PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is **NOT TRUE** ?
(a) Det A = Det B (b) Trace of A = Trace of B
(c) A and B have the same eigenvectors (d) A and B have the same eigenvalues

2. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that
(a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla} \cdot \vec{F} = 0$ (c) $\vec{\nabla} V = 0$ (d) $\nabla^2 V = 0$

3. The quantum mechanical operator for the momentum of a particle moving in one dimension is given by
(a) $i\hbar \frac{d}{dx}$ (b) $-i\hbar \frac{d}{dx}$ (c) $i\hbar \frac{\partial}{\partial t}$ (d) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

4. A Carnot cycle operates on a working substance between two reservoirs at temperatures T_1 and T_2, with $T_1 > T_2$. During each cycle, an amout of heat Q_1 is extracted from the reservoir at T_1 and an amount Q_2 is delivered to the reservoir at T_2. Which of the following statements is **INCORRECT** ?
(a) work done in one cycle is $Q_1 - Q_2$
(b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$
(c) entropy of the hotter reservoir decreases
(d) entropy of the universe (consisting of the working substance and the two reservoirs) increases

5. In a first order phase transition, at the transition temperature, specfic heat of the system
(a) diverges and its entropy remains the same
(b) diverges and its entropy has finite discontinuity
(c) remains unchanged and its entropy has finite discontinuity
(d) has finite discontinuity and its entropy diverges

6. The semi-empirical mass formula for the binding energy of nucleus contains a surface correction term. This term depends on the mass number A of the nucleus as
(a) $A^{-1/3}$ (b) $A^{1/3}$ (c) $A^{2/3}$ (d) A

7. The population inversion in a two level laser material cannot be achieved by optical pumping because
(a) the rate of upward transitions is equal to the rate of downward transitions.
(b) the upward transitions are forbidden but downward transitions are allowed.
(c) the upward transitions are allowed but downward transitions are forbidden.
(d) the spontaneous decay rate of the higher level is very low.

8. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with a Curie temperature (T_c) is given by
(a) $\frac{C}{T - T_c}$, for $T < T_c$ (b) $\frac{C}{T - T_c}$, for $T > T_c$ (c) $\frac{C}{T + T_c}$, for $T > T_c$ (d) $\frac{C}{T + T_c}$, for all temperatures

9. The order of magnitude of the energy gap of a typical superconductor is
(a) 1 MeV (b) 1 KeV (c) 1 eV (d) 1 meV

10. Which of the following statements is **CORRECT** for a common emitter amplifier circuit ?
(a) The output is taken from the emitter
(b) There is 180° phase shift between input and output voltages
(c) There is no phase shift between input and output voltages
(d) Both *p-n* junctions are forward biased

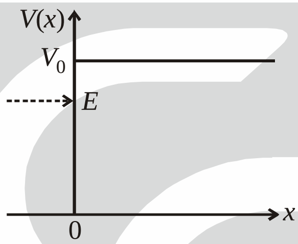
11. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is
(a) 18 (b) 12 (c) 9 (d) 6

12. A heavy symmetrical top is rotating about its own axis of symmetry (the z-axis). If I_1, I_2 and I_3 are the principal moments of inertia along x, y and z respectively, then
(a) $I_2 = I_3 ; I_1 \neq I_2$ (b) $I_1 = I_3 ; I_1 \neq I_2$ (c) $I_1 = I_2 ; I_1 \neq I_3$ (d) $I_1 \neq I_2 \neq I_3$

13. An electron with energy E is incident from left on a potential barrier, given by

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases}$$

as shown in the figure.

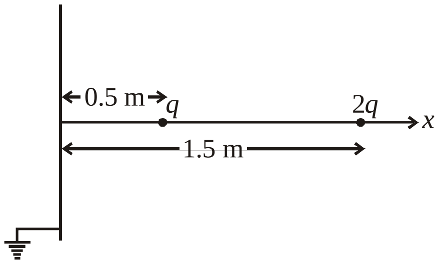


For $E < V_0$, the space part of the wavefunction for $x > 0$ is of the form
(a) $e^{\alpha x}$ (b) $e^{-\alpha x}$ (c) $e^{i\alpha x}$ (d) $e^{-i\alpha x}$

14. If L_x, L_y and L_z are respectively the x, y and z components of angular momentum operator L, the commutator $\left[L_x L_y, L_z\right]$ is equal to
(a) $i\hbar(L_x^2 + L_y^2)$ (b) $2i\hbar L_z$ (c) $i\hbar(L_x^2 - L_y^2)$ (d) 0

15. The normalized ground state wavefunction of a hydrogen atom is given by $\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{3/2}} e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from the nucleus, located at the origin. The expectation value $\langle 1/r^2 \rangle$ is
(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$ (c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

16. Two charges q and $2q$ are placed along the x-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is



(a) $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ (b) $\frac{1}{4\pi\varepsilon_0}2q^2$ (c) $\frac{1}{4\pi\varepsilon_0}q^2$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

17. A uniform surface current is flowing in the positive y-direction over an infinite sheet lying in xy-plane. The direction of the magnetic field is

(a) along $\hat{i}$ for $z > 0$ and along $-\hat{i}$ for $z < 0$ (b) along $\hat{k}$ for $z > 0$ and along $-\hat{k}$ for $z < 0$

(c) along $-\hat{i}$ for $z > 0$ and along $\hat{i}$ for $z < 0$ (d) along $-\hat{k}$ for $z > 0$ and along $\hat{k}$ for $z < 0$

18. A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform magnetic field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is

(a) $\vec{r}\times(m\times\vec{B})$ (b) $\vec{r}\times\vec{\nabla}(m\cdot\vec{B})$ (c) $\vec{m}\times\vec{B}$ (d) $\vec{m}\times\vec{B}+\vec{r}\times\vec{\nabla}(\vec{m}\cdot\vec{B})$

19. Which of the following expressions for a vector potential $\vec{A}$ **DOES NOT** represent a uniform magnetic field of magnitude B_0 along the z-direction?

(a) $\vec{A}=(0, B_0x, 0)$ (b) $\vec{A}=(-B_0y, 0, 0)$

(c) $\vec{A}=\left(\frac{B_0x}{2}, \frac{B_0y}{2}, 0\right)$ (d) $\vec{A}=\left(-\frac{B_0y}{2}, \frac{B_0x}{2}, 0\right)$

20. A neutron passing through a detector is detected because of

(a) the ionization it produces

(b) the scintillation light it produces

(c) the electron-hole pair it produces

(d) the secondary particles produced in a nuclear reaction in the detector medium.

21. An atom with one outer electron having orbital angular momentum ℓ is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is

(a) $2\ell+2$ (b) $2\ell+1$ (c) 2ℓ (d) $2\ell-1$

22. For a multi electron atom ℓ, L and S specify the one electron orbital angular momentum, total angular momentum and total spin angular momentum respectively. The selection rules for electric dipole transition between the two electronic energy levels, specified by ℓ, L and S are

(a) $\Delta L = 0, \pm 1$; $\Delta S = 0$; $\Delta\ell = 0, \pm 1$ (b) $\Delta L = 0, \pm 1$; $\Delta S = 0$; $\Delta\ell = \pm 1$

(c) $\Delta L = 0, \pm 1$; $\Delta S = \pm 1$; $\Delta\ell = 0, \pm 1$ (d) $\Delta L = 0, \pm 1$; $\Delta S = \pm 1$; $\Delta\ell = \pm 1$

23. For a three-dimensional crystal having N primitive unit cells with a basis of p atoms, the number of optical branches is

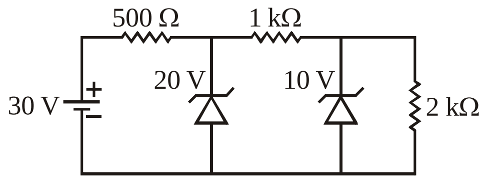
(a) 3 (b) $3p$ (c) $3p-3$ (d) $3N-3p$

24. For an intrinsic semiconductor, m_e^* and m_h^* are respectively the effective masses of electrons and holes near the corresponding band edge. At a finite temperature, the position of the Fermi level

(a) depends on m_e^* but not on m_h^* (b) depends on m_h^* but not on m_e^*

(c) depends on both m_e^* and m_h^* (d) depends neither on m_e^* nor m_h^*

25. In the following circuit, the voltage across and the current through the 2 kΩ resistance are



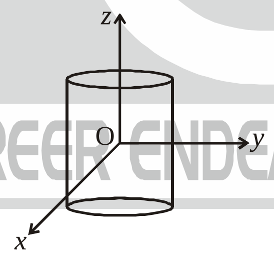
(a) 20 V, 10 mA (b) 20 V, 5 mA (c) 10 V, 10 mA (d) 10 V, 5 mA

Q.26 – Q.55 : Carry TWO marks each.

26. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point $P(1,1,1)$ is

(a) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i}+2\hat{j}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

27. Consider a cylinder of height h and a, closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_s \vec{r}\cdot\hat{n}\,ds$ over the closed surface of the cylinder is



(a) $2\pi a^2(a+h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) zero

28. The solutions to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

(a) circles with different radii (b) circles with different centers

(c) straight lines with different slopes (d) straight lines with different intercepts on the y-axis

29. A particle is moving under the action of a generalized potential

$$V(q,\dot{q}) = \frac{(1+\dot{q})}{q^2}$$

The magnitude of the generalized force is

(a) $\frac{2(1+\dot{q})}{q^3}$ (b) $\frac{2(1-\dot{q})}{q^3}$ (c) $\frac{2}{q^3}$ (d) $\frac{\dot{q}}{q^3}$

30. Two bodies of mass m and $2m$ are connected by a spring constant k. The frequency of the normal mode is

(a) $\sqrt{\frac{3k}{2m}}$ (b) $\sqrt{\frac{k}{m}}$ (c) $\sqrt{\frac{2k}{3m}}$ (d) $\sqrt{\frac{k}{2m}}$

31. Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation

$$Q = q^{\alpha}\cos(\beta p)$$
$$P = q^{\alpha}\sin(\beta p)$$

is canonical for

(a) $\alpha = 2,\ \beta = 1/2$ (b) $\alpha = 2,\ \beta = 2$ (c) $\alpha = 1,\ \beta = 1$ (d) $\alpha = 1/2,\ \beta = 2$

32. Two particles, each of rest mass m collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of the final entity is

(a) $\frac{5m}{4}$ (b) $2m$ (c) $\frac{5m}{2}$ (d) $\frac{25m}{8}$

33. The normalized eigenstates of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0 & ;\ \text{if } 0 \le x \le a \\ \infty & ;\ \text{otherwise} \end{cases}$$

are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), \text{ where } n = 1, 2, 3, ...$$

The particle is subjected to a perturbation

$$V'(x) = V_0\cos\left(\frac{\pi x}{a}\right) \text{ for } 0 \le x \le \frac{a}{2}$$
$$= 0 \qquad \text{otherwise}$$

The shift in the ground state energy due to the perturbation, in the first order perturbation theory, is

(a) $\frac{2V_0}{3\pi}$ (b) $\frac{V_0}{3\pi}$ (c) $-\frac{V_0}{3\pi}$ (d) $-\frac{2V_0}{3\pi}$

34. If the isothermal compressibility of a solid is $\kappa_T = 10^{-10}\ (\text{Pa})^{-1}$, the pressure required to increase its density by 1 % is approximately

(a) 10^4 Pa (b) 10^6 Pa (c) 10^8 Pa (d) 10^{10} Pa

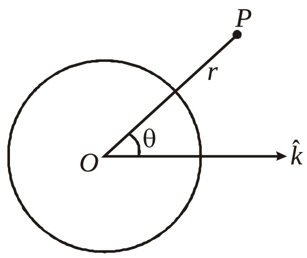
35. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is

(a) $2k_B \ln N$ (b) $3k_B \ln N$ (c) $Nk_B \ln 2$ (d) $Nk_B \ln 3$

36. A system has two energy levels with energies ε and 2ε. The lower level is 4-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is

(a) $\frac{1}{1+e^{-\varepsilon/k_BT}}$ (b) $\frac{1}{1+2e^{\varepsilon/k_BT}}$ (c) $\frac{1}{2e^{\varepsilon/k_BT}+4e^{2\varepsilon/k_BT}}$ (d) $\frac{1}{2e^{\varepsilon/k_BT}-4e^{2\varepsilon/k_BT}}$

37. A spherical conductor of radius a is placed in a electric field $\vec{E} = E_0\hat{k}$. The potential at a point $P(r, \theta)$ for $r > a$, is given by $\varphi(r,\theta) = \text{constant} - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$, where r is the distance of P from the center O of the sphere and θ is the angle OP makes with the z-axis.



The charge density on the sphere at $\theta = 30°$ is

(a) $3\sqrt{3}\varepsilon_0 E_0/2$ (b) $3\varepsilon_0 E_0/2$ (c) $\sqrt{3}\varepsilon_0 E_0/2$ (d) $\varepsilon_0 E_0/2$

38. According to the single particle nuclear shell model, the spin parity of the ground state of ${}^{17}_{8}O$ is

(a) $\frac{1}{2}^-$ (b) $\frac{3}{2}^-$ (c) $\frac{3}{2}^+$ (d) $\frac{5}{2}^+$

39. In the β-decay of neutron $n \to p + e^- + \bar{\nu}_e$, the anti-neutrino $\bar{\nu}_e$ escapes detection. Its existence is inferred from the measurement of

(a) energy distribution of electrons (b) angular distribution of electrons
(c) helicity distribution of electrons (d) forward-backward asymmetry of electrons

40. The isospin and the strangeness of Ω^- baryon are

(a) 1, –3 (b) 0, –3 (c) 1, 3 (d) 0, 3

41. The life time of an atomic state is 1 nanosecond. The natural line width of the spectral line in the emission spectrum of this state is of the order of

(a) $10^{-10}eV$ (b) $10^{-9}eV$ (c) $10^{-6}eV$ (d) $10^{-4}eV$

42. The degeneracy of an excited state of nitrogen atom having electronic configuration $1s^2\ 2s^2\ 2p^2\ 3d^1$ is

(a) 6 (b) 10 (c) 15 (d) 150

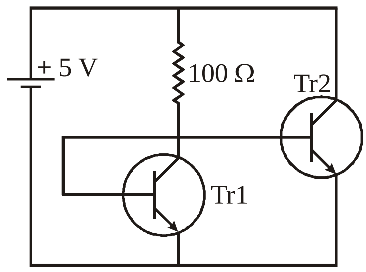
43. The far infrared rotational absorption spectrum of a diatomic molecule shows equidistant lines with spacing 20 cm^{-1}. The position of the first Stokes line in the rotational Raman spectrum of this molecule is

(a) 20 cm^{-1} (b) 40 cm^{-1} (c) 60 cm^{-1} (d) 120 cm^{-1}

44. A metal with body centered cubic (bcc) structure shows the first (i.e. smallest angle) diffraction peak at a Bragg angle of $\theta = 30°$. The wavelength of X-ray used is 2.1 Å. The volume of the PRIMITIVE unit cell of the metal is

(a) 26.2 $(Å)^3$ (b) 13.1 $(Å)^3$ (c) 9.3 $(Å)^3$ (d) 4.6 $(Å)^3$

45. In the following circuit, Tr1 and Tr2 are identical transistors having $V_{BE} = 0.7$ V. The current passing through the transistor Tr2 is



(a) 57 mA (b) 50 mA (c) 48 mA (d) 43 mA

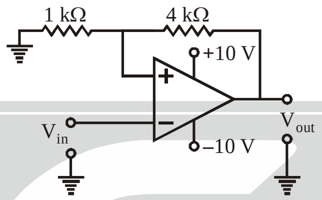
46. The following Boolean expression

$Y = A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D} + \overline{A}\cdot B\cdot\overline{C}\cdot D + \overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D + \overline{A}\cdot\overline{B}\cdot C\cdot D + \overline{A}\cdot B\cdot C\cdot D + A\cdot\overline{B}\cdot\overline{C}\cdot D$

can be simplified to

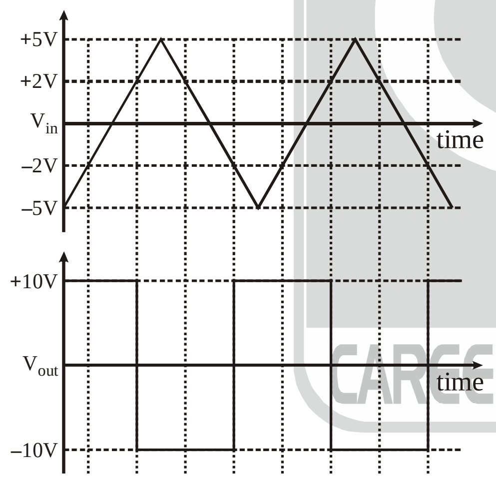
(a) $\overline{A}\cdot\overline{B}\cdot C + A\cdot\overline{D}$ (b) $\overline{A}\cdot B\cdot\overline{C} + A\cdot\overline{D}$ (c) $A\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot D$ (d) $A\cdot\overline{B}\cdot C + \overline{A}\cdot D$

47. Consider the following circuit.

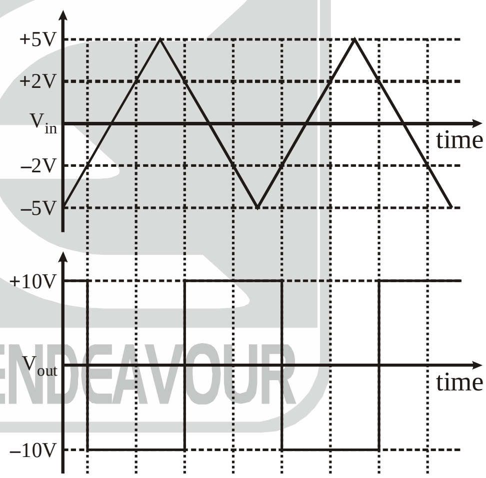


Which of the following correctly represents the output V_{out} corresponding to the input V_{in} ?

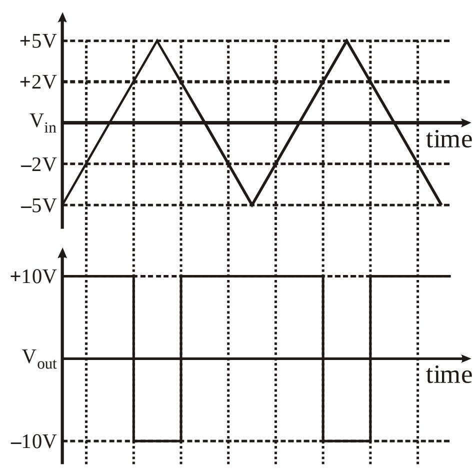
(a)



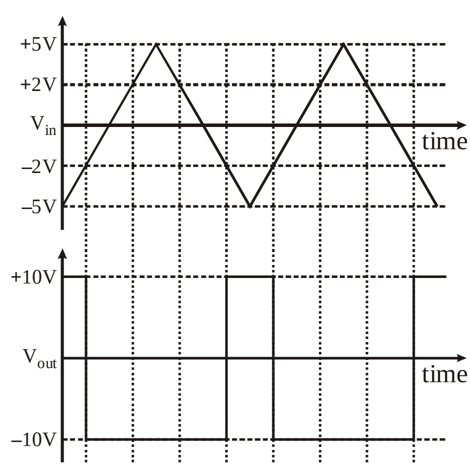
(b)



(c)



(d)



Common Data for Questions 48 and 49 :

Consider a function $f(z)=\dfrac{z\sin z}{(z-\pi)^2}$ of a complex variable z.

48. Which of the following statements is **TRUE** for the function $f(z)$?

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z=\pi$

(c) $f(z)$ has a pole of order 2 at $z=\pi$

(d) $f(z)$ has a simple pole at $z=\pi$

49. Consider a counterclockwise circular contour $|z|=1$ about the origin. The integral $\oint f(z)\,dz$ over this contour is

(a) $-i\pi$ (b) zero (c) $i\pi$ (d) $2i\pi$

Common Data for Questions 50 and 51 :

The tight binding energy dispersion (*E-k*) relation for electrons in a one-dimensional array of atoms having lattice constant *a* and total length *L* is $E=E_0-\beta-2\gamma\cos(ka)$, where E_0, β and γ are constant and *k* is the wave-vector.

50. The density of states of electrons (including spin degeneracy) in the band is given by

(a) $\dfrac{L}{\pi\gamma a\sin(ka)}$ (b) $\dfrac{L}{2\pi\gamma a\sin(ka)}$ (c) $\dfrac{L}{2\pi\gamma a\cos(ka)}$ (d) $\dfrac{L}{\pi\gamma a\cos(ka)}$

51. The effective mass of electrons in the band is given by

(a) $\dfrac{\hbar^2}{\gamma a^2\cos(ka)}$ (b) $\dfrac{\hbar^2}{2\gamma a^2\cos(ka)}$ (c) $\dfrac{\hbar^2}{\gamma a^2\sin(ka)}$ (d) $\dfrac{\hbar^2}{2\gamma a^2\sin(ka)}$

Statement for Linked Answer Question 52 and 53 :

In a one-dimensional harmonic oscillator, φ_0, φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by

$$\psi_1=\varphi_0-2\varphi_1+3\varphi_2$$
$$\psi_2=\varphi_0-\varphi_1+\alpha\varphi_2$$

where α is a constant.

52. The value of α for which ψ_2 is orthogonal to ψ_1 is

(a) 2 (b) 1 (c) –1 (d) –2

53. For the value of α determined in Q. 52, the expression value of energy of the oscillator in the state ψ_2 is

(a) $\hbar\omega$ (b) $\dfrac{3\hbar\omega}{2}$ (c) $3\hbar\omega$ (d) $\dfrac{9\hbar\omega}{2}$

Statement for Linked Answer Question 54 and 55 :

A plane electromagnetic wave has the magnetic field given by

$$\vec{B}(x, y, z, t) = B_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{k}$$

where k is the wave number and $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the Cartesian unit vectors in x, y and z direction, respectively.

54. The electric field $\vec{E}(x, y, z, t)$ corresponding to the above wave is given by

(a) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$

(b) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

(c) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{i}$

(d) $cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{j}$

55. The average Poynting vector is given by

(a) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (c) $\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ (d) $-\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$