TIFR-2010

Section-A: 2×3 = 60 Marks

1. The matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

can be relatede by a similarity transformation to the matrix.

(a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

2. A car tyre is slowly pumped up to a pressure of 2 atmospheres in an environment at 15°C. At this point, it bursts. Assumng the sudden expansion of the air (a mixture of O_2 and N_2) that was inside the tyre to be adiabatic, its temperature after the burst is

(a) –55°C (b) –37°C (c) –26°C (d) +9°C

3. A small meteor approaches the Earth. When it is at a large distance, it have velocity v_∞ and impact parameter b. If R_e is the radius of the Earth and v_0 is the escape velocity, the condition for the meteor to strike the Earth is

(a) $b < R_e\sqrt{1-\left(\frac{v_0}{v_\infty}\right)^2}$ (b) $b > R_e\sqrt{1+\left(\frac{v_0}{v_\infty}\right)^2}$

(c) $b < R_e\sqrt{1+\left(\frac{v_0}{v_\infty}\right)^2}$ (d) $b = R_e\left(\frac{v_0}{v_\infty}\right)$

4. Consider a very, very thin wire of uniformly circular cros section. The diameter of the wire is of the order of microns. The correct equipment required to measure the precise value of resistivity of this wire is

(a) ammeter, voltmeter, scale, slide calipers (b) ammeter, magnet, screw gauge, thermometer

(c) voltmeter, magnet, screw gauge, scale (d) ammeter, voltmeter, scale, monochromatice laser source

5. A function f(x) is defined in the range $-1 \le x \le 1$ by

$$f(x) = \begin{matrix} 1-x & \text{for} & x \ge 0 \\ 1+x & \text{for} & x < 0 \end{matrix}$$

The first few terms in the Fourier series approximating this function are

(a) $\frac{1}{2}+\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x+\ldots$ (b) $\frac{1}{2}+\frac{4}{\pi^2}\sin\pi x+\frac{4}{9\pi^2}\sin 3\pi x+\ldots$

(c) $\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x+\ldots$ (d) $\frac{1}{2}-\frac{4}{\pi^2}\cos\pi x+\frac{4}{9\pi^2}\cos 3\pi x-\ldots$

6. A lead container contains 1 gm of a ${}^{60}_{27}Co$ radioactive source. It is known that a ${}^{60}_{27}Co$ nucleus emits a β particle of energy 316 KeV followed by two γ emissions 1173 and 1333 KeV respectively. Which of the following experimental methods would be the best way to determine the lifetime of the ${}^{60}_{27}Co$ source?

(a) Measure the change in temperature of the source

(b) Measure the weight of the source now and again after one year

(c) Measure the recoil momentum of the nucleus during β emission

(d) Measure the number of γ photons emitted by this source

7. A beam of hydrogen molecules travels in the z direction with a kinetic energy of 1 eV. The molecules are in an excited state, from which they decay and dissociate into two hydrogen atoms. When one of the dissociated atoms has its final velocity perpendicular to the z direction, its kinetic energy is always 0.8 eV. The energy released in the dissociative reaction is

(a) 0.26 eV (b) 2.6 eV (c) 0.36 eV (d) 3.6 eV

8. Two parallel plates of metal sandwich a dielectric pad of thickness d, forming an ideal capacitor of capacitance C. The dielectric pad is elastic, having a spring constant k. If an ideal battery of voltage V across its terminals is connected to the two plates of this cpacitor, the fractional change $\delta d/d << 1$ in the gap between the plates is

(a) zero (b) $+\frac{\frac{1}{2}CV^2}{kd^2}$ (c) $-\frac{\frac{1}{2}CV^2}{kd^2+CV^2}$ (d) $-\frac{\frac{1}{2}CV^2}{kd^2-CV^2}$

9. When white light is scattered from a liquid, a strong absorption line is seen at 400 nm, and two emission lines are observed, one of which is at 500 nm, and another in the infra-red portion of the spectrum. The wavelength of this second emission line is

(a) 900 nm (b) 2000 nm (c) 100 nm (d) 222 nm

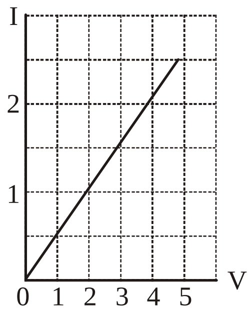
10. A detector is used to count the numbr of γ rays emitted by a radioactive source. If the number of counts recorded in exactly 20 seconds is 10000, the error in the counting rate per second is

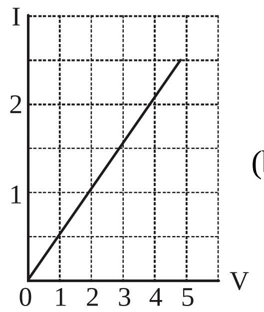
(a) $\pm$ 5.0 (b) $\pm$ 22.4 (c) $\pm$ 44.7 (d) $\pm$ 220.0

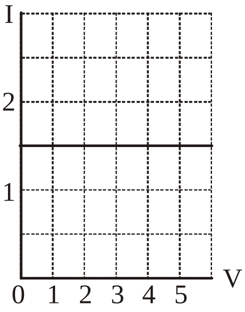
11. Consider a standard chess board with 8×8 squares. A piece starts from the lower left corner, which we shall call Square (1, 1). A single move of this piece corresponds to either one step right, i.e. to Square (1, 2) or one step forwards, i.e. to Square (2, 1). If it continues to move according to these rules, the number of different paths by which the piece can reach the Square (5,5) starting from the Square (1,1) is

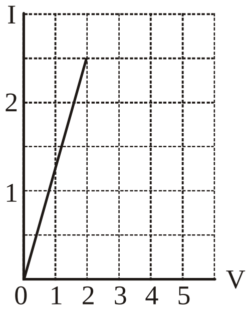
(a) 120 (b) 72 (c) 70 (d) 45

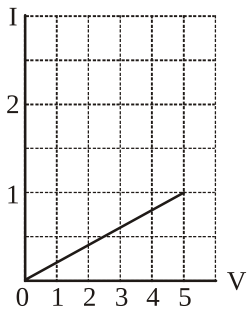
12. The uppermost graph in the set below shows the variation of current v/s voltage applied across a copper conductor at temperature T_1. Which of the graphs below — marked (a), (b), (c) or (d) — will show the possible variation of the I–V curve for the sameconductor at anothr temperature $T_2 > T_1$?



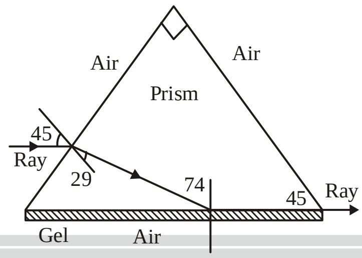
(a)


(b)


(c)


(d)


13. A ray of light is incident on a right-angled prism as shown in the figure below. The lower surface of his prism is coated with a gel. If the incident ray makes angles (marked in degrees) as shown in the figure, the refractive index of the gel must be



(a) 1.40 (b) 1.46 (c) 1.50 (d) 1.52

14. A particle $_1$ is confined in a one-dimensional infinite potential well with walls at x = ±1. Another Particle P_2 is confined in a one-dimensional infinite potential well with walls at x = 0, 1. Comparing the two particles, one can conclude that

(a) the no. of nodes in the n^{th} excited state of P_1 is twice that of P_2

(b) the no. of nodes in the n^{th} excited state of P_1 is half that of P_2

(c) the energy of the n^{th} level of P_1 is the same as that of P_2

(d) the energy of the n^{th} level of P_1 is one quarter of that of P_2

15. A charged particle is in the ground state of a one-dimensional harmonic oscillator potential, generated by electrical means. If the power is suddenly switched off, so that the potential disappears, then, according to quantum mechanics,

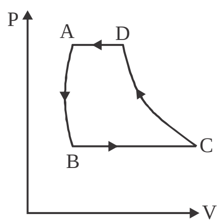
(a) the particle will shoot out of the well and move out towards infinity in one of the two possible direction as

(b) the particle will stop oscillating and as time increases it may be found farther and farther way from the centre of the well

(c) the particle will keep oscillating about the same mean position but with increasing amplitude as time increases

(d) the particle will undergo a transition to one of the higher excited states of the harmonic oscillator

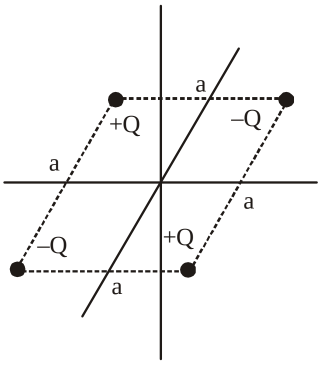
16. The pV diagram given below represents a



(a) Carnot refrigerator (b) Carnot engine

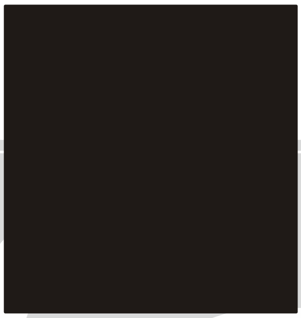
(c) gas turbine refrigerator (d) gas turbine engine

17. In the laboratory, four point charges +Q, –Q, +Q, –Q are placed at the four ends of a horiznotal square of side a, as shown in the figure below. The number of neutral points (where the electric field vanishes) is

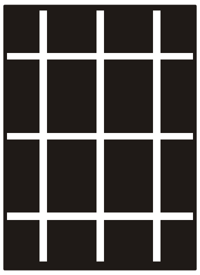
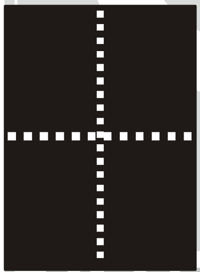
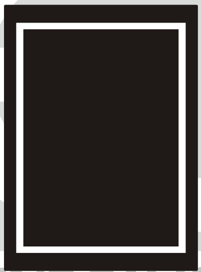
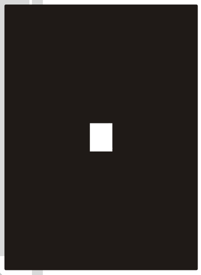


(a) ∞ (b) 4 (c) 1 (d) zero

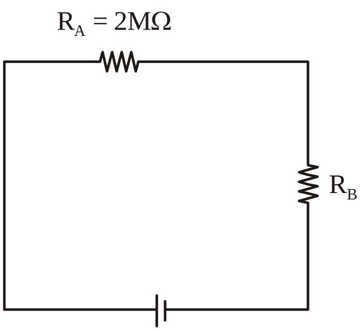
18. Coherent monochromatic light falling throught a small aperture produced a Fraunhofer diffraction pattern as shown below



By looking at this diffraction pattern carefully one can guess that the shape of the aperture was

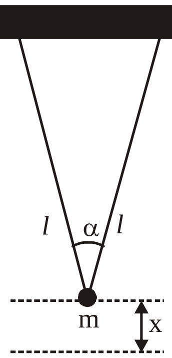
(a)  (b)  (c)  (d) 

19. In the circuit given below, a person measures 9.0 V across the battery, 3.0 V across the 2MΩ resistor R_A and 4.5 V across the unknown resistor R_B, using an ordinary voltmeter which has a finite input resistance r. Assuming that the battery has negligible internal resistance, it follows that (i) the resistance R_B and (ii) the input resistance r of the voltmeter are, in MΩ



(a) $R_B = 3.0$, r = 6.0 (b) $R_B = 2.5$, r = 7.5 (c) $R_B = 4.0$, r = 12.0 (d) $R_B = 4.5$, r = 10.0

20. A heavy mass m is suspended from two identical steel wires of length l, radius r and Young's modulus, Y, as shown in the figure below. When the mass is pulled down by distance $x(x << l)$ and released, it undergoes elastic oscillations in the vertical direction with a time period



(a) $\frac{2\pi}{r}\sqrt{\frac{ml}{2Y\cos^2\left(\frac{\alpha}{2}\right)}}$ (b) $2\pi\sqrt{\frac{l\cos\left(\frac{\alpha}{2}\right)}{g}}$

(c) $\sqrt{\frac{2\pi ml}{Yr^2}}$ (d) $\frac{2\pi}{r}\sqrt{\frac{mgl}{2Y}}$

B Section : 20 × 3 = 60 Marks

1. The wave function Ψ of a quantum mechanical system described by a Hamiltonian $\hat{H}$ can be written as a linear combination of Φ_1 and Φ_2 which are the eigenfunctions of $\hat{H}$ with eigenvalues E_1 and E_2 respectively At t = 0, the system is prepared is the state $\Psi_0 = \frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ and then allowed to evolve with time. The wavefunction at time $T = \frac{\frac{1}{2}h}{(E_1 - E_2)}$ will be (accurate to within a phase).

(a) $\frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ (b) Φ_1 (c) $\frac{4}{5}\Phi_1 - \frac{3}{5}\Phi_2$ (d) Φ_2

(e) $\frac{3}{5}\Phi_1 + \frac{4}{5}\Phi_2$ (f) $\frac{3}{5}\Phi_1 - \frac{4}{5}\Phi_2$

2. Light transmitted along an optical fibre incurs losses due to Rayleigh Scattering from inhomogeneities. If a Fibre of given length transmits 50% of the monochromatic light coupled into it at a wavelength of 1350 nm, the transmitted fraction for the same fibre at 1550 nm will be

(a) 55% (b) 57% (c) 62% (d) 67%

(e) 74% (f) 87%

3. A quantum system has three energy leels –0.12 eV, –0.2 eV and –0.44 eV respectively. Three electrons are distributed among these levels. At a temperature of 1727°C the system has total energy –0.68 eV. The free energy of the system is approximately

(a) +1.5 eV (b) +0.3 eV (c) –0.1 eV (d) –0.3 eV

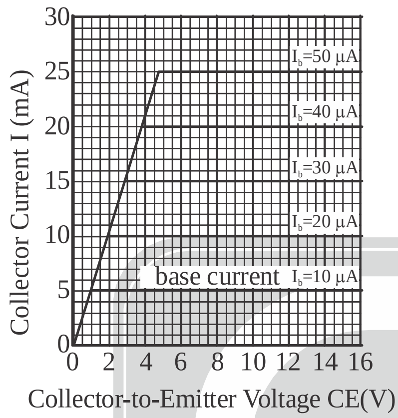
(e) –1.0 eV (f) –1.5 eV

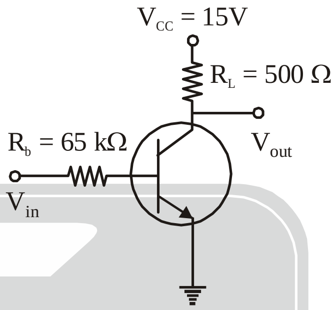
4. An atom is capable of existing in two states: a ground state of mass M and an excited state of mass $M + \Delta$. If the trranstion from the ground state tothe excited state proceeds by the absorption of a photon, the photon frequency in the laboratory frame (where the atom is initially at rest) is

(a) $\frac{\Delta c^2}{h}$ (b) $\frac{\Delta c^2}{h}\left(1+\frac{\Delta}{2M}\right)$ (c) $\frac{Mc^2}{h}$ (d) $\frac{\Delta c^2}{h}\left(1-\frac{\Delta}{2M}\right)$

(e) $\frac{Mc^2}{h}\left(1+\frac{\Delta}{2M}\right)$ (f) $\frac{Mc^2}{h}\left(1-\frac{\Delta}{2M}\right)$

5. A plot of the common-emitter characteristics of a silicon n-p-n transistor is shown below. Given this information, and assuming that there will be a 0.7 V drop across a forward biased silicon p-n junction, the approximate value of the output voltage V_{out} for an input voltage $V_{in} = 2$ V in the adjacent circuit will be





(a) 4 V (b) 6 V (c) 8 V (d) 10 V

(e) 12 V (f) 14 V

6. Measurement of the electric field (E) and the magnetic field (B) in a plane-polarized electromagnetic wave in vacuum led to the following:

$$\frac{\partial E}{\partial x} = \frac{\partial E}{\partial y} = 0 \qquad \frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial y} = 0 \qquad \frac{\partial B}{\partial z} = +\frac{\partial E}{\partial t}$$

It follows that

(a) $\vec{E} = E\hat{i}, \vec{B} = B\hat{j}$ and the wave was travelling along $\hat{k}$

(b) $\vec{E} = E\hat{j}, \vec{B} = B\hat{i}$ and the wave was travelling along $\hat{k}$

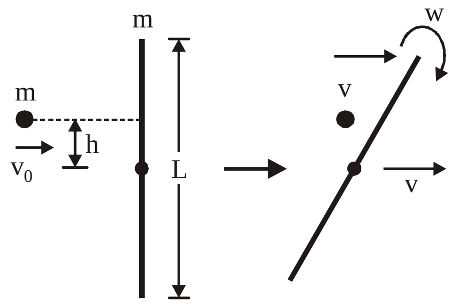
(c) $\vec{E} = E\hat{j}, \vec{B} = B\hat{k}$ and the wave was travelling along $-\hat{i}$

(d) $\vec{E} = E\hat{k}, \vec{B} = B\hat{i}$ and the wave was travelling along $\hat{j}$

(e) $\vec{E} = E\hat{i}, \vec{B} = B\hat{k}$ and the wave was travelling along $-\hat{j}$

(f) the wave was travelling along $\pm\hat{k}$ but directions of $\vec{E}$ and $\vec{B}$ are not uniquely defined

7. A mass m travels in a straight line with velocity v_0 perpendicular to a uniform stick of mass m and length L, which is initially at rest. The distance from the centre of the stick to the path of the travelling mass is h(see figure). Now the travelling mass m collides elastically with the stick, and the centre of the stick and the mass 'm' are observed to move with equal speed v after the collision. Assuming that the travelling mass m can be treated as a point mass, and the moment of inertia of the stick about its center is $I = \frac{mL^2}{12}$, it follows that the distance h must be



(a) $\frac{L}{2}$ (b) $\frac{L}{4}$ (c) $\frac{L}{\sqrt{6}}$ (d) $\frac{L}{\sqrt{3}}$

(e) $\frac{L}{3}$ (f) zero

8. The binding energy per uncleon fo ^{235}U is 7.6 MeV. The ^{235}U nucleus undergoes fission to produce two fragments, both having binding energy per nucleon 8.5 MeV. the energy released, in Joules, from the complete fission of 1 kG of ^{235}U is therefore,

(a) 8000 (b) 10^{35} (c) 450 (d) 20000

(e) 8.7×10^{13} (f) 5.0×10^{8}