SECTION-A

1. ***This question consists of TWENTY FIVE sub-questions (1.1 to 1.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×1 = 25]**

1.1. If two matrices A and B can be diagonalized simultaneously, which of the following is true?

(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$ (c) $AB = BA$ (d) $AB^2AB = BABA^2$

1.2. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

1.3. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true?

(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$

(b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$

(d) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$

1.4. The homogeneity of time leads to the law of conservation of

(a) linear momentum (b) angular momentum

(c) energy (d) parity

1.5. Hamilton canonical equations of motion for a conservative system are

(a) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(b) $\frac{dp_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(c) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

(d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}$ and $-\frac{dp_i}{dt} = \frac{\partial H}{\partial q_i}$

1.6. If R_1 is the value of the Rydberg constant assuming mass of the nucleus to be infinitely large compared to that of an electron, and if R_2 is the Rydberg constant taking nuclear mass to be 7500 times the mass of the electron, then the ratio R_2/R_1 is

(a) a little less than unity (b) a little more than unity

(c) infinitely small (d) infinitely large

1.7. Consider an infinitely long straight cylindrical conductor of radius R with its axis along the z-direction, which carries a current of 1A uniformly distributed over its cross section. Which of the following statements is correct?

(a) $\vec{\nabla} \times \vec{B} = 0$ everywhere

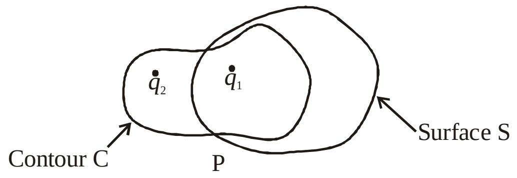
(b) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ everywhere,

(c) $\vec{\nabla} \times \vec{B} = 0$ for $r > R$,

(d) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ for $r > R$

where r is the radial distance from the axis of the cylinder.

1.8. Consider a set of two stationary point charges q_1 and q_2 as shown in the figure. Which of the following statements is correct?



(a) The electric field at P is independent of q_2

(b) The electric flux crossing the closed surface S is independent of q_2

(c) The line integral of the electric field $\vec{E}$ over the closed contour C depends on q_1 and q_2.

(d) $\vec{\nabla}\cdot\vec{E} = 0$ everywhere

1.9. If the wave function of a particle trapped in space between $x = 0$ and $x = L$ is given by $\psi(x) = A\sin\left(\frac{2\pi x}{L}\right)$, where A is a constant, for which value(s) of x will the probability of finding the particle be the maximum?

(a) $\frac{L}{4}$ (b) $\frac{L}{2}$ (c) $\frac{L}{6}$ and $\frac{L}{3}$ (d) $\frac{L}{4}$ and $\frac{3L}{4}$

1.10. In a Stern-Gerlach experiment, the magnetic field is in +z direction. A particle comes out of this experiment in $\left|+\hat{z}\uparrow\right\rangle$ state. Which of the following statements is true?

(a) The particle has a definite value of the *y*-component of the spin angular momentum

(b) The particle has a definite value of the square of the spin angular momentum

(c) The particle has a definite value of the *x*-components of spin angular momentum

(d) The particle has definite values of *x*-and *y*-components of spin angular momentum

1.11. If σ is the total cross-section and $f(\theta), \theta$ being the angle of scattering, is the scattering amplitude for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that k is the magnitude of the wave vector along the $\hat{z}$ direction.

(a) $\sigma = |f(\theta)|^2$

(b) $\sigma = \frac{4\pi}{k}|f(\theta = 0)|^2$

(c) $\sigma = \frac{4\pi}{k}\times$ Imaginary part of $[f(\theta = 0)]$

(d) $\sigma = \frac{4\pi}{k}|f(\theta)|^2$

1.12. In a classical micro-canonical ensemble for a system of N non-interacting particles, the fundamental volume in phase space which is regarded as "equivalent to one micro-state" is

(a) h^{3N} (b) h^{2N} (c) h^{N} (d) h

where h is the Planck's constant

1.13. Which of the following conditions should be satisfied by the temperature T of a system of N non-interacting particles occupying a volume V, for Bose-Einstein condensation to take place?

(a) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{3/2}$

(b) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{V}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{3/2}$

(c) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

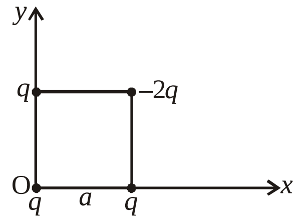
(d) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{V}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

where m is the mass of each particle of the system, k_B is the Boltzmann constant, h is the Planck's constant and ς is the well known Zeta function.

1.14. A large circular coil of N turns and radius R carries a time varying current $I = I_0 \sin(\omega t)$. A small circular coil of n turns and radius $r(r << R)$ is placed at the center of the large coil such that the coils are concentric and coplanar. The induced emf in the small coil

(a) leads the current in the large coil by $\pi/2$
(b) lags the current in the large coil by π
(c) is the phase with the current in the large coil
(d) lags the current in the large coil by $\pi/2$

1.15. Four charges are placed at the four corners of a square of side a as shown in the figure. The electric dipole moment of this configuration is



(a) $\vec{p} = qa\hat{i} + qa\hat{j}$
(b) $\vec{p} = -qa\hat{i} + qa\hat{j}$
(c) $\vec{p} = -qa\hat{i} - qa\hat{j}$
(d) $\vec{p} = qa\hat{i} - qa\hat{j}$

1.16. Which of the following statements is true?

(a) In a micro-canonical ensemble the total number of particles N and the energy E are constants while in a canonical ensemble N and temperature T are constants
(b) In a micro-canonical ensemble the total number of particles N is a constant but the energy E is variable while in a canonical ensemble N and T are constants
(c) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N and T both vary
(d) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N is a constant but T varies

1.17. In a one-dimensional Kronig Penny model, the total number of possible wave functions is equal to

(a) twice the number of unit cells
(b) number of unit cells
(c) half the number of unit cells
(d) independent of the number of unit cells

1.18. The potential in a divalent solid at a particular temperature is represented by a one-dimensional periodic model. The solid should behave electrically as

(a) a semiconductor
(b) a conductor
(c) an insulator
(d) a superconductor

1.19. In a cubic system with cell edge a, two phonons with wave vectors $\vec{q}_1$ and $\vec{q}_2$ collide and produce a third phonon with a wave vector $\vec{q}_3$ such that

$$\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{R},$$

where $\vec{R}$ is a lattice vector. Such a collision process will lead to

(a) finite thermal resistance
(b) zero thermal resistance
(c) an infinite thermal resistance
(d) a finite thermal resistance for certain $|\vec{R}|$ only

1.20. The baryon number of proton, the lepton number of proton, the baryon number of electron and the lepton number of electron are respectively

(a) zero, zero, one and zero
(b) one, one, zero and one
(c) one, zero, zero and one
(d) zero, one, one and zero

1.21. Typical energies released in a nuclear fission and a nuclear fusion reaction are respectively

(a) 50 MeV and 1000 MeV
(b) 200 MeV and 1000 MeV
(c) 1000 MeV and 50 MeV
(d) 200 MeV and 10 MeV

1.22. Nuclear forces are

(a) spin dependent and have no non-central part
(b) spin dependent and have a non-central part
(c) spin independent and have no non-central part
(d) spin independent and have a non-central part

1.23. The nuclear spins of ${}_6C^{14}$ and ${}_{12}Mg^{25}$ nuclei are respectively

(a) zero and half-integer
(b) half-integer and zero
(c) an integer and half-integer
(d) both half-integers

1.24. The asymmetry terms in the Weizsacker semi-empirical mass formula is because of

(a) non-spherical shape of the nucleus
(b) non-zero spin of nucleus
(c) unequal number of protons and neutrons inside the nucleus
(d) odd number of protons inside the nucleus

1.25. Which of the following options is true for a two input XOR gate?

	Input A	Input B	Output
(a)	0	1	1
(b)	1	0	0
(c)	0	0	1
(d)	1	1	1

2. ***This question consists of TWENTY FIVE sub-questions (2.1 to 2.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×2 = 50]**

2.1. Which of the following vectors is orthogonal to the vector $(a\hat{i}+b\hat{j})$, where a and b $(a \neq b)$ are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors?

(a) $-b\hat{i}+a\hat{j}$
(b) $-a\hat{i}+b\hat{j}$
(c) $-a\hat{i}-b\hat{j}$
(d) $-b\hat{i}-a\hat{j}$

2.2. Fourier transform of which of the following functions does not exist?

(a) $e^{-|x|}$
(b) xe^{-x^2}
(c) e^{x^2}
(d) e^{-x^2}

2.3. The unit vector normal to the surface $3x^2+4y=z$ at the point (1, 1, 7) is

(a) $(-6\hat{i}+4\hat{j}+\hat{k})/\sqrt{53}$
(b) $(4\hat{i}+6\hat{j}-\hat{k})/\sqrt{53}$
(c) $(6\hat{i}+4\hat{j}-\hat{k})/\sqrt{53}$
(d) $(4\hat{i}+6\hat{j}+\hat{k})/\sqrt{53}$

2.4. The solution of the differential equation

$$(1+x)\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}-y(x)=0 \text{ is}$$

(a) Ax^2+B
(b) $Ax+Be^{-x}$
(c) $Ax+Be^{x}$
(d) $Ax+Bx^2$

where A and B are constants

2.5. A particle of mass M moving in a straight line with speed v collides with a stationary particle of the same mass. In the center of mass coordinate system, the first particle is deflected by 90º.The speed of the second particle, after collision, in the laboratory system will be

(a) $v/\sqrt{2}$
(b) $\sqrt{2}\,v$
(c) v
(d) $v/2$

2.6. The scalar potential corresponding to the force field $\vec{F} = \hat{i}(y+z)$

(a) is $y^2/2$ (b) is 1 (c) is zero (d) does not exist

2.7. Two particles of equal mass are connected by an inextensible string of length L. One of the masses is constrained to move on the surface of a horizontal table. The string passes through a small hole in the table and the other mass is hanging below the table. The only constraint is that the first mass moves on the surface of the table. The number of degrees of freedom of the masses-string system is

(a) five (b) four (c) two (d) one

2.8. An electron is accelerated from rest by 10.2 million volts. The percent increase in its mass is

(a) 20,000 (b) 2,000 (c) 200 (d) 20

2.9. An infinitely long closely wound solenoid carries a sinusoidally varying current. The induced electric field is

(a) zero everywhere

(b) non-zero inside and zero outside the solenoid

(c) non-zero inside as well as outside the solenoid

(d) zero inside and non-zero outside the solenoid

2.10. A laser beam of wavelength 600 nm with a circular cross section having a radius of 10mm falls normally on a lens of radius 20 mm and focal length 10 cm. The radius of the focussed spot is approximately

(a) $0.3\mu m$ (b) $0.6\mu m$ (c) $3.0\mu m$ (d) $6.0\mu m$

2.11. A left circularly polarized light beam of wavelength 600 nm is incident on a crystal of thickness *d* and propagates perpendicular to its optic axis. The ordinary and extraordinary refractive indices of the crystal are n_0 = 1.54 and n_e = 1.55 respectively. The emergent light will be right circularly polarized if *d* is

(a) $120\mu m$ (b) $60\mu m$ (c) $30\mu m$ (d) $15\mu m$

2.12. In a two beam interference pattern, the maximum and minimum intensity values are found to be $25I_0$ and $9I_0$ respectively, where I_0 is a constant. The intensities of the two interfering beams are

(a) $16I_0$ and I_0 (b) $5I_0$ and $3I_0$ (c) $17I_0$ and $8I_0$ (d) $8I_0$ and $2I_0$

2.13. An electron propagating along the *x*-axis passes through a slit of width $\Delta y = 1\text{nm}$. The uncertainty in the *y*-component of its velocity after passing through the slit is

(a) 7.322×10^5 m/s (b) 1.166×10^5 m/s

(c) 3.436×10^5 m/s (d) 2.326×10^4 m/s

2.14. $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $[\hat{A},\hat{B}]$ stands for the commutator of $\hat{A}$ and $\hat{B}$, then $\left[[\hat{A},\hat{B}],[\hat{B},\hat{A}]\right]$ is equal to

(a) $\hat{A}\hat{B}\hat{A}\hat{B} - \hat{B}\hat{A}\hat{B}\hat{A}$ (b) $\hat{A}(\hat{A}\hat{B}-\hat{B}\hat{A}) - \hat{B}(\hat{B}\hat{A}-\hat{A}\hat{B})$

(c) zero (d) $\left([\hat{A},\hat{B}]\right)^2$

2.15. An electron is in a state with spin wave function $\phi_s = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$ in the S_z representation. What is the probability of finding the z-component of its spin along the $-\hat{z}$ direction?

(a) 0.75 (b) 0.50 (c) 0.35 (d) 0.25

2.16. If the wavelength of the first line of the Balmer series in the hydrogen spectrum is λ, then the wavelength of the first line of the Lyman series is

(a) $(27/5)\lambda$ (b) $(5/27)\lambda$ (c) $(32/27)\lambda$ (d) $(27/32)\lambda$

2.17. A vector $\vec{A}=(5x+2y)\hat{i}+(3y-z)\hat{j}+(2x-az)\hat{k}$ is solenoidal if the constant a has a value

(a) 4 (b) –4 (c) 8 (d) –8

2.18. The mean free path of the particles of a gas at a temperature T_0 and pressure p_0 has a value λ_0. If the pressure is increased to $1.5p_0$ and the temperature is reduced to $0.75\ T_0$, the mean free path

(a) remains unchanged (b) is reduced to half

(c) is doubled (d) is equal to $1.125\lambda_0$

2.19. Which of the following relations between the particle number density n and temperature T must hold good for a gas consisting of non-interacting particles to be described by quantum statistics?

(a) $n/T^{1/2} << 1$ (b) $n/T^{3/2} << 1$

(c) $n/T^{3/2} >> 1$ (d) $n/T^{1/2}$ and $n/T^{3/2}$ can have any value

2.20. For a prefect free-electron gas in a metal, the magnitudes of phase velocity (v_p) and group velocity (v_g) are such that

(a) $v_p = v_g$ (b) $v_p = \frac{1}{2}v_g$

(c) $v_p = \sqrt{2}v_g$ (d) $v_p = 2v_g$

2.21. A metal has free-electron density $n = 10^{29}\ \text{m}^{-3}$. Which of the following wavelengths will excite plasma oscillations?

(a) $0.033\,\mu\text{m}$ (b) $0.330\,\mu\text{m}$ (c) $3.300\,\mu\text{m}$ (d) $33.000\,\mu\text{m}$

2.22. For an NaCl crystal, the cell-edge a = 0.563 nm. The smallest angle at which Bragg reflection can occur corresponds to a set of planes whose incides are

(a) 100 (b) 110 (c) 111 (d) 200

2.23. Silicon has diamond structure with unit-cell-edge a = 0.563 nm. The interatomic separation is

(a) 0.122 nm (b) 0.234 nm (c) 0.383 nm (d) 0.542 nm

2.24. The spin and parity of ${}_4Be^9$ nucleus, as predicted by the shell model, are respectively.

(a) 3/2 and odd (b) 1/2 and odd (c) 3/2 and even (d) 1/2 and even

2.25. The feedback ratio of an amplifier, which on application of a negative feedback, changes the voltage gain from –250 to –100 is

(a) –0.250 (b) –0.025 (c) –0.060 (d) –0.006

SECTION-B

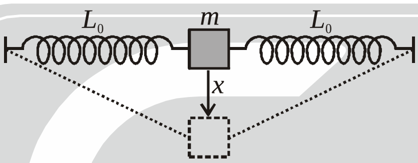
***This section consists of TWENTY* questions of *FIVE* marks each. *ANY FIFTEEN* out of these questions have to answered on the Answer Book provided.** **[75 Marks]**

3. Given the differential equation $\frac{d^2y(x)}{dx^2}+2\frac{dy(x)}{dx}+5y(x)=0$ find its solution that satisfies the initial conditions $y=0$ and $x=0$ and $\frac{dy}{dx}=1$ at $x=0$

4. Find the matrix that diagonalizes the matrix $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

5. Using the residue theorem, compute the integral $I=\int_0^\infty \frac{dx}{(1+x)^4}$.

6. A particle of mass M is attached to two identical springs of unstretched length L_0 and spring constant k. The entire system is placed on a horizontal frictionless table as shown in the figure. The mass is slightly puled along the surface of the table and perpendicular to the lengths of the springs and then let go. Using the Lagrangian equation (s) of motion, show whether the mass will execute simple harmonic motion. If so, find the time period.



7. A uniform thin circular disc of mass M and radius R lies in the X-Y plane with its centre at the origin. Find the moments of inertia tensor. What are the values of the principal moments of inertia? Find the principal axes.

8. Two events, 10^{-7} s apart in time, take place at two points 50 m apart on the X-axis. Find the speed of an observer moving along the X-axis who observes the two events simultaneously. What will be the spatial separation between these two events as seen by this observer?

9. Consider a parallel plate air filled capacitor with a plate area of 10 cm^2 separated by a distance of 2 mm. The potential difference across the plates varies as

   $V = 360\sin(2\pi\times 10^6 t)$ volts,

   where t is measured in seconds. Neglecting fringe effects, calculate the displacement current flowing through the capacitor.

10. The potential of a spherically symmetric charge distribution is given by

$$V(r)=\frac{a}{3}\left(4-\frac{r^2}{R^2}\right);\ \text{for } r\le R$$

$$=\frac{aR}{r};\ \text{for } r>R,$$

   a and R being constants. Find the corresponding charge distribution.

11. Consider a plane electromagnetic wave propagating in free space and having an electric field distribution given by

$$\vec{E}=E_0\left(\frac{\sqrt{3}}{2}\hat{j}-\frac{1}{2}\hat{i}\right)\exp\left[i\left(\omega t-\frac{\sqrt{3}}{2}\alpha x-\frac{1}{2}\alpha y\right)\right],$$

   where E_0,ω and a are constants. Calculate the corresponding magnetic field $\vec{B}$.

12. A particle in the ground state of an infinitely deep one dimensional potential well of width a is subject to a perturbation of the form

$$V = V_0 \cos^2\left(\frac{\pi x}{a}\right)$$

where V_0 is a constant. Find the shift in energy of the particle in the lowest order perturbation theory.

13. A quantum particle is in a state which is the superposition of the eigenstates of the momentum operator $p_x = -i\hbar\frac{\partial}{\partial x}$. If the probability of finding the momentum $\hbar k$ of the particle is 90%, compute its wave function.

14. The wave function of a free particle is given by $\psi(\vec{r}) = Ce^{-(x^2+y^2+z^2)}$, where C is a constant. Compute the momentum space probability density, normalize it to 1 and hence find the value of C.

15. Carbon monoxide has a bond length of 0.1132 nm. What will be the frequency of rotation of the molecule for its lowest excited state?

16. 1 Kg of water at a temperature of 353 K is mixed adiabatically with an equal mass of water at 293 K. Find the change in entropy of the universe assuming the specific heat of water to be constant equal to 238 $J.kg^{-1}.\ K^{-1}$.

17. A conductor having a free electron gas is maintained at a very low temperature ($T \to 0K$). Find the average energy per electron in terms of the electrons density and the electron mass.

18. A small concentration of minority carries is injected into a homogeneous semiconductor held at 300K. An electric field of 30 V/cm is applied across the width of the crystal. As a result, the minority carriers move a distance of 1.5 cm in a time of 300 μs. What is the diffusion coefficient of the minority carriers in the semiconductor?

19. A phonon with wave vector $\vec{q}$ gets absorbed on collision with an electron of wave vector $\vec{k}$. The electron is considered free and its energy is much larger than that of the phonon. If the electron is scattered at an angle α, show that $\alpha = 2\sin^{-1}(q/2k)$.

20. In spherical coordinates, the wave function describing a state of a system is

$$\psi(r,\theta,\phi) = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2}\frac{r}{a_0}e^{-4/2a_0}\sin\theta e^{-i\phi}$$

where a_0 is a constant. Find the parity of the system in this state.

21. Calculate the minimum kinetic energy that the neutron should have in order to induce the reaction

$O^{16}(n^1,\ He^4)C^{13}$

in which C^{13} is left in an excited state of energy 1.79 MeV. Given:

Mass of O^{16} = 16.000000 amu
Mass of n^1 = 1.008986 amu
Mass of He^4 = 4.003874 amu
Mass of C^{13} = 13.007490 amu

22. Calculate the dc collector voltage (V_c) with respect to ground in the amplifier circuit shown in the figure. The current gain β for the transistor is 200.