JEST - 2012

OBJECTIVE TYPE

1. A monoatomic ideal gas at 170ºC is adiabatically compressed to 1/8 of its original volume. The temperature after compression is
(a) 2.1ºC (b) 17ºC (c) –200.5ºC (d) 887ºC

2. An observer in an inertial frame finds that at a point P the electric field vanishes but the magnetic field does not. This implies that in any other inertial frame the electric field $\vec{E}$ and the magnetic field $\vec{B}$ satisfy
(a) $|\vec{E}|^2 = |\vec{B}|^2$ (b) $\vec{E}.\vec{B} = 0$ (c) $\vec{E}\times\vec{B} = 0$ (d) $\vec{E} = 0$

3. Consider a system of particles in three dimension with momentum $\vec{p}$ and energy $E = c|\vec{p}|$, c being a constant. The system is maintained at inverse temperature β, volume V and chemical potential μ. What is the grand partition function of the system?
(a) $\exp\left[e^{\beta\mu}8\pi V/(\beta ch)^3\right]$ (b) $e^{\beta\mu}6\pi V/(\beta ch)^2$
(c) $\exp\left[e^{\beta\mu}6\pi V/(\beta ch)^3\right]$ (d) $e^{\beta\mu}8\pi V/(\beta ch)^2$

4. The ground state (apart from normalization) of a particle of unit mass moving in a one-dimensional potential $V(x)$ is $\exp(-x^2/2)\cosh(\sqrt{2}x)$. The potential $V(x)$, in suitable units so that $\hbar = 1$, is (up to an additive constant).
(a) $\frac{x^2}{2}$ (b) $\frac{x^2}{2} - \sqrt{2}x\tanh(\sqrt{2}x)$
(c) $\frac{x^2}{2} - \sqrt{2}x\tan(\sqrt{2}x)$ (d) $\frac{x^2}{2} - \sqrt{2}x\coth(\sqrt{2}x)$

5. A magnetic field $\vec{B} = B_0(\hat{i} + 2\hat{j} - 4\hat{k})$ exists at point. If a test charge moving with a velocity, $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$ experiences no force at a certain point, the electric field at that point in SI units is
(a) $\vec{E} = -v_0B_0(3\hat{i} - 2\hat{j} - 4\hat{k})$ (b) $\vec{E} = -v_0B_0(\hat{i} + \hat{j} + 7\hat{k})$
(c) $\vec{E} = v_0B_0(14\hat{j} + 7\hat{k})$ (d) $\vec{E} = -v_0B_0(14\hat{j} + 7\hat{k})$

6. For small angular displacement (i.e. $\sin\theta \approx \theta$), a simple pendulum oscillates harmonically. For larger displacements, the motion
(a) beomes a periodic (b) remains periodic with the same period
(c) remains periodic with a higher period (d) remains periodic with a lower period.

7. A plant orbits a massive star in a highly elliptical orbit, i.e. the total energy E is close to zero. The initial distance of closest approach is R_0. Energy is dissipated through tidal motions until the orbit is circularized with a final radius of R_f. Assume that orbital angular momentum is conserved during the circularization process. Then
(a) $R_f = R_0/2$ (b) $R_f = R_0$ (c) $R_f = \sqrt{2}R_0$ (d) $R_f = 2R_0$

8. A binary system consists of two stars of equal mass 'm' orbitting each other in a circular orbit under the influence of gravitational forces. The period of the orbit is τ. At $t = 0$, the motion is stopped and the stars are allowed to fall to towards each other. After what time *t*, expressed in terms of τ, do they collide? The following integral may be useful $\left(x = r^{1/2}\right)$.

$$\int \frac{x^2 dx}{\sqrt{\alpha - x^2}} \frac{x}{2}\sqrt{\alpha - x^2} + \frac{\alpha}{2}\sin^{-1}\left(\frac{x}{\sqrt{\alpha}}\right)$$

(a) $\sqrt{2}\tau$ (b) $\frac{\tau}{\sqrt{2}}$ (c) $\frac{\tau}{2\sqrt{2}}$ (d) $\frac{\tau}{4\sqrt{2}}$

9. Consider a system maintained at temperature T, with two available energy states E_1 and E_2 each with degeneracies g_1 and g_2. If p_1 and p_2 are probabilities of occupancy of the two energy states, what is the entropy of the system?

(a) $S = -k_B\left[p_1 \ln\left(p_1/g_1\right) + p_2 \ln\left(p_2/g_2\right)\right]$ (b) $S = -k_B\left[p_1 \ln\left(p_1 g_1\right) + p_2 \ln\left(p_2 g_2\right)\right]$

(c) $S = -k_B\left[p_1 \ln\left(p_1^{g_1}\right) + p_2 \ln\left(p_2^{g_2}\right)\right]$ (d) $S = -k_B\left[\left(1/p_1\right)\ln\left(p_1/g_1\right) + \left(1/p_2\right)\ln\left(p_2/g_2\right)\right]$

10. Consider the Bohr model of the hydrogen atom. If α is the fine-structure constant, the velocity of the electron in its lowest orbit is

(a) $\frac{c}{1+\alpha}$ (b) $\frac{c}{1+\alpha^2}$ or $(1-\alpha)c$

(c) $\alpha^2 c$ (d) αc

11. Consider a particle of mass 'm' moving inside a two-dimensional square box whose sides are described by the equations $x = 0,\ x = L,\ y = 0,\ y = L$. What is the lowest eigenvalue of an eigenstate which changes sign under the exchange of x and y?

(a) $\frac{\hbar^2}{mL^2}$ (b) $\frac{3\hbar^2}{2mL^2}$ (c) $\frac{5\hbar^2}{2mL^2}$ (d) $\frac{7\hbar^2}{2mL^2}$

12. Consider a ideal gas of mass 'm' at temperature T1 which is mixed isobarically (i.e. at constant pressure) with an equal mass of same gas at temperature T2 in a thermally insusulated container. What is the change of entropy of the universe?

(a) $2mC_p \ln\left(\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right)$ (b) $2mC_p \ln\left(\frac{T_1-T_2}{2\sqrt{T_1T_2}}\right)$

(c) $2mC_p \ln\left(\frac{T_1+T_2}{2T_1T_2}\right)$ (d) $2mC_p \ln\left(\frac{T_1-T_2}{2\sqrt{T_1T_2}}\right)$

13. In a certain intertial frame two light pulses are emitted at point 5 km apart and separated in time by 5 μs. An observer moving at a speed V along the line joining these points notes that the pulses are simultaneous. Therefore V is

(a) 0.7c (b) 0.8c (c) 0.3c (d) 0.9c

14. A circular conducting ring of radius R rotates with constant angular velocity ω about its diameter placed along the *x*-axis. A uniform magnetic field B is applied along the *y*-axis. If at time $t = 0$ the ring is entirely in the xy-plane, the emf induced in the ring at time $t > 0$ is

(a) $B\omega^2\pi R^2 t$ (b) $B\omega^2\pi R^2 \tan(\omega t)$ (c) $B\omega^2\pi R^2 \sin(\omega t)$ (d) $B\omega^2\pi R^2 \cos(\omega t)$

15. Define $\sigma_x = \left(f^\dagger + f\right)$ and $\sigma_y = -i\left(f^\dagger - f\right)$, where the σ' are Pauli spin matrices and $f, f^\dagger$ obey anticommutation relations $\{f, f\} = 0, \{f, f^\dagger = 1\}$. Then σ_z is given by

(a) $f^\dagger f - 1$ (b) $2f^\dagger f - 1$ (c) $2f^\dagger f + 1$ (d) $f^\dagger f$

16. The value of integral $\int_0^\infty \frac{\ln x}{\left(x^2+1\right)^2} dx$ is

(a) 0 (b) $-\frac{\pi}{4}$ (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

17. An electric field in a region is given by $\vec{E}(x, y, z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$. For which values of a, b, c does this represent an electrostatic field?

(a) 13, 1, 12 (b) 17, 6, 1 (c) 13, 1, 16 (d) 45, 6, 1

18. If $[x]$ denotes the greatest integer not exceeding x, then $\int_0^\infty [x] e^{-x} dx$

(a) $\frac{1}{e-1}$ (b) 1 (c) $\frac{e-1}{e}$ (d) $\frac{e}{e^2-1}$

19. Consider a system of two spin-1/2 particles with total spin $\vec{S} = \vec{s}_1 + \vec{s}_2$, where s_1 and s_2 are in terms of Pauli matrices σ_i. The spin triplet projection operator is

(a) $\frac{1}{4} + s_1.s_2$ (b) $\frac{3}{4} - s_1.s_2$ (c) $\frac{3}{4} + s_1.s_2$ (d) $\frac{1}{4} - s_1.s_2$

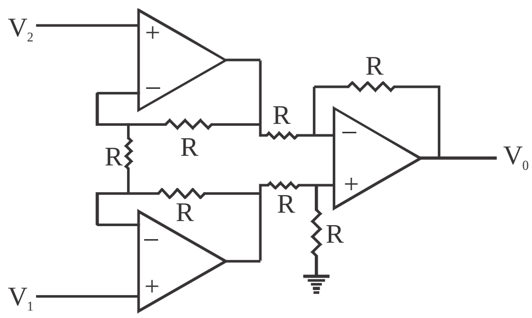
20. A collection of N two-level systems with energies 0 and E > 0 is in thermal equilibrium at temperatures T. For $T \to \infty$, the specific heat approaches.

(a) 0 (b) Nk_B (c) $3Nk_B/2$ (d) ∞

21. Consider a spin-1/2 particle in the presence of homogeneous magnetic field of magnitude B along z-axis which is prepared initially in a state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$ at time t = 0. At what time 't' will the particles be in the state $-|\psi\rangle$ (μ_B is Bohr magneton)?

(a) $t = \frac{\pi\hbar}{\mu_B B}$ (b) $t = \frac{2\pi\hbar}{\mu_B B}$ (c) $t = \frac{\pi\hbar}{2\mu_B B}$ (d) Never

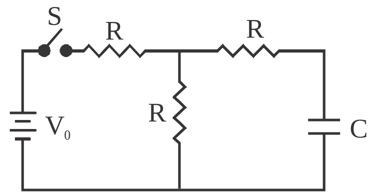
22. The classic three op-amp instrumentation amplifier configuration is shown below:



The op-amp are ideal and all resistors are of equal value R. The gain, defined as the output voltage V_0 divided by the differential input voltage $V_1 - V_2$, is equal to

(a) 2 (b) 3 (c) 4 (d) 6

23. A capacitor C is connected to a battery V_0 through three equal resistors R and a switch S as shown below:



The capacitor is initially uncharged. At time t = 0, the switch S is closed. The voltage across the capacitor as a function of time 't' for t > 0 is given by

(a) $(V_0/2)(1-\exp)(-t/2RC)$ (b) $(V_0/3)(1-\exp)(-t/3RC)$

(c) $(V_0/3)(1-\exp)(-3t/2RC)$ (d) $(V_0/2)(1-\exp)(-2t/3RC)$

24. As $x \to 1$, the infinite series $x - \frac{1}{3}x^3 + \frac{1}{5}x^4 - \frac{1}{7}x^7 + ...$

(a) diverges (b) converges to unity (c) converges to $\frac{\pi}{4}$ (d) none of the above.

25. A beam of X-rays is incident on a BCC crystal. If the difference between the incident and scatered wavevectors is $\vec{K} = h\hat{x} + k + \ell\hat{z}$ where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors of the associated cubic lattice, the necessary condition for the scattered beam to give a Laue maximum is

(a) $h + k + =$ even (b) $h = k = \ell$ (c) h, k, ℓ are distinct (d) $h + k + \ell = odd$

Part-B: One Mark Questions:

26. A small magnet is dropped down a long vertical copper tube in a uniform gravitational field. After a long time, the magnet

(a) attains a constant velocity (b) moves with a constant acceleration

(c) moves with a constant decelaration (d) executes simple harmonic motion

27. The grond state energy of 5 identical spin-1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency ω is

(a) $\frac{(15/2)}{\hbar\omega}$ (b) $\frac{(13/2)}{\hbar\omega}$ (c) $\frac{(1/2)}{\hbar\omega}$ (d) $5\hbar\omega$

28. Efficiency of a perfectly reversible (Carrot) heat engine operating between absolute temperature T and zero is equal to

(a) 0 (b) 0.5 (c) 0.75 (d) 1

29. Consider a particle of electric charge 'e' and mass 'm' moving under the influence of a constant horizontal electric field E and constant vertical gravitational field described by acceleration due to gravity g. If the particle starts from rest, what will be its trajectory?

(a) parabolic (b) elliptic (c) straight line (d) circular

30. A thermally insulated ideal gas of volume V1 and temperature T expands to another enclosure of volume V2 through a prous plug. What is the change in the temperature of the gas?

(a) 0 (b) $T \ln \frac{V_1}{V_2}$ (c) $T \ln \frac{V_2}{V_1}$ (d) $T \ln\left[(V_2 - V_1)/V_2\right]$

31. The spatial part of a two-electron state is symmetric under exchange . If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and spin-down states respectively of each particle, the spin-part of the two-particle state is

(a) $|\uparrow\rangle|\uparrow\rangle$ (b) $|\uparrow\rangle|\downarrow\rangle$

(c) $(|\downarrow\rangle|\uparrow\rangle - |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$ (d) $(|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle)/\sqrt{2}$

32. The ratio of maximum to minimum resistance that can be obtained with $N 1-\Omega$ resistors is
(a) N (b) N^2 (c) 1 (d) ∞

33. The wave function of a free particle in one dimension is given by $\psi(x) = A\sin x + B\sin 3x$. Then $\psi(x)$ is an eigenstate of
(a) the position operator (b) the Hamiltonian
(c) the momentum operator (d) the parity operator

34. A point charge +q is placed at (0, 0, d) above a grounded infinite coducting plane defined by z = 0. There are no charges present anywhere else. What is the magnitude of electric field at (0, 0, –d)?
(a) $\frac{q}{(8\pi\varepsilon_0 d^2)}$ (b) $-\infty$ (c) 0 (d) $\frac{q}{(16\pi\varepsilon_0 d^2)}$

35. What is the value of the following series?

$$\left(1-\frac{1}{2!}+\frac{1}{4!}-........\right)^2+\left(1-\frac{1}{3!}+\frac{1}{5!}-....\right)^2$$

(a) 0 (b) e (c) e^2 (d) 1

36. The second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle of thirty degrees of the crystal plane. The distance between the lattice planes is
(a) 1 Angstrom (b) 2 Angstrom (c) 4 Angstrom (d) 8 Angstrom

37. A jet of gas consists of molecules of mass m, speed v and number density 'n' all moving co-linearly. This jet hits a wall at an angle θ to the normal. The pressure exerted on the wall at an angle θ to the normal. The pressure exerted on the wall by the jet assuming elastic collision will be
(a) $p = 2mnv^2\cos^2\theta$ (b) $p = 2mnv^2\cos\theta$ (c) $p = \sqrt{(3/2)}mnv\cos^2\theta$ (d) $p = mnv^2$

38. A time-dependent magnetic field $\vec{B}(t)$ is produced in a circular region of space, infinitely long and of radius R. The magnetic field is given as $\vec{B} = B_0 t\hat{z}$ for $0 \le r < R$ and is zero for $r > R$, where B_0 is a positive constant. The electric field for $r > R$ is
(a) $\frac{B_0R^2}{r}\hat{r}$ (b) $\frac{B_0R^2}{r}\hat{\theta}$ (c) $-\frac{B_0R^2}{r}\hat{r}$ (d) $-\frac{B_0R^2}{r}\hat{\theta}$

39. If the coordinate q and the momentum p from a canonical pair (q, p), which one of the sets given below also forms a canonical?
(a) $\frac{B_0R^2}{r}\hat{r}$ (b) $\frac{B_0R^2}{2r}\hat{\theta}$ (c) $-\frac{B_0R^2}{r}\hat{r}$ (d) $-\frac{B_0R^2}{2r}\hat{\theta}$

40. The net charge of an n-type semiconductor is
(a) positive (b) zero (c) negative (d) dependent on the dopant density

41. The length and radius of a perfectr cylinder are each measured with an RMS error of 1%. The RMS error on the inferred volume of the cylinder is roughtly
(a) 1.7% (b) 3.3% (c) 0.5% (d) 1%

42. When unpolarised light is incident on a glass plate at a particular angle, it is observed that the reflected beam is linearly polarized. What is the angle of the refracted beam with respect to the surface normal? Refractive index of glass is 1.52
(a) 56.7º (b) 33.4º (c) 23.3º
(d) The light is completely reflected and there is no refracted beam.

43. The Dulong-Petit law fails near room temperature (300K) for many light elements (such as boron and beryllium) because their Debye temperature is

(a) >> 300 K (b) ~ 300 K (c) << 300 K (d) 0 K

44. An unbiased die is cast twice. The probability that the positive difference (bigger-smaller) between the two numbers is 2 is

(a) 1/9 (b) 2/9 (c) 1/6 (d) 1/3

45. A cube has a constant electric potential V on its surface. If there are no charges inside the cube, the potential at the centre of the cube is

(a) V (b) V/8 (c) 0 (d) V/6

46. A girl measurses the period of simple pendulum inside a stationary lift and finds it to be T seconds. If the lift accelerates upward with an accleration g/4, then the period of the pendulum will be

(a) T (b) T/4 (c) $2T/\sqrt{5}$ (d) $2T\sqrt{5}$

47. The quantum state $\sin x|\uparrow\rangle + \exp(i\phi)\cos x|\downarrow\rangle$, where $\langle\uparrow||\downarrow\rangle = 0$ and x, ϕ are, real, is orthogonal to:

(a) $\sin x|\uparrow\rangle$ (b) $\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$

(c) $-\cos x|\uparrow\rangle - \exp(i\phi)\sin x|\downarrow\rangle$ (d) $-\exp(i\phi)\cos x|\uparrow\rangle + \sin x|\downarrow\rangle$

48. The binding energy of the hydrogen atom (electron bound to proton) is 13.6 eV. The binding energy of positronium (electron bound to positron) is

(a) 13.6/2 eV (b) 13.6/180 eV (c) 13.6×1810 eV (d) 13.6×2 eV

49. For an N×N matrix consisting of all ones,

(a) all eigenvalues = 1 (b) all eigenvalues = 0

(c) the eigenvalues are 1, 2, ..., N (d) one eigenvalue = N, the others = 0

50. A sodium atom in the first excited 3P states has a lifetime of 16 ns for decaying to the ground 3S state. The wavelength of the emitted photon is 589 nm. The corresponding line width of the transition (in frequency units) is about

(a) 1.7×10^{6} Hz (b) 1×10^{7} Hz (c) 6.3×10^{7} Hz (d) 5×10^{14} Hz