PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is

(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

2. Which one of the following quantities is invariant under Lorentz transformation?

(a) Charge density (b) Charge (c) Current (d) Electric field

3. The number of normal Zeeman splitting components of $^1P \rightarrow {}^1D$ transition is

(a) 3 (b) 4 (c) 8 (d) 9

4. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is 5×10^{-8}s, then the proper half-life is ______$\times10^{-8}$ s. ($c = 3\times10^8$ m/s)

5. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is

(a) 0º (b) 45º (c) 90º (d) 120º

6. Two masses m and 3m are attached to the two ends of a massless spring with force constant K. If m = 100g and K = 0.3 N/m, then the natural angular frequency of oscillation is ________ Hz.

7. The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by,

$$\vec{E} = \hat{x}10\cos\left(6\pi\times10^7\, t - 0.4\pi z\right)V/m$$

The phase velocity of the wave is __________$\times10^8$ m/s.

8. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is

(a) orthogonal (b) symmetric (c) anti-symmetric (d) unitary

9. The recoil momemtum of an atom is p_A when it emits an infrared photon of wavelength 1500 nm, and it is p_B when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is

(a) 1 : 1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

10. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to

(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

11. Which one of the following high energy processes is allowed by conservation laws?

(a) $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$ (b) $\pi + p \rightarrow \pi^0 + n$

(c) $n \rightarrow p + e^- + \nu_e$ (d) $\mu^+ \rightarrow e^- + \gamma$

12. The length element ds of an arc is given by, $(ds)^2 = 2\left(dx^1\right)^2 + \left(dx^2\right)^2 + \sqrt{3}\, dx^1 dx^2$. The metric tensor g_{ij} is

(a) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

13. The ground state and the first excited state wave functions of a one dimensional infinite potential well are ψ_1 and ψ_2, respectively. When two spin-up electrons are placed in this potential, which one of the following, with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(a) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)-\psi_1(x_2)\psi_2(x_2)\right]$ (b) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)+\psi_1(x_2)\psi_2(x_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1)+\psi_1(x_2)\psi_2(x_2)\right]$ (d) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2)-\psi_1(x_2)\psi_2(x_1)\right]$

14. If the vector potential,

$$\vec{A}=\alpha x\hat{x}+2y\hat{y}-3z\hat{z}$$

satisfies the Coulomb gauge, the value of the constant α is ________________

15. At a given temperature, T, the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillator is _____________ $k_B T$.

16. Which one of the following is a fermion?

(a) α particle (b) ${}_4Be^7$ nucleus (c) hydrogen atom (d) deuteron

17. Which one of the following three-quark states (qqq), denoted by X, CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript

(a) X^{++} (b) X^{+} (c) X^{-} (d) X^{--}

18. The Hamilton's canonical equations of motion in terms of Poisson Brackets are

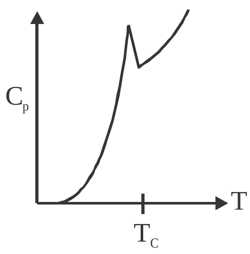
(a) $\dot{q}=\{q,H\}$; $\dot{p}=\{p,H\}$ (b) $\dot{q}=\{H,q\}$; $\dot{p}=\{H,q\}$

(c) $\dot{q}=\{H,p\}$; $\dot{p}=\{H,q\}$ (d) $\dot{q}=\{p,H\}$; $\dot{p}=\{q,H\}$

19. The Miller indices of a plane passing through the three points having coordinates (0, 0, 1), (1, 0, 0), $\left(\frac{1}{2},\frac{1}{2},\frac{1}{4}\right)$ are

(a) (2 1 2) (b) (1 1 1) (c) (1 2 1) (d) (2 1 1)

20. The plot of specific heat versus temperature across the superconducting transition temperature (T_C) is most appropriately represented by

(a)


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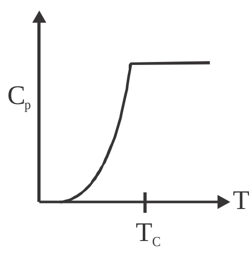

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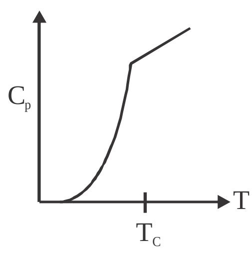

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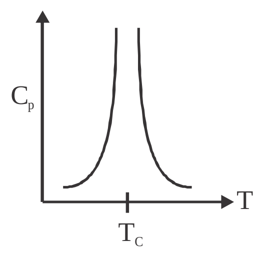
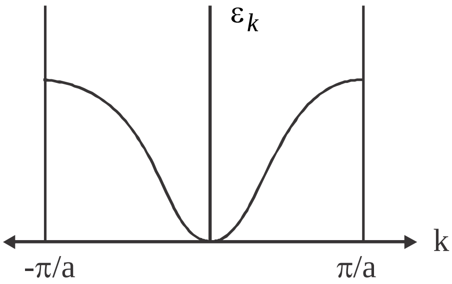
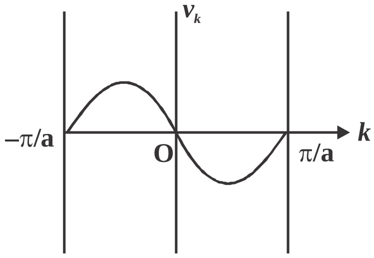


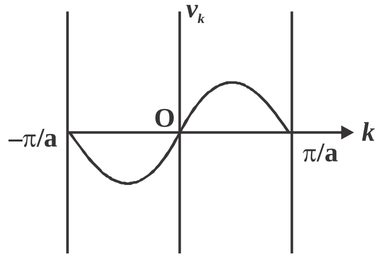
21. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does NOT commute with

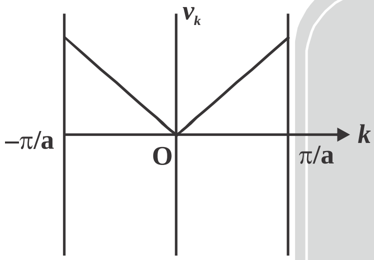
(a) S_z (b) L^2 (c) S^2 (d) $\left(\vec{L}+\vec{S}\right)^2$

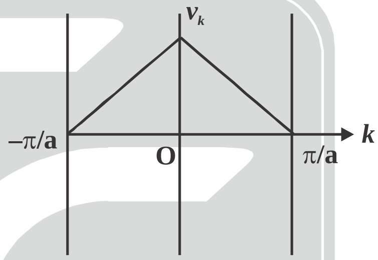
22. The energy, ε_k for band electrons as a function of the wave vector, k in the first Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as ('a' is lattice constant)



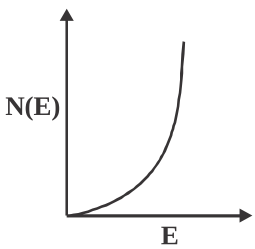
(a) 

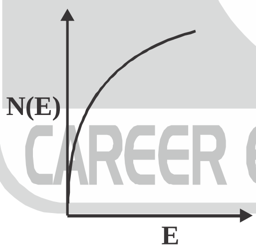
(b) 

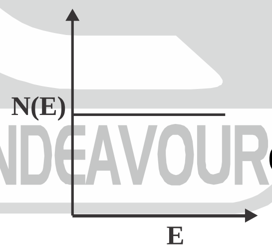
(c) 

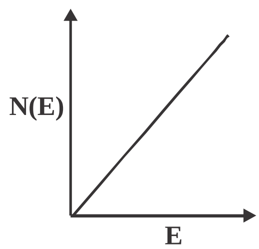
(d) 

23. For a free electron gas in two dimensions, the variation of the density of states, N(E) as a function of energy E, is the best represented by

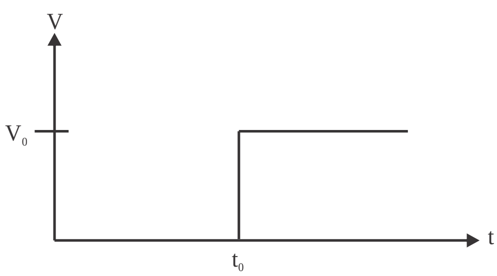
(a) 

(b) 

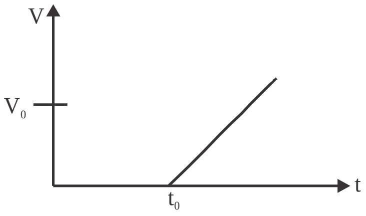
(c) 

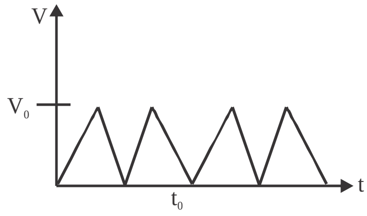
(d) 

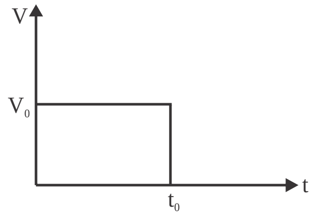
24. The input given to be an ideal OP-AMP integrator circuit is

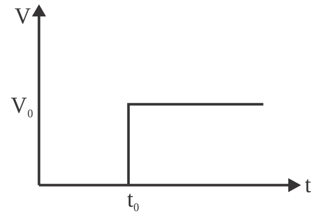


The correct output of the integrator circuit is

(a) 

(b) 

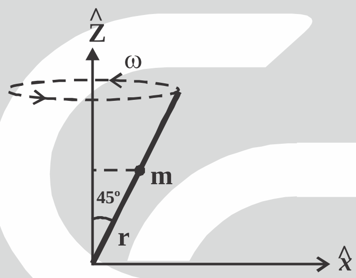
(c) 

(d) 

25. The minimum number of flip-flops required to construct a mod-75 counter is _________

Q.26 – Q.55 : Carry TWO marks each.

26. A bead of mass 'm' can slide without friction along a massless rod kept at 45º with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant, r is the distance of the bead from the origin. The momentum conjugate to 'r' is



(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$ (c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

27. An electron in the ground state of the hydrogen atom has the wave function

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-(r/a_0)}$$

where a_0 is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is:

(Hint: $\int_0^\infty e^{-\alpha r} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+1}}$)

(a) $-a_0^2/2$ (b) $-a_0^2$ (c) $-3a_0^2/2$ (d) $-2a_0^2$

28. For Nickel, the number density is 8×10^{23} atoms/cm^3 and electronic configuration is $1s^2\ 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic states is ________ $\times 10^9$ A/m.

(Given the value of Bohr magneton $\mu_B = 9.21\times10^{-21}\ Am^2$)

29. A particle of mass 'm' is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$$

where a and r_0 are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

30. The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non degenerate is ________ meV.

$\left(k_B T = 25meV \text{ at } 300\text{K}\right)$

31. A particle of mass m is subjected to a potential

$$V(x,y) = \frac{1}{2}m\omega^2\left(x^2+y^2\right), \quad -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of 'g' is ____________________

32. A hydrogen atom is in the state

$$\Psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$$

where n, ℓ, m in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is ______________ $\hbar^2$.

33. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$ where k is a positive constant. The orbital angular momentum of the planet is

(a) $2r_0 km$ (b) $\sqrt{2r_0 km}$ (c) $r_0 km$ (d) $\sqrt{r_0 km}$

34. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

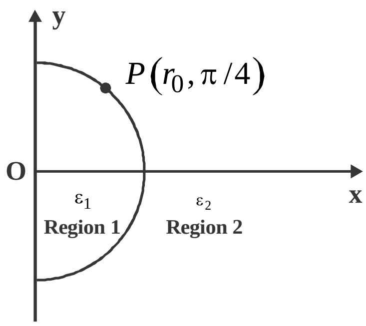
35. The value of the integral

$$\oint_C \frac{z^2}{e^z+1}dz$$

where C is the circle $|z| = 4$, is

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

36. A ray of light insid Region 1 in the xy-plane is incident at the semicircle boundary that carries no free charges. The electric field at the point $P(r_0, \pi/4)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to x-axis. If ε_1 and ε_2 are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\varepsilon_2}{\varepsilon_1}$ is ______________



37. The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0$$

subject to the boundary conditions $y(0)=1$ and $y(\infty)=0$, is

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$ (c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

38. Given that the linear transformation of a generalized coordinate 'q' and the corresponding momentum p,

$$Q = q + 4ap$$
$$p = q + 2p$$

is canonical, the value of the constant 'a' is ___________

39. The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is ________ Tesla.

(Given: $m_p = 1.67\times10^{-27} kg, e = 1.6\times10^{-19} C$)

40. For a system of two bosons, each of which can occupy any of the two energy levels 0 and ε, the mean energy of the system at a temperature T with $\beta = \frac{1}{k_BT}$ is given by

(a) $\frac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon\, e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (b) $\frac{1}{2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon\, e^{-\beta\varepsilon} + \varepsilon\, e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon\, e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}}$

41. In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and I_0, respectively. The intensities of the individual waves are

(a) $3I_0$ and I_0 (b) $4I_0$ and I_0 (c) $5I_0$ and $4I_0$ (d) $9I_0$ and I_0

42. ψ_1 and ψ_2 are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

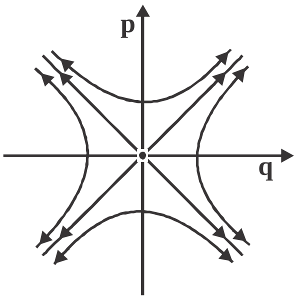
$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the system is in the state ψ_2, its probability to be in spin-up state is ___________

43. Neutrons moving with speed 10^3 m/s are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is 30º, the interplanar spacing of the crystal is ________Å.

(Given: $m_n = 1.675\times10^{-27}\, kg,\, h = 6.626\times10^{-34}\ J.s$)

44. The Hamiltonian of a particle of mass 'm' is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which of the following figures describes the motion of the particle in phase space?

(a)


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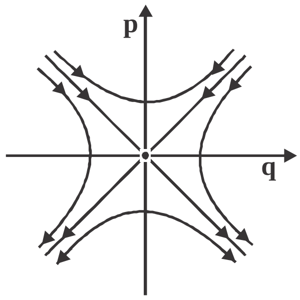
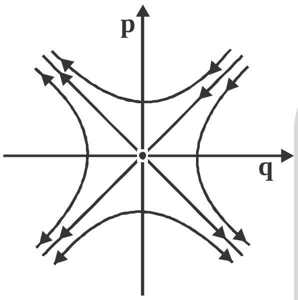
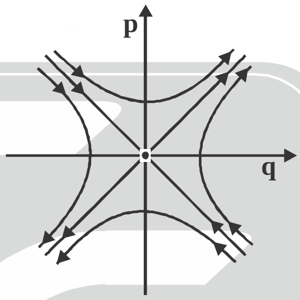


(c)


(d)


45. The intensity of a laser in free space is 150 mW/m^2. The corresponding amplitude of the electric field of the laser is ______ V/m. $\left(\varepsilon_0 = 8.854\times10^{-12}\ C^2/N.m^2\right)$

46. The emission wavelength for the transition ${}^1D_2 \rightarrow {}^1F_3$ is 3122 Å. The ratio of populations of the final to the initial states at a temperature 5000 K is $(h = 6.626\times10^{-34}\ J.s,\, c = 3\times10^8\ m/s,\ k_B = 1.380\times10^{-23}\ J/K)$

(a) 2.03×10^{-5} (b) 4.02×10^{-5} (c) 7.02×10^{-5} (d) 9.83×10^{-5}

47. Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(a) $k_B \ln 2$ (b) $2k_B \ln 2$ (c) $2k_B \ln 2$ (d) $3k_B \ln 4$

48. A particle is confined to a one dimensional potential box with potential

$$V(x) = 0, \qquad 0 < x < a$$
$$= \infty, \qquad \text{otherwise}$$

If the particle is subjected to a perturbation, within the box, $W = \beta x$, where β is a small constant, the first order correction to the ground state energy is

(a) 0 (b) $a\beta/4$ (c) $a\beta/2$ (d) $a\beta$

49. Consider the process $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$. The minimum kinetic energy of the muons (μ) in the centre of mass frame required to produce the pion (π) pairs at rest is _________ MeV. (Given : $m_\mu = 105\ MeV/c^2, m_\pi = 140\ MeV/c^2$)

50. A one dimensional harmonic oscillator is in the superposition of number states, $|n\rangle$, given by

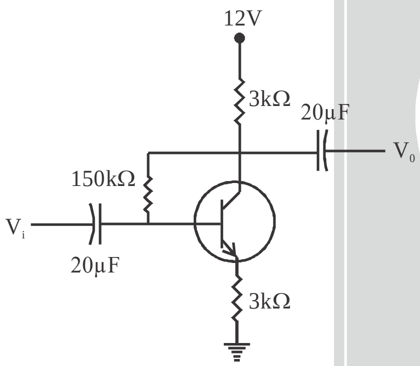
$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _____________ $\hbar\omega$.

51. A nucleus X undergoes a first forbidden β-decay to a nucleus Y. If the angular momentum (I) and parity (P), denoted by I^P as $\frac{7}{2}^-$ for X, which of the following is a possible I^P value for Y?

(a) $\frac{1}{2}^+$ (b) $\frac{1}{2}^-$ (c) $\frac{3}{2}^+$ (d) $\frac{3}{2}^-$

52. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current I_C is ______________ mA.



53. In order to measure a maximum of 1V with a resolution of 1mV using a n-bit A/D converter, working under the principle of ladder network, the minimum value of n is ___________________________

54. If L_+ and L_- are the angular momentum ladder operators, then, the expectation value of $(L_+L_- + L_-L_+)$, in the state $|\ell = 1, m = 1\rangle$ of an atom is ___________ $\hbar^2$

55. A low pass filter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively, are

(a) 0.71 and 45º (b) 0.71 and –45º (c) 0.5 and –90º (d) 0.5 and 90º