SECTION-A

1. ***This question consists of FIFTEEN sub-questions (1.1 to 1.15) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.***
[15×1 = 15]

1.1. A square matrix A is unitary if:

(a) $A^\dagger = A$ (b) $A^\dagger = A^{-1}$ (c) Tr (A) = 1 (d) det (A) = 1

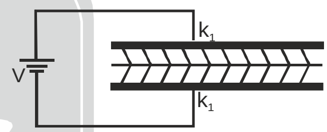
1.2. A planet moves around the Sun in an elliptical orbit with semi-major axis a and time period T. T is proportional to

(a) a^2 (b) $a^{1/2}$ (c) $a^{3/2}$ (d) a^3

1.3. A particle moves in a central force field $\vec{f} = -kr^n\,\hat{r}$, where k is a constant, r, the distance of the particle from the origin and $\hat{r}$ is the unit vector in the direction of position vector $\vec{r}$. Closed stable orbits are possible for:

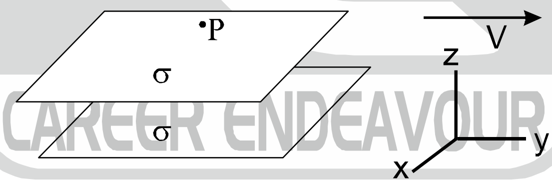
(a) n = 1 and n = 2 (b) n = 1 and n = –1 (c) n = 2 and n = –2 (d) n = 1 and n = –2

1.4. The space between the plates of a parallel plate capacitor is filled with two dielectric slabs of dielectric constants k_1 and k_2 as shown in the figure. If the capacitor is charged to a potential V, then at the interface between the two dielectrics.



(a) $\vec{E}$ is discontinuous and $\vec{D}$ is continuous
(b) $\vec{E}$ is discontinuous and $\vec{D}$ is discontinuous
(c) $\vec{E}$ is continuous and $\vec{D}$ is continuous
(d) $\vec{E}$ is continuous and $\vec{D}$ is discontinous

1.5. Two large parallel plates move with a constant speed v in the positive y-direction as shown in the figure. If both the plates have a surface charge density $\sigma > 0$, the magnetic field at the point P just above the top plate will have :

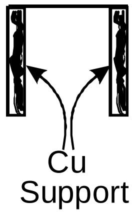


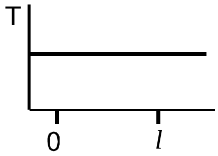
(a) Larger magnitude than the field at the mid-point between the plates and point towards $-\hat{x}$
(b) Smaller magnitude than the field at the mid-point between the plates and point towards $+\hat{x}$.
(c) Larger magnitude than the field at the mid-point between the plates and point towards $+\hat{x}$
(d) Smaller magnitude than the field at the mid-point between the plates and point towards $-\hat{x}$

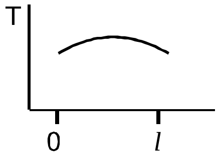
1.6. Which of the following is an example of a first order phase transition?

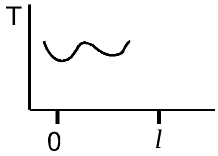
(a) A liquid - gas phase transition at the critical point.
(b) A paramagnet - ferromagnet phase transition
(c) A normal metal - superconductor phase transition.
(d) A liquid - gas phase transition away from the critical point.

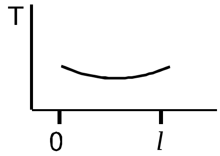
1.7. A tungsten wire of uniform cross section and high resistance is supported by two copper supports of low resistance in vacuum. The length of the wire is l. A constant current I is sent through the wire. The temperature profile (T vs. x) of the wire will look like.



(a) 

(b) 

(c) 

(d) 

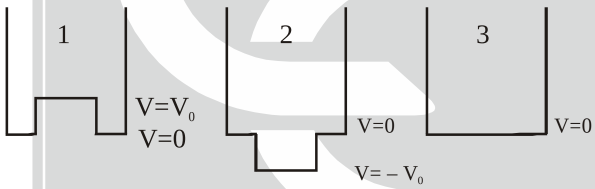
1.8. The energy E of K_α X-rays emitted from targets of different atomic number Z varies as

(a) Z^2 (b) $Z^{2/3}$ (c) Z (d) $Z^{1/2}$

1.9. In a Stern-Gerlach experiment the atomic beam whose angular momentum state is to be determined, must travel through

(a) homogeneous radio frequency magnetic field
(b) homogeneous static magnetic field
(c) inhomogeneous static magnetic field
(d) inhomogeneous radio frequency magnetic field

1.10. Let E_1, E_2, E_3 be the respective ground state energies of the following potentials: Which one of the following is correct?



(a) $E_1 < E_2 < E_3$ (b) $E_3 < E_1 < E_2$ (c) $E_2 < E_3 < E_1$ (d) $E_2 < E_1 < E_3$.

1.11. The mean momentum of a nucleon in a nucleus with mass number A varies as

(a) A (b) A^2 (c) $A^{-2/3}$ (d) $A^{-1/3}$

1.12. A particle is scatterd by a central potential. If the dominant contribution to the scattering is from the p-wave, the differential cross-section is

(a) isotropic (b) proportional to $\cos^2\theta$
(c) proportional to $\cos\theta\cos^2\phi$ (d) proportional to $\sin^2\theta\sin^2\phi$

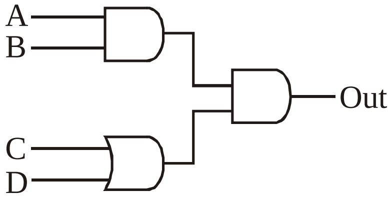
1.13. Magnons in ferromagnets

(a) decrease the magnetization (b) increase the magnetization
(c) stabilize the magnetization (d) cause critical magnetic fluctuations

1.14. The Fermi energy of a free electrong as depends on the electron density ρ as

(a) $\rho^{1/3}$ (b) $\rho^{2/3}$ (c) $\rho^{-1/3}$ (d) $\rho^{-2/3}$

1.15. If the output of the logic circuit shown in the figure is 1, the input could be



(a) A = 1, B = 1, C = 1, D = 0 (b) A =1, B = 1, C = 0, D = 0
(c) A = 1, B = 0, C = 1, D = 1 (d) A = 0, B = 1, C = 1, D = 1.

2. ***This question consists of THIRTY sub-questions (2.1 to 2.30) of TWO marks each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.***
[30×2 = 60]

2.1. If Z_1 and Z_2 are complex numbers, which of the following is always true?
(a) $Z_1^* Z_2 + Z_2^* Z_1 = 2 Z_1^* Z_2$ (b) $Z_1^* Z_2 + Z_2^* Z_1 = 2 Z_1 Z_2$
(c) $Z_1^* Z_2 + Z_2^* Z_1 \le 2 |Z_1| \, |Z_2|$ (d) $Z_1^* Z_2 + Z_2^* Z_1 \ge 2 |Z_1| \, |Z_2|$

2.2. The eigenvalues of the matrix $M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ are
(a) 1, 0 (b) 1, 1 (c) 1, 2 (d) 0, 2

2.3. The Dirac delta function $\delta(x)$ can be represented by:
(a) $\frac{1}{\pi}\lim_{N\to 0}\frac{\sin(Nx)}{x}$ (b) $\frac{1}{\pi}\lim_{\varepsilon\to 0+}\frac{\varepsilon}{x^2+\varepsilon^2}$
(c) $\frac{1}{\sqrt{x}}\lim_{\varepsilon\to+\infty}\frac{1}{\sqrt{\varepsilon}}\exp\left(-x^2/\varepsilon\right)$ (d) $\frac{1}{2p}\int_{-\infty}^{+\infty}\exp\left(i\left|x-x^4\right|u\right)du$

2.4. If $\vec{A}(t)$ is a vector of constant magnitude, which of the following is true?
(a) $\frac{d\vec{A}}{dt}=0$ (b) $\frac{d^2\vec{A}}{dt^2}=0$ (c) $\frac{d\vec{A}}{dt}\bullet\vec{A}=0$ (d) $\frac{d\vec{A}}{dt}\times\vec{A}=0$

2.5. A particle constrained to move along the x-axis in a potential $V = kx^2$, is subjected to an external time-dependent force $\vec{f}(t)$. Here k is a constant, x is the distance from the origin, and t the time. At some time T, when the particle has zero velocity $x = 0$, the external force is removed. The particle will then,
(a) execute simple harmonic motion (b) move along $+x$ direction
(c) move along $-x$ direction (d) remain at rest

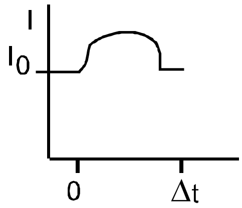
2.6. Consider two particles with position vectors $\bar{r}_1$ and $\bar{r}_2$. The force exerted by particle 2 on particle 1 is, $\bar{f}(\bar{r}_1,\bar{r}_2)=(\dot{r}_2-\dot{r}_1)(\bar{r}_2-\bar{r}_1)$. The force is:
(a) Central and conservative (b) Non-central and conservative
(c) Central and non-conservative (d) Non-central and non-conservative.

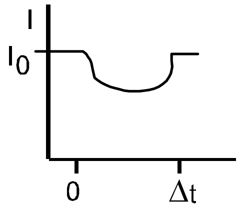
2.7. A closed tall jar containing air and a fly is placed on a sensitive weighing machine. When the fly is stationary, the reading of the weighing machine is W. If the fly starts flying with some upwards acceleration, the reading of the machine will be

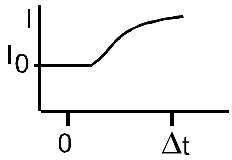
(a) W (b) > W

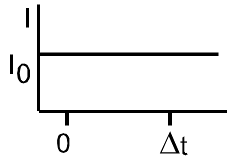
(c) < W (d) directly proportional to the acceleration

2.8. A solenoid with an iron core is connected in series with a battery of emf V and it is found that a constant current I_0 passes through the solenoid. If at t = 0, the iron core is pulled out from the solenoid quickly in a time Δt, which one of the following could be a correct description of the current passing through the solenoid?

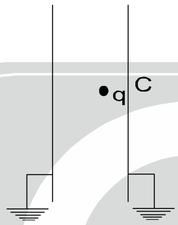
(a) 

(b) 

(c) 

(d) 

2.9. A point charge q is kept at the mid-point between two large parallel grounded conducting plates. Assume no gravity. The charge is displaced a little towards the right plate. The charge will now,



(a) Stay where it is (b) Move towards the right plate
(c) Move towards the left plate (d) Oscillate between the plates

2.10. A very long solenoid with n turns per unit length carries a current I. The magnetic field at a point, which is on its axis and its end face, is

(a) $\mu_0 nI$ (b) $(2/3)\mu_0 nI$ (c) $(1/3)\mu_0 nI$ (d) $(1/22)\mu_0 nI$

2.11. Three plane waves are given as $\bar{y}_1 = A_1\,\hat{y}.\cos(kx - \omega t + \delta_1)$, $\bar{y}_2 = A_2\,\hat{z}\cos(kx - \omega t + \delta_2)$

$\bar{y}_3 = A_3\,\hat{y}\cos(kx - \omega t + \delta_3)$, where $\delta_1, \delta_2, \delta_3$ and A_1, A_2, A_3 are constants. If these waves are superposed pairwise, which superposition will lead to interference?

(a) $\bar{y}_1$ and $\bar{y}_2$ (b) $\bar{y}_2$ and $\bar{y}_3$ (c) $\bar{y}_1$ and $\bar{y}_3$ (d) No interference in any pair

2.12. If a spin 1 particle is in the state $|m = 0\rangle$ with respect to a quantization axis $\hat{n}$, which of the following is correct?

(a) $\langle \vec{S} \rangle = 0$ (b) $\langle \vec{S} \rangle = \hat{n}$ (c) $\langle \vec{S} \rangle = \sqrt{2}\hat{n}$ (d) $\langle \vec{S} \rangle = -\hat{n}$

2.13. Let a particle move in a potential field given by

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \le 0 \end{cases}$$

The allowed energies of this particle are

(a) $(n + 1/2)\hbar\omega$ (b) $(2n + 3/2)\hbar\omega$ (c) $(2n + 1/2)\hbar\omega$ (d) $(n + 5/2)\hbar\omega$

2.14. A spin $\frac{1}{2}$ particle with g>0, is subjected to a magnetic field $\vec{H} = H_0\hat{z}$. If the quantization axis is along $+\hat{z}$, then the minimum energy eigenstate is given by,

(a) $|\downarrow\rangle$ (b) $\frac{1}{\sqrt{2}}|\uparrow\rangle - |\downarrow\rangle$ (c) $|\uparrow\rangle$ (d) $\frac{1}{\sqrt{2}}|\uparrow\rangle + |\downarrow\rangle$

2.15. Bound state eigenfunctions of an attractive finite range smooth potential behave for larger r as:

(a) $\exp(-r/r_0)$ where r_0 is a positive constant. (b) (I/r^n) where n>0

(c) constant (d) $\exp(ikr)/r$

2.16. The order of magnitude of the number of air molecules in a room of volume 50 m^3 at STP is

(a) 10^{27} (b) 10^{24} (c) 10^{30} (d) 10^{20}

2.17. An amount of heat Q is transferred from a heat reservoir at temperature T_A to another heat reservoir at temperature T_B. What is the change in the entropy ΔS of the combined system?

(a) $Q\left(\frac{1}{T_B} - \frac{1}{T_A}\right)$ (b) $Q\left(\frac{1}{T_B} + \frac{1}{T_A}\right)$ (c) $\frac{Q}{\sqrt{T_B T_A}} \ln\frac{T_B}{T_A}$ (d) $Q\left(\frac{1}{T_A} - \frac{1}{T_B}\right)$

2.18. In a canonical ensemble

(a) the energy and the temperature are constants.
(b) the entropy and the energy are constans.
(c) the temperature and the density are constants.
(d) the density and the entropy are constants.

2.19. What is the second nearest-neighbour distance in a face centred cubic lattice whose conventional unit cell parameter is *a*?

(a) $a/\sqrt{2}$ (b) $a/2$ (c) a (d) $\sqrt{2}/a$

2.20. Magnetic long range order is typically exhibited by

(a) noble metals (b) alkali metals (c) inert gas solids (d) transition metals

2.21. X-rays with a wave vector $\vec{K}$ are scattered from a simple cubic lattice with lattice spacing $a = 2\pi$. The scattered X-rays have wave vector $\vec{K}$. The possible values of $\Delta K_1 = K_x - K'_x$ for which there are peaks in the scattered intensity are:

(a) $0 \le \Delta K_1 \le \frac{\pi}{4}$ (b) ΔK_x = integer

(c) ΔK_x = integer multiple of 2π (d) ΔK_x = integer/ 2π

2.22. Under the LS coupling scheme, the possible spectral terms $^{2s+1}L_J$ for the electronic configuration 2s3s are

(a) $^2S_{1/2}, {}^2P_{3/2}, {}^2P_{1/2}$ (b) $^1S_0, {}^3P_1$ (c) $^1S_0, {}^3S_1$ (d) $^3S_0, {}^3S_1$

2.23. Which of the following is the spectroscopic ground state ($^{2s+1}L_J$)(for Mn^{3+} ions of electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^4$ predicted by Hund's rule?

(a) $^5D_0$ (b) $^5D_4$ (c) $^5D_3$ (d) $^5D_2$

2.24. The Bohr model gives the value for the ionisation potential of Li^{2+} ion as

(a) 13.6 eV (b) 27.2 eV (c) 40.8 eV (d) 122.4 eV

2.25. An admissible potential between the proton and the neutron in a deutron is

(a) Coulomb (b) Harmonic oscillator

(c) Finite square potential (d) Infinite square well.

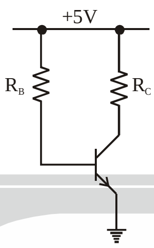
2.26. The deutron is known to be in a state with $S = 1$, $J = 1$, $I = 0$ where S, J, I refer to spin, total angular momentum and isospin quantum numbers respectively. The allowed values of the orbital angular momentum quantum number L are

(a) 0 (b) 0, 1, 2 (c) 1 (d) 0, 2

2.27. Identify the reaction which has the same transition probability as $\pi^+ + p \to \pi^+ + p$

(a) $\pi^+ + n \to \pi^+ + n$ (b) $\pi^- + p \to \pi^0 + n$ (c) $\pi + n \to \pi + n$ (d) $\pi^0 + p \to \pi^+ + n$

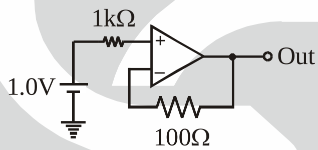
2.28. In the circuit shown below, $R_B = 1$ kΩ and $R_C = 100Ω$. If the transistor β(h_{FE}) is 100, the current through R_C will be



(a) ≈ 0.43 A (b) ≈ 50 mA

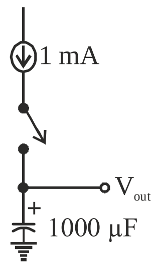
(c) zero (d) Oscillating between 0 and 50 mA.

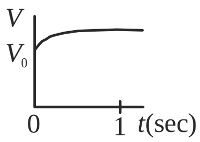
2.29. The output of the circuit on the right will be



(a) 1V (b) 11V (c) –10V (d) 0 V

2.30. The circuit in the figure on the right shows a constant current source charging in a capacitor. The initial voltage across the capacitor is V_0. The switch is closed at time $t = 0$. The voltage across the capacitor is best described by



(a)


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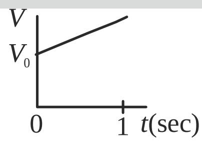

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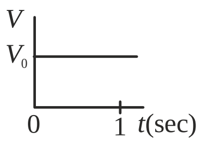

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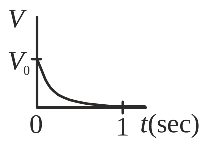


SECTION-B

Answer any FIFTEEEN questions. Each question carries 5 marks.

3. Evaluate the following integral by using the method of residues:

$$\int_{-\infty}^{\infty} \frac{xe^{-ix}}{x^2+a^2}\,dx$$

4. The height of a hill at a point (x, y) in metres is given by

$$h(x, y) = \exp\left[2xy - x^2 - 2y^2 - 4x + 8y + 1\right]$$

where x and y are in km with respect to a certain origin.

(a) where is the top of the hill located?

(b) what is the unit vector in the direction of steepest ascent at the origin?

5. A particle of mass m is constrained to move on the surface of a sphere of radius R. The sphere is resting on ground.

(a) Set up the Lagrangian for the particle by clearly identifying the kinetic energy and potential energy

(b) What are the constants of the motion for the particle?

6. A particle of mass m and velocity $\vec{u}$ collides elastically with another particle of the same mass at rest in laboratory frame. The scattering angle in the centre of mass frame is found to be 90º. Find the velocities of the scattered particles in the centre of mass frame and the laboratory frame.

7. A pion (rest mass m_0 = 135 MeV/c^2) is moving with a velocity $\vec{v} = 0.8c\hat{z}$. If it decays by emitting two photons both of which move in z-direction, find their energies and frequencies in units of MeV and MeV/h respectively.

8. Let ψ_0 and ψ_2 denote respectively the ground state and second excited state energy eigenfunctions of a particle moving in harmonic oscillator potential with frequency ω. If at time $t = 0$ the particle has the wavefunction,

$$\psi(x) = -\frac{1}{\sqrt{3}}\psi_0(x) + \sqrt{\frac{2}{3}}\psi_2(x)$$

(a) Find $\psi(x, t)$ for $t \neq 0$

(b) Determine the expectation value of the energy as a function of time

(c) Determine momentum and position expectation values as functions of time

9. Let $H_0 = p^2/2m + \left(\frac{1}{2}\right)m\omega^2 x^2$ be the unperturbed Hamitonian and

$$V' = \begin{cases} \lambda x & for\ x > 0 \\ 0 & for\ x \leq 0 \end{cases}$$

be a small perturbation. Determine the first order correction to the ground state energy. What is the condition on λ for the first order perturbation theory to be valid? The ground state wave function is given as

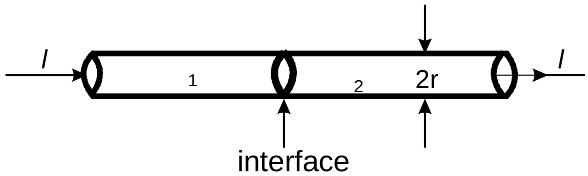
$$\psi_{g_\ell} = \frac{1}{\sqrt{\left(\sqrt{\pi}\,x_0\right)}}\exp\left(\frac{-x2}{2x_0^2}\right), \text{ where } x_0 = \sqrt{\hbar/m\omega}$$

10. A helium - neon laser beam $(\lambda = 632\text{ nm})$ of intensity 1.0 W/cm^2 is travelling along x-axis in vacuum. Find :

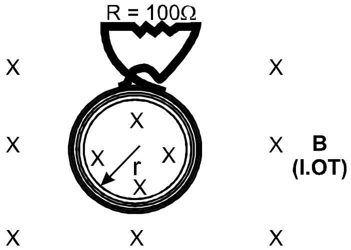
(a) Amplitudes of E and B fields associated with the laser beam

(b) Expression for $\vec{E}(x, t)$ and $\vec{B}(x, t)$ if the E-field is polarized along $\hat{y}$ direction.

11. Two wires of the same cross sectional area and electrical conductivity σ_1 and σ_2 are connected as shown in the figure. If a constnt current I is made to pass through the wires, find the induced charge density at the junction between the two wires.



12. The ends of a circular coil of radius r (= 2 cm) and N(=100) turns are connected to a 100Ω resistor. The coil is placed such that a uniform magnetic field of strength 1.0 Tesla is perpendicular to the plane of the coil. If the coil is rotated by 180°, find the charge that will flow through the resistor.



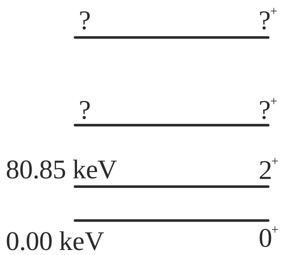
13. Consider the stellar stmosphere consisting of hydrogen atoms, which is in thermal equilibrium. Let the average kinetic energy of hydrogen atoms be 1.0 eV. Find,
(a) the temperature of the atmosphere
(b) the ratio of the number of atoms in the second excited state to those in the ground state

14. Consider a system with ground state energy E_0 and an excited state with energy E_1. Determine the partition function and internal energy of the at a temperature T. Also, find the specific heat of the system in the limit $T \to 0$.

15. Consider the two branches of a phonon spectrum of a cubic lattice, $\omega_1(\vec{k}) = A|\sin(k)|$ and $\omega_2(\vec{k}) = B(2+\cos(k))$. In the Debye approximation give the phonon dispersion relations and the density of phonon levels for each branch.

16. The band structure of an electron in a one-dimensional periodic potential is given by $E_1(k) = A[1-\cos(k)]$ and $E_2(k) = B$. Find the elective mass of the electron for each band $(\text{for } k \approx 0)$. Mention which branch could contribute to electrical conduction.

17. The first two energies and spin particles for Er^{166} is shown in the figure. Using the rotational model, determine the spin, parity and energy of the next two levels. Also, determine the moment of inertia of the nucleus in the units $\left(h^2 / keV\right)$

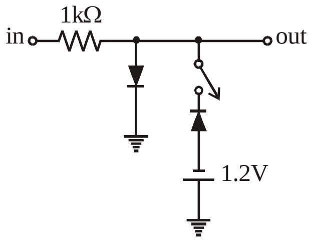


18. For the processes given below identify those that are allowed and those disallowed. State the intersection responsible for the former and the symmetry that forbids the latter.

(1) $p \to n + e^+ + \nu_e$ (2) $n \to p + e^- + \overline{\nu}_e$ (3) $p + \overline{p} \to \pi^+ + \pi^-$
(4) $\pi^+ \to \mu^+ + \gamma$ (5) $\mu^+ + \mu^- \to e^+ + e^-$

19. Draw the circuit diagram of an inverting amplifier of gain –10 and input impedance 10 $k\Omega$. The circuit should work for dc as well as ac signals. You are allowed to use only one op-amp and two resistors. If the circuit should amplify signals in the range –1.6 V to +1.6 V, what should be the op-amp supply voltages?

20. The circuit on the right is given a 3V amplitude triangle wave input. The switch is open to begin with. What are the maximum and minimum output voltages? Sketch the output voltage vs. time, for two cycles, marking the voltage axis clearly.



Also sketch the output voltage when the switch is closed.

21. The moment of inertia of HBr molecule is 3.30×10^{-47} kg m^2. Find the wave number (in cm^{-1}) of absorption lines involving transitions between the rotational ground state and the first two excited states of the molecule under electric dipole approximation.

22. Consider Zeeman effect in alkali metal spectra :

(a) Sketch the Zeeman split components of the terms $^2P_{3/2}$ and $^2S_{1/2}$ and find the energy difference in units of $\mu_B B$ between each Zeeman component and the unperturbed position of the term. The Lade g factor for $^2P_{3/2}$ and $^2S_{1/2}$ is 4/3 and 2 respectively. Here μ_B and B are Bohr magneton and magnetic field respectively.

(b) How many scparate lines occur in the multiple arising from $^2P_{3/2} \rightarrow {}^2S_{1/2}$ transition in the presence of weak magnetic field?