\right)dx=1$$

$$\Rightarrow \quad |A|^2\left[2a^5+\frac{2a^5}{5}-\frac{4a^5}{3}\right]=1\Rightarrow|A|^2\cdot\frac{16a^5}{15}=1\Rightarrow|A|=\sqrt{\frac{15}{16a^5}}$$

$$\therefore \quad \psi=\sqrt{\frac{15}{16a^5}}\left(a^2-x^2\right)\text{ for } -a<x<a$$

The energy is given by

$$E=\langle\psi|H|\psi\rangle=\frac{15}{16a^5}\int_{-a}^{a}\left[a^2-x^2\left(-\frac{\hbar^2}{2m}\right)\frac{d}{dx^2}\left(a^2-x^2\right)+\frac{1}{2}m\omega^2x^2\left(a^2-x^2\right)^2\right]dx$$

$$=\frac{15}{16a^5}\int_{-a}^{a}\frac{\hbar^2}{2m}\left(2a^2-2x^2\right)+\frac{1}{2}m\omega^2\left(a^4x^2+x^6-2a^2x^4\right)dx$$

$$=\frac{15}{16a^5}\left[\frac{\hbar^2}{2m}\left(4a^3-\frac{4a^3}{3}\right)+\frac{1}{2}m\omega^2\left(\frac{2a^7}{3}+\frac{x^7}{7}-\frac{2a^2x^5}{5}\right)\right]$$

$$=\frac{15}{16a^5}\left[\frac{4\hbar^2a^3}{3m}+\frac{1}{2}m\omega^2\left(\frac{70+30-84}{105}\right)a^7\right]$$

$$=\frac{15}{16a^5}\left[\frac{4\hbar^2a^3}{3m}+\frac{1}{2}m\omega^2\frac{16a^7}{105}\right]=\frac{15}{16}\left[\frac{4\hbar^2}{3ma^2}+\frac{8m\omega^2}{105}a^2\right]$$

for minimum energy, $\frac{dE}{da}=0 \Rightarrow -\frac{8\hbar^2}{3ma^3}+\frac{16m\omega^2}{105}a=0$

$$\Rightarrow \quad a^4=\frac{8\hbar^2}{3m}\times\frac{105}{16m\omega^2} \Rightarrow a^4=\frac{35\hbar^2}{2m^2\omega^2} \Rightarrow a^2=\sqrt{\frac{35\hbar^2}{2m^2\omega^2}}=\sqrt{\frac{35}{2}}\left(\frac{\hbar}{m\omega}\right)$$

Therefore, ground state energy,

$$=\frac{15}{16}\left[\frac{4\hbar^2}{3m}\frac{m\omega}{\hbar}\sqrt{\frac{2}{35}}+\frac{8m\omega^2}{105}\times\sqrt{\frac{35}{2}}\frac{\hbar}{m\omega}\right]$$

$$=\frac{15}{16}\left[\frac{4}{3}\sqrt{\frac{2}{35}}+\frac{8}{105}\sqrt{\frac{35}{2}}\right]\hbar\omega=\frac{4}{3}\cdot\frac{15}{16}\left[\sqrt{\frac{2}{35}}+\frac{2}{35}\sqrt{\frac{35}{2}}\right]\hbar\omega$$

$$=\frac{4}{3}\times\frac{15}{16}\times\sqrt{\frac{2}{35}}\times\hbar\omega\times 2=\sqrt{\frac{25\times 2\times 4}{16\times 35}}\times\hbar\omega=\sqrt{\frac{5}{14}}\hbar\omega$$

Correct option is (b)

59. Rate of spontaneous emission (in dipole approx.)

$$\Gamma_{if}=\frac{4\omega^3\alpha}{3c^2}\left|\langle\psi_f|\vec{r}|\psi_i\rangle\right|^2 \qquad \text{where, } \omega=\left(E_i-E_f\right)/\hbar,\ \alpha = \text{fine structure constant}$$

Mean lifetime of the decaying state is $t=\frac{1}{\Gamma_{if}}$

Note : There is no need of solving the integration for θ,ϕ part as values of ℓ, m_ℓ for the initial and final state are same for both transitions

Transition 1 : $|210\rangle \rightarrow |100\rangle$

$$\left|\langle\psi_f|\vec{r}|-\psi_i\rangle\right|=\langle R_{10}|r|R_{21}\rangle=\int_0^\infty\left[\frac{2}{\sqrt{a_0^3}}e^{-r/a_0}\right]r\left[\frac{1}{\sqrt{6a_0^3}}\frac{r}{2a_0}e^{-r/2a_0}\right]\left(r^2dr\right)$$

$$=\frac{1}{\sqrt{6}\,a_0^4}\int_0^\infty e^{-3r/2a_0}\cdot r^4dr=\frac{1}{\sqrt{6}\cdot a_0^4}\cdot\frac{4!}{\left(3/2a_0\right)^5}=\frac{2^8}{3^4\sqrt{6}}a_0$$

$$\omega_1=\left(E_{210}-E_{100}\right)/\hbar=-\frac{3}{4\hbar}E_{100}$$

Transition 2 : $|310\rangle \rightarrow |100\rangle$

$$\left|\langle\psi_f|\vec{r}|-\psi_i\rangle\right|=\langle R_{10}|r|R_{31}\rangle=\int_0^\infty\left[\frac{2}{\sqrt{a_0^3}}e^{-r/a_0}\right]r\left[\frac{8}{9\sqrt{6a_0^3}}\left(1-\frac{r}{6a_0}\right)\left(\frac{T}{3a_0}\right)e^{-r/2a_0}\right]\left(r^2dr\right)$$

$$=\frac{16}{27\sqrt{6}a_0^4}\int_0^\infty e^{-4r/3a_0}r^4dr-\frac{16}{\sqrt{6}\,162a_0^5}\int_0^\infty e^{-4r/3a_0}r^5dr$$

$$=\frac{16}{27\sqrt{6}\,a_0^4}\frac{4!}{\left(\frac{4}{3}a_0\right)^5}-\frac{16}{162\sqrt{6}\,a_0^5}\frac{5!}{\left(\frac{4}{3}a_0\right)^6}$$

$$=\frac{16\times 24\times 3^5}{27\times 4^5\times\sqrt{6}}a_0-\frac{16\times 120\times 3^6}{162\times 4^6\times\sqrt{6}}a_0=\frac{3^3}{2^3\sqrt{6}}a_0-\frac{135}{64}a_0=\frac{3^4}{4^3\sqrt{6}}a_0$$

$$\omega_2=\left(E_{310}-E_{100}\right)/\hbar=-\frac{8}{9\hbar}E_{100}$$

So, $$\frac{t_1}{t_2}-\infty\frac{\Gamma_{310\to 100}}{\Gamma_{210\to 100}}\infty\frac{(\omega_2)^3}{(\omega_1)^3}\frac{\left|\langle\psi_{100}|\vec{r}|-y_{310}\rangle\right|^2}{\left|\langle\psi_{100}|\vec{r}|-y_{210}\rangle\right|^2}\infty\frac{\left(8^3/9^3\right)}{\left(3^3/4^3\right)}\times\frac{\left(\left(3^8/4^6\right)\times 6\right)a_0^2}{\left(\left(2^{16}/3^8\right)\times 6\right)a_0^2}$$

$$\Rightarrow\quad \frac{t_1}{t_2}\infty\frac{4^3\times 2^3\times 3^8\times 4^3\times 3^8}{3^3\times 3^3\times 4^6\times 3^3\times 2^{16}}\infty\frac{3^7}{2^{13}}$$

60. For Poission distribution, probability the 'r' number of successes will occur, is

$$P(r)=\frac{e^{-\langle r\rangle}\cdot\langle r\rangle^r}{r!}$$

where, r = number of successes (Random variable n)

$$\Rightarrow\quad P(n=r=0)=\frac{e^{-\langle n\rangle}\cdot\langle n\rangle^0}{0!}=10^{-6}$$

$$\Rightarrow\quad -\langle n\rangle\ln(e)=-6\cdot\ln(10)$$

$$\Rightarrow\quad \langle n\rangle=6\times(2.303)\simeq 14$$

Correct option is (a)

61. If the energy is proportional to n^{th} power of momentum k, i.e.

$$E\propto k^n$$

Then, $\bar{p}=\frac{n}{3}\in=\frac{n}{3}\frac{E}{V}$ where, E is the energy and $\in$ is the energy density

Since, $E\propto k^3\,(given), n=3$

$\therefore\quad \bar{p}=\frac{3}{3}\in$ or $\frac{\bar{p}}{\in}=1$

Correct option is (c)

62. The hamiltonian is,

$$H=-jS_0\left(S_1+S_2\right);\ S_0, S_1, S_2=\pm 1$$

The possible values of Hamiltonian are

S_0	S_1	S_2	$S_0(S_1+S_2)$	H
1	1	1	2	$-2j$
1	1	-1	0	0
1	-1	1	0	0
-1	1	1	-2	$+2j$
1	-1	-1	-2	$+2j$
-1	1	-1	0	0
-1	-1	1	0	0
-1	-1	-1	$+2$	$-2j$

The average energy is,

$$\langle H\rangle = \frac{\sum H_i e^{-\beta H_i}}{\sum e^{-\beta H_i}} = \frac{-4j\, e^{2\beta j} + 4\, j\, e^{-2\beta j}}{2\, e^{2\beta j} + 2\, e^{-2\beta j} + 4} = \frac{-2j\left(e^{+2\beta j} - e^{-2\beta j}\right)}{2 + e^{2\beta j} + e^{-2\beta j}}$$

$$= \frac{-4j\sinh(2\beta j)}{2 + 2\cosh(2\beta j)} = -2j\frac{\sinh(2\beta j)}{1+\cosh(2\beta j)}$$

Correct option is (d)

63. $$I_F = I_0\left(e^{\frac{V_F}{\eta V_T}} - 1\right)$$

Forward $$R_F = \frac{V_F}{I_F} = \frac{V_F}{I_0\left(e^{\frac{V}{\eta V_T}} - 1\right)}$$

Reverse $$R_R = \frac{V_R}{I_0}$$ → Reverse Saturation current

$$\frac{R_R}{R_F} = \frac{V_R}{I_0} \times \frac{I_0\left(e^{\frac{V_F}{\eta V_T}} - 1\right)}{V_F}$$

Put value $$\frac{R_R}{R_F} = \frac{V_R}{V_F}\cdot\left(e^{\frac{V_F}{\eta V_T}} - 1\right)$$

Thermal voltage → $V_T = \frac{KT}{q}$

$e^{\frac{V}{\eta V_T}} >> 1$ For forward bias

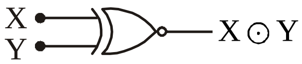
$$\frac{R_R}{R_F} = \frac{V_R}{V_F}\cdot e^{\frac{qV_F}{\eta kT}}$$

Correct option is (b)

64. Given 1-bit comparator

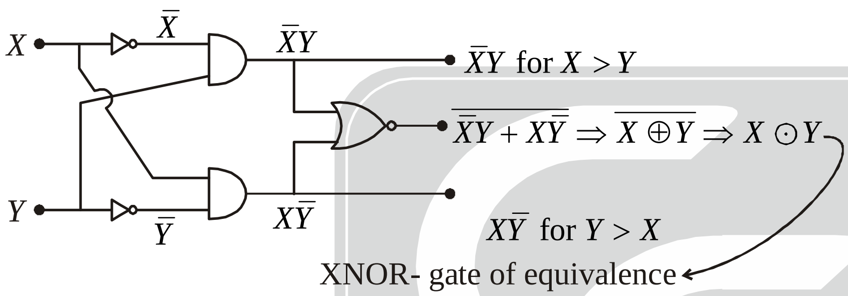
Input X	Input Y	Possible output $X>Y$	$X=Y$	$X<Y$
0	0	0	1	0
0	1	0	0	$1 \rightarrow X\bar{Y}$
1	0	1	0	0
1	1	0	1	0

$X>Y$: $\bar{X}Y$ $X=Y$: $\bar{X}\bar{Y}+XY$, $X\odot Y$



Gate of equivalence

CKT - (3)



This circuit is satisfying above truth table

Correct option is (c)

65. According to Ohm's law:

$$V = IR \quad \Rightarrow \quad R = \frac{V}{I} = \frac{25}{1} = 25\,\Omega$$

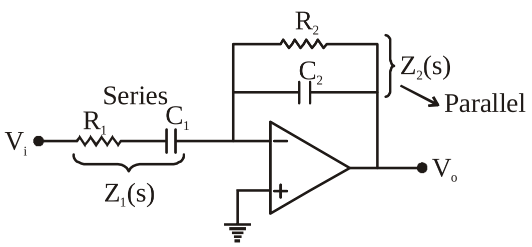
Let S is the slope of given graph

$$\therefore \quad R = \frac{1}{S} \Rightarrow \frac{\Delta R}{R} = \frac{\Delta S}{S} \Rightarrow \Delta R = \frac{R}{S} \times \Delta S = 25 \times 25 \times 1.255 \times 10^{-3} = 0.8\,\Omega$$

$$\therefore \quad R = R \pm \Delta R = (25 \pm 0.8)\,\Omega$$

Correct option is (b)

66.



$$Z_1(s) = R_1 + \frac{1}{C_1 S} \Rightarrow \frac{R_1 C_1 S + 1}{C_1 S}$$

Put value

↓

$$Z_1(j\omega) = \frac{R_1 C_1 j\omega + 1}{j\omega C_1}$$

$$Z_1(j\omega) = \frac{20\times10^3\times10\times10^{-9}\times j\times2\pi\times16\times10^3+1}{j\times2\pi\times16\times10^3\times10\times10^{-9}}$$

$$Z_1(j\omega) = 20000 - 994.72\, j$$

Magnitude = 20024.72

$$Z_2(s) = \frac{R_2\times\frac{1}{C_2S}}{R_2+\frac{1}{C_2S}} \qquad \text{"Parallel"}$$

$$Z_2(s) = \frac{R_2}{R_2C_2S+1} \qquad \text{Put value } S = j\omega$$

$$Z_2(j\omega) = \frac{R_2}{R_2C_2\,j\omega+1} \quad \text{Put } R_2 \text{ and } C_2 \text{ value}$$

$$Z_2(j\omega) = \frac{200\times10^3}{200\times10^3\times100\times10^{-12}\times j\times2\pi\times16\times10^3+1}$$

$$Z_2(j\omega) = 39662.158 - 79745.5\, j$$

Magnitude = 89064.199

Gain = $\frac{V_0}{V_i} = \frac{Z_2}{Z_1} = \frac{89064.2}{20024.72} = 4.44$

Go for nearest value

All option are incorrect

Correct option is (d)

67. The upper state term is $^3P_2$, that is J = 2 and M_J = –2, –1, 0, 1, 2

There are 9 Zeeman transition, according to the selection rule

$$\Delta M_J = 0 \pm 1 \left(M_J = 0 \not\leftrightarrow M_J = 0 \;\; if \;\; \Delta_J = 0\right)$$

The lower state must split into three components, i.e.

$$M_J = 1, 0, -1 \Rightarrow J = 1$$

the multiplicity selection rule, $\Delta S = 0$ demands the lower state term also, to be a triplet that is, S=1

The values J = 1 and S = 1 lead L = 1 i.e. lower state is

$^3P_1$

Correct option is (c)

68. We know that, $\frac{A_{21}}{B_{21}} = 8\pi h\left(\frac{v}{c}\right)^3$

where, A_{21} = Einstein coefficient of spotaneous emission of radiation

B_{21} = Eienstein coefficient of stimulated emission radiation

Also, spontaneous life time of the upper level is $t_{sp} = \frac{1}{A_{21}}$

Relation between Einstein coefficient is

$$\frac{A_{21}}{B_{21}} = 8\pi h\left(\frac{v}{c}\right)^3$$

$$\Rightarrow \quad t_{sp} = \frac{1}{A_{21}} = \frac{1}{8\pi hB_{21}}\left(\frac{c}{v}\right)^3 = \frac{\lambda^3}{8\pi hB_{21}}$$

$$\Rightarrow \quad t_{sp} = \frac{\left(3000\times10^{-10}\right)^3}{8\times3.14\times2.1\times10^{19}\times6.062\times10^{-34}} \simeq 80\ ns$$

Correct option is (c)

69. The kinetic energy of the Auger electron will be

$$K.E. = (E_K - E_L) - E_L = \left[(25.4-3.34)-3.34\right] KeV = 18.7\ KeV$$

Correct option is (d)

70. We know the the k_{max} in 3D solid is given by

$$k_{\max} = \sqrt{3}\,\frac{\pi}{a} = \frac{\sqrt{3}}{5.2}\times3.14\mathring{A}^{-1} = 0.6038\mathring{A}^{-1}\times\sqrt{3}$$

And the wave vector of electron,

$$k_e = \frac{2\pi}{\lambda} = \frac{2\pi\times E_g\,(eV)}{12400}\,\mathring{A}^{-1} = \frac{2\times3.14\times1.875}{12400} = 9.495\times10^{-4}\ \mathring{A}^{-1}$$

$$\therefore \quad \frac{k_e}{k_{\max}} = \frac{9.495\times10^{-4}}{0.6038\times\sqrt{3}} \simeq 10^{-3} \Rightarrow k_e = \frac{k_{\max}}{1000}$$

Correct option is (c)

71. According to the Drude-Model, the conductivity

$$\sigma = \frac{n\,e^2 e}{m}$$

Therefore, the mean collision time,

$\tau \propto \frac{\sigma}{n}$, where σ = conductivity and n = electron density

$$\therefore \quad \frac{\tau_{Cu}}{\tau_{Ag}} = \frac{\sigma_{Cu}}{n_{Cu}}\times\frac{n_{Ag}}{\sigma_{Ac}}$$

Given : $\sigma_{Cu} = 95\%$ of $\sigma_{Ag} = 0.95\,\sigma_{Ag}$

$n_{Ag} = 70\%$ of $n_{Cu} = 0.70\,n_{Cu}$

$$\frac{\tau_{Cu}}{\tau_{Ag}} = \frac{\sigma_{Cu}}{\sigma_{Ag}}\times\frac{n_{Ag}}{n_{Cu}} = 0.95\times0.70 = 0.66$$

Correct option is (d)

72. We know equation of continuety

$$\bar{\nabla}\cdot\bar{J} + \frac{d\rho}{dt} = 0\ ;$$

$$\bar{\nabla}\cdot\left(\sigma\bar{E}\right) + \frac{d\rho}{dt} = 0$$

$$\sigma\left(\bar{\nabla}\cdot\bar{E}\right)+\frac{d\rho}{dt}=0$$

$$\frac{\sigma\rho}{\varepsilon_0}+\frac{d\rho}{dt}=0 \Rightarrow \int\frac{d\rho}{\rho}=-\frac{\sigma}{\varepsilon_0}\int dt$$

$$\ln\rho=-\frac{\sigma}{\varepsilon_0}t+c$$

$$t=0,\ \rho(r,0)=\rho_0$$

$$\ln\left(\frac{\rho}{\rho_0}\right)=-\frac{\sigma}{\varepsilon_0}t$$

$$\rho=\rho_0 e^{-\frac{\sigma}{\varepsilon_0}t}$$

$$\rho(r,t)=\rho(r,0)e^{-\frac{\sigma}{\varepsilon_0}t}$$

Correct option is (a)

73. In spontaneous α-decay

$$Q=E_\alpha\frac{A}{A-4}$$

Here, A = 232, atomic mass number

$$\Rightarrow \quad Q=E_\alpha\frac{232}{232-4}=E_\alpha\frac{232}{228}$$

$$\Rightarrow \quad E_\alpha=\frac{228}{232}Q=\frac{57}{58}Q$$

Correct option is (a)

74. Range $R=\frac{\hbar C}{mC^2} \Rightarrow R\propto\frac{1}{m}$

$$\frac{R_\pi}{R_{rho}}=\frac{m_{rho}}{m_\pi} \Rightarrow \frac{1.40}{R_{rho}}=\frac{770}{14}$$

$$\Rightarrow \quad R_{rho}=\frac{140\times1.40}{770}=0.25\,fm$$

Correct option is (c)

75. $X\rightarrow\Sigma^{+}+\pi^{-}+\pi^{0}$

$I_3\rightarrow+1-1+0 \Rightarrow I_3=0$

Correct option is (a)