TIFR-2011

Section-A: 2×3 = 60 Marks

1. The infinite series

$$x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + ...$$

where $-1 < x < +1$, can be summed to the value

(a) tan hx (b) $\ln\left(1 - \frac{4}{\pi}\tan^{-1} x\right)$ (c) $\frac{1}{2}\ln\left[\frac{(1+x)}{(1-x)}\right]$ (d) $\frac{1}{2}\ln\left[\frac{(1-x)}{(1+x)}\right]$

2. A 100 page book is known to have 200 printing errors distributed randomly through the pages. The probability that one of the pages will be found to be completely free of errors is closest to

(a) 67% (b) 50% (c) 25% (d) 13%

3. Consider the matrix

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

A 3-dimensional basis formed by eigenvectors of M is

(a) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

4. Two solid spheres S_1 and S_2 of the same uniform density fall from rest under gravity in a viscous medium and, after some time, each terminal velocities, v_1 and v_2 respectively. If the masses of S_1 and S_2 are m_1 and m_2 respectively, and $v_1 = 4\,v_2$, then the ratio m_1/m_2 is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) 4 (d) 8

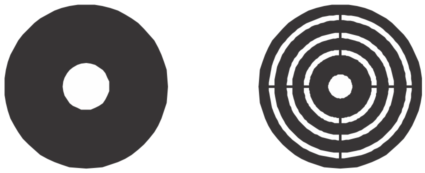
5. The dynamics of a particle of mass m is described in terms of three generalized co-ordinates ξ, η and φ. If the Lagrangian of the system is

$$L = \frac{1}{8}m\left[(\xi+\eta)\left(\frac{\dot{\xi}^2}{\xi^2} + \frac{\dot{\eta}^2}{\eta^2}\right) + 4\xi\eta\dot{\varphi}^2\right] + \frac{1}{8}k(\xi+\eta)^2$$

where k is a constant, then a conserved quantity in the system will be

(a) $(m+k)\left(\dot{\xi}+\dot{\eta}\right)$ (b) $m\,\xi\eta\,\dot{\varphi}$ (c) $m\left(\frac{\dot{\xi}^2}{\eta^2}+\frac{\dot{\eta}^2}{\xi^2}\right)$ (d) $m(\xi+\eta)\left(\frac{\dot{\xi}}{\xi^2}+\frac{\dot{\eta}}{\eta^2}\right)$

6. A scientist is given two heavy spheres made of the same metal, which have the same diameter and weight, and is asked to distinguish the spheres, without damaging them in any way. Though the spheres look identical, one of them is actually a hollow spherical shell, while the other is a set of concentric shells mounted on four thin rods of the same metal (see figure).



To make this distinction, the scientist must perform an experiment where each sphere is

(a) set rotating under the action of a constant torque

(b) made into the bob of a long simple pendulum and set oscillating

(c) immersed fully in a non-corrosive liquid and then weighed

(d) given the same electric charge Q and the potential is measured

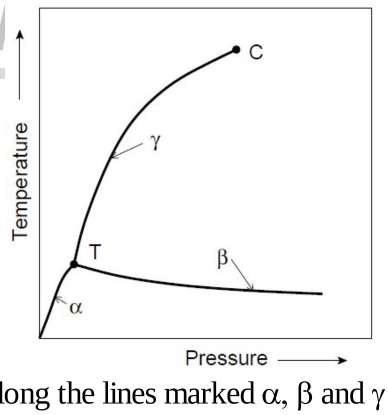
7. A narrow beam of light of wavelength 589.3 nm from a sodium lamp is incident normally on a diffraction grating of transmission type. If the grating constant is 1000000 m^{-1}, the number of principal maxima observed in the transmitted light will be

(a) 7 (b) 5 (c 3 (d) 1

8. A closed, thermally-insulated box contains one mole of an ideal monatomic gas G in thermodynamic equilibrium with blackbody radiation B. The total internal energy of the system is $U = U_G + U_B$ where U_G and $U_B(\alpha T^4)$ are the energies of the ideal gas and the radiation respectively. If $U_G = U_B$ at a certain temperature T_0 K, then the energy required to raise the temperature from T_0 K to $(T_0 + 1)$ K, in terms of the gas constant R, is

(a) 7.5 R (b) 6 R (c) 1.5 R (d) 0.33 R

9. The phase diagram of a pure substance is given in the figure below, where 'T' denotes the triple point and 'C' denotes the critical point.



The phase transitions occurring along the lines marked α, β and γ are

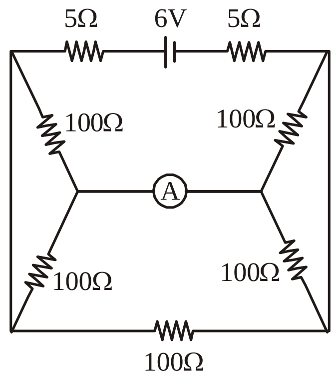
(a) α = melting; β = condensation; γ = sublimation

(b) α = sublimation; β = vaporisation; γ = melting

(c) α = melting; β = vaporisation; γ = condensation

(d) α = sublimation; β = melting; γ = vaporisation

10. The current read by the ammeter (A) in the circuit given below is



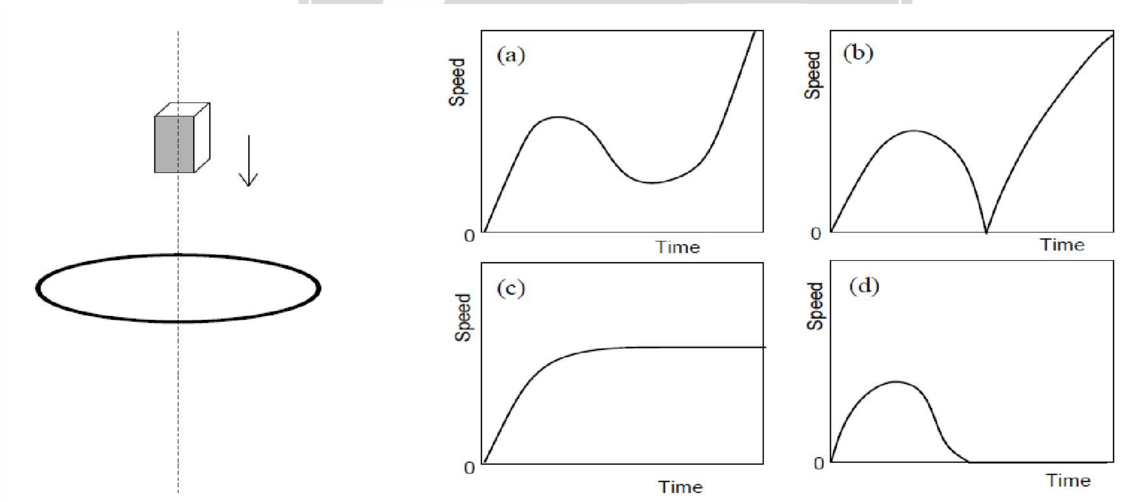
(a) 27.3 mA (b) 100.0 mA (c) 54.5 mA (d) 50.0 mA

11. The sign of the majority charge carriers in a doped silicon crystal is to be determined experimentally. In addition to a voltage supply, the combination of instruments needed to perform the experiment is

(a) thermometer, Voltmeter and Ammeter (b) Pickup Coil, Voltameter and Ammeter

(c) Magnet, voltmeter and Ammeter (d) Heater, Magnet and Thermometer

12. A small but very powerful bar magnet falls from rest under gravity through the centre of a horizontal ring of conducting wire, as shown in the figure below (on the left). The speed-versus-time graph, in arbitrary units, of the magnet will correspond most closely to which of the four plots below (on the right)?



13. The spectra of electromagnetic radiation emitted by distant objects like stars and galaxies given important clues about their physical properties. In this context, a correct statement is that

(a) the nuclear structure of the distant objects cannot be determined from lines in the visible region of the spectrum

(b) absorption lines in the spectra of distant objects do not carry information about their motion in a direction transverse to the line of sight

(c) the wavelengths in the emission spectrum of an element in a star are always the same as those found in laboratory experiments

(d) absorption spectra cannot be used to determine which molecules are present in the distant objects

14. Given that the ionization energies of Hydrogen (^1H) and Lithium (^3Li) are 13.6 eV and 5.39 eV, respectively, the effective nuclear charge experienced by the valence electron of a ^3Li atom may be estimated in terms of the proton charge e as

(a) 0.63 e (b) 1.26 e (c) 1.59 e (d) 3.00 e

15. Two identical non-interacting particles, each of mass m and spin 1/2, are placed in a one-dimensional box of length L. In quantum mechanics, the lowest possible value of the total energy of these two particles is ε_0. If, instead, four such particles are introduced into a similar one-dimensional box of length 2L, then the lowest possible value of their total energy will be

(a) $2\varepsilon_0$ (b) $5\varepsilon_0/4$ (c) $3\varepsilon_0/2$ (d) ε_0

16. An excited atomic electron undergoes a spontaneous transition, $3d_{3/2} \rightarrow 2p_{1/2}$

The interaction responsible for this transition must be of the type

(a) electric dipole (E1) OR magnetic quadrupole (M2)

(b) electric dipole (E1) OR magnetic dipole (M1)

(c) electric quadrupole (E2) OR magnetic quadrupole (M2)

(d) electric quadrupole (E2) OR magnetic dipole (M1)

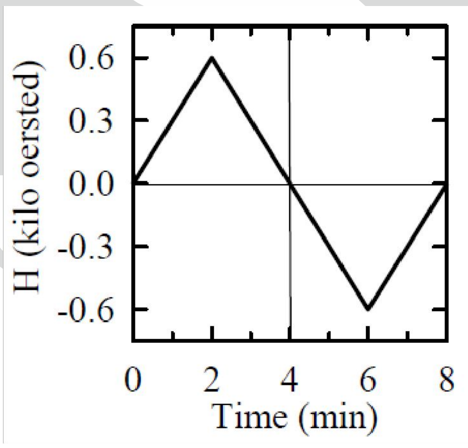
17. A fast-moving ^{14}N nucleus collides with an α particle at rest in the laboratory frame, giving rise to the reaction

$$^{14}N + \alpha \rightarrow {}^{17}O + p$$

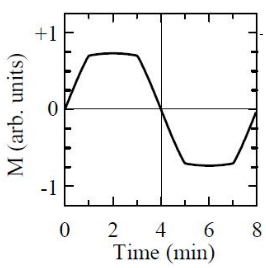
Given the masses 14.00307 a.m.u. and 16.99913 a.m.u. for ^{14}N and ^{17}O nuclei respectively, and 4.00260 a.m.u. and 1.00783 a.m.u. for α and p respectively, the minimum kinetic energy in the laboratory frame of the ^{14}N nucleus must be

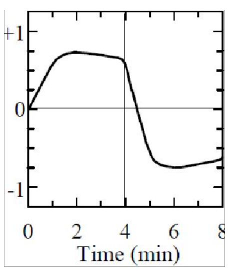
(a) 4.20 MeV (b) 1.20 MeV (c) 5.41 MeV (d) 1.55 MeV

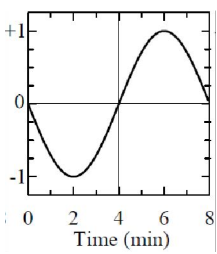
18. An unmagnetised sample of iron is placed in a magnetic field H which varies with time as shown in the plot below.

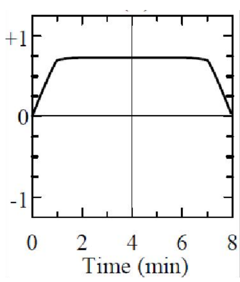


The magnetisation M of this iron sample is continuously measured and also plotted as a function of time. The appearance of this plot will be closest to

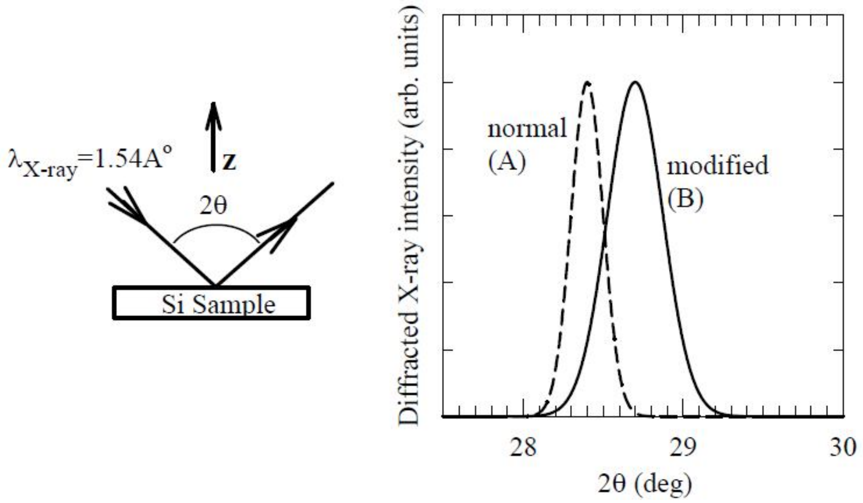
(a)


(b)


(c)


(d)


19. The figure below shows the Bragg diffraction pattern for X-rays of wavelength 1.54 A incident on two crystalline Silicon thin film Samples A and B. The dashed line corresponds to a normal Sample A and the continuous line corresponds to another Sample B, which is modified due to differences in the growth conditions.



These plots suggest that the modified sample B is

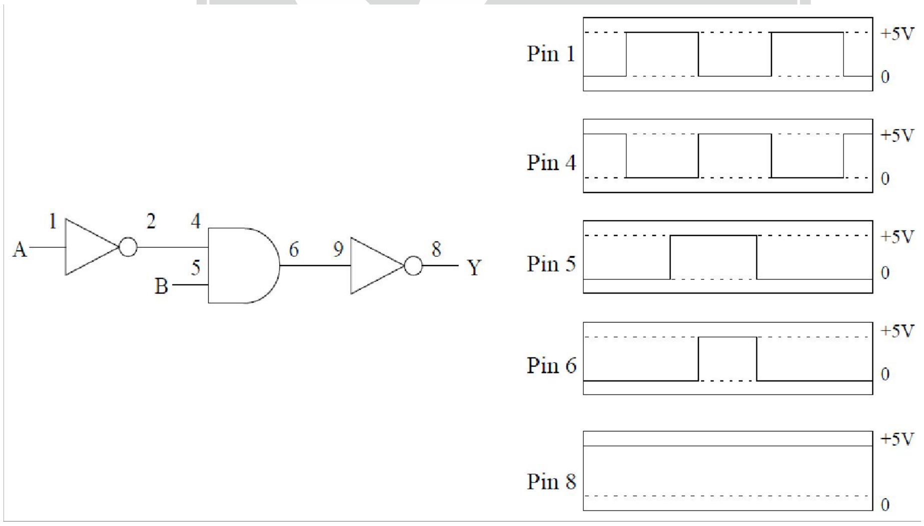
(a) stretched in all directions by 3%

(b) compressed in all directions by 3%

(c) stretched in the z direction by 1% and possible compressed in x and y directions

(d) compressed in the z direction by 1% and possibly stretched out in x and y directions

20. The digital electronic circuit shown below (left side) has some problem and is not performing as intended. The voltage at each pin as a function of time is shown in the adjacent figures.



The problem in the about circuit may be that

(a) the Pin 6 is shorted to ground

(b) the input inverter is shorted

(c) the Pin 8 is clamped to +5 V

(d) OR gate is used instead of AND gate

B Section : 8 × 5 = 40 Marks

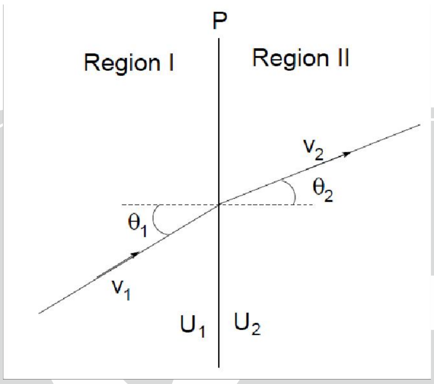
1. The trace of the real 4 × 4 matrix U = ex(A), where

$$A = \begin{pmatrix} 0 & 0 & 0 & \pi/4 \\ 0 & 0 & -\pi/4 & 0 \\ 0 & \pi/4 & 0 & 0 \\ -\pi/4 & 0 & 0 & 0 \end{pmatrix}$$

is equal to

(a) $2\sqrt{2}$ (b) $\pi/4$ (c) $\exp(i\varphi)$ for $\varphi = 0, \pi$ (d) zero

2. A region of space is divided into two parts by a plane P, as shown in the figure below. A particle of mass m passes from Region I to Region II, where it has speed v_1 and v_2 respectively. There is a constant potential U_1 in Region I and U_2 in Region II.



Let T_1 be the kinetic energy of the particles in Region I. If the trajectory of the particle is inclined to the normal to the plane P by angles θ_1 and θ_2, as shown in the figure then the ratio $\sin\theta_1/\sin\theta_2$ is given by

(a) $\sqrt{\dfrac{1-T_1}{(U_1-U_2)}}$ (b) $\sqrt{\dfrac{1+T_1}{(U_1-U_2)}}$ (c) $\sqrt{\dfrac{1-(U_1+U_2)}{T_1}}$ (d) $\sqrt{\dfrac{1+(U_1+U_2)}{T_1}}$

(e) $\sqrt{\dfrac{1-T_1}{(U_1-U_2)}}$ (f) $\sqrt{\dfrac{1+(U_1-U_2)}{T_1}}$

3. The electric field of an electromagnetic wave of angular frequency ω propagating in a medium with conductivity σ, permittivity ε and permeability μ is given by

$$E = E_0 \exp[-i(wt - kx)]$$

where the imaginary part of the complex propagation constant k is $\omega\sqrt{\dfrac{\mu\varepsilon}{2}}$ multiplied by the factor

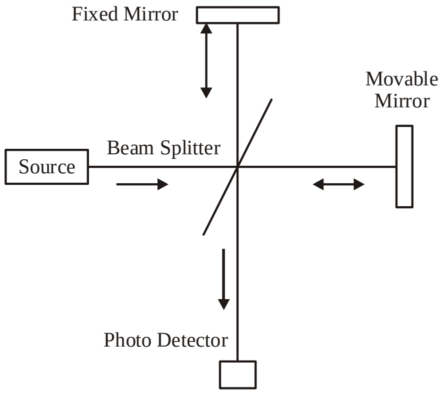
(a) $\left[\sqrt{1+\sqrt{\dfrac{\sigma}{\omega\varepsilon}}}+1\right]^{1/2}$ (b) $\left[\sqrt{1+\dfrac{\sigma}{\omega\varepsilon}}+1\right]^{1/2}$ (c) $\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2}-1\right]^{1/2}$

(d) $\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2}+1\right]^{1/2}$ (e) $\left[\sqrt{1+\sqrt{\dfrac{\sigma}{\omega\varepsilon}}}-1\right]^{1/2}$ (f) $\left[\sqrt{1+\sqrt{\dfrac{\sigma}{\omega\varepsilon}}}-1\right]^{1/2}$

4. A system having N non-degenerate energy eigenstates is populated by N identical spinzero particles and 2N identical spin-half particles. There are no interactions between any of these particles. If N = 1000, the entropy of the system is closet to

(a) 13.82 k_B (b) zero (c) 693.1 k_B (d) 1000 k_B
(e) 5909.693 k_B (f) 6909 k_B

5. The Michelson interferometer in the figure below can be used to study properties of light emitted by distant sources.



A source S_1, when at rest, is known to emit light of wavelength 632.8 nm. In this case, if the Movable Mirror is translated through a distance d, it is seen that 99,565 interference fringes pass across the Photo-Detector. For another Source S_2, moving at an uniform speed of 1.5×10^7 ms^{-1} towards the interferometer along the straight line joining it to the beam Splitter, one sees 100,068 interference fringes pass across the Photo-Detector for the same displacement d of the Movable Mirror. It follows that S_2, in its own rest frame, must be emitting light of wavelength.

(a) 661.9 nm (b) 662.8 nm (c) 598.9 nm (d) 631.2 nm
(e) 599.6 nm (f) 628.0 nm

6. A particle of mass m is placed in the ground state of a one-dimensional harmonic oscillator potential of the form.

$$V(x) = \frac{1}{2}kx^2$$

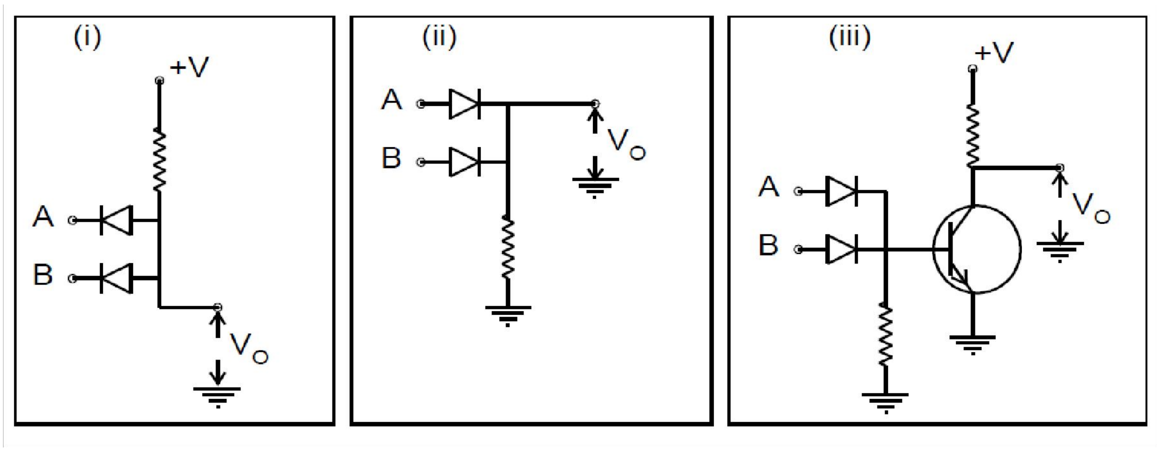
where the stiffness constant k can be varied externally. The ground state wavefunction has the form $\psi(x) \propto \exp(-ax^2\sqrt{k})$ where a is a constant. If, suddenly, the parameter k is changed to 4k, the probability that the particle will remain in the ground state of the new potential is

(a) 0.47 (b) 0.06 (c) 0.53 (d) 0.67
(e) 0.33 (f) 0.94

7. A cloud Chamber of width 0.01 m is filled with pure nitrogen gas (N_2) at normal temperature and pressure. A beam of α–particles, when incident normally no the chamber, make tracks which are visible under strong illumination. Whenever an α–particle $\left({}^4_2He\right)$ has a nuclear collision with a ${}^{14}_7N$ nucleus, the track shows a distinct bend. The radius of a nucleus is given by $r = r_0A^{1/3}$ where $r_0 = 1.217 \times 10^{-15}$ m and A is the atomic mass number. If the α particles move at non-relativistic speeds, and the total number of incident α particles is 10^7, the number of such distinct bends is approximately.

(a) 100 (b) 200 (c) 300 (d) 400
(e) 500 (f) 600

8. The three electronic circuits marked (i), (ii) and (iii) in the figure below can all work as logic gates, where the input signals are either 0V or 5 V and the output is V_0.



Identify the correct combination of logic gates (i), (ii), (iii), in the options given below.

(a) NOR, XOR, AND

(b) OR, NAND, NOR

(c) NAND, AND, XOR

(d) XOR, AND, NAND

(e) AND, OR, NOR

(f) NOR, NAND, OR