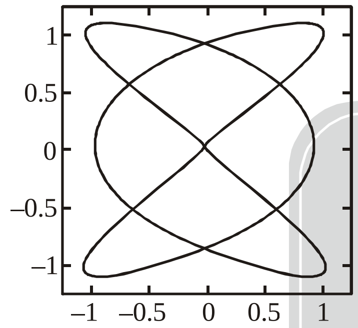
TIFR-PHYSICS-2013
Section-A

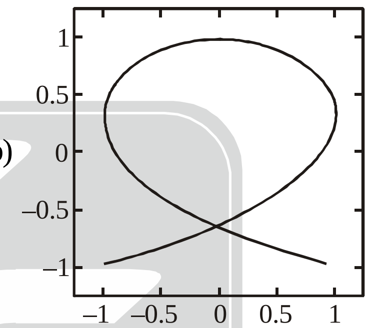
1. The value of the integral $\int_0^\infty dx\, x^9 \exp(-x^2)$ is

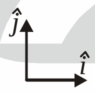
(a) 20160 (b) 12 (c) 18 (d) 24

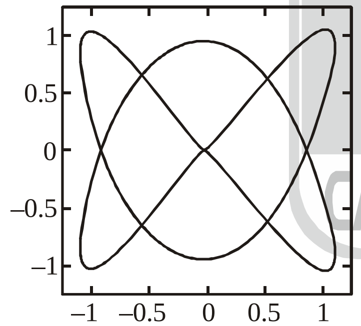
2. A two-dimensional vector $\vec{A}(t)$ is given by $\vec{A}(t) = \hat{i}\sin 2t + \hat{j}\cos 3t$

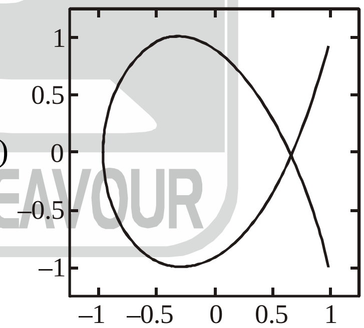
Which of the following graphs best describes the locus of the tip of the vector, as t is varied from 0 to 2π?

(a)


(b)




(c)


(d)


3. The differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ has the complete solution, in terms of arbitrary constants A and B.

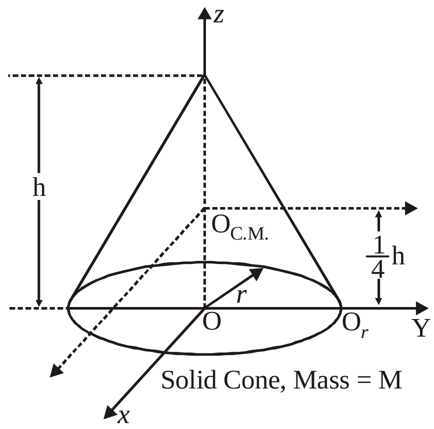
(a) $A\exp x + Bx\exp x$ (b) $A\exp x + B\exp(-x)$

(c) $A\exp x + Bx\exp(-x)$ (d) $x\{A\exp x + B\exp(-x)\}$

4. A stone is dropped vertically from the top of a tower of height 40m. At the same time a gun is aimed directly at the stone from the ground at a horizontal distance 30m from the base of the tower and fired. If the bullet from the gun is to hit the stone before it reaches the ground, the minimum velocity of the bullet must be, approximately,

(a) 57.4 ms^{-1} (b) 27.7 ms^{-1} (c) 17.7 ms^{-1} (d) 7.4 ms^{-1}

5. Consider the uniform solid right cone depicted in the figure on the right. This cone has mass M and a circular base of radius r. If the moment of inertia of the cone about an axis parallel to the X axis passing through the centre of mass $O_{C.M.}$ (see figure) is given by $\frac{3}{80}M(4r^2+h^2)$ then the moment of inertia about another axis parallel to the X axis, but passing through the point O_r (see figure), is



(a) $\frac{3}{80}M(4r^2+h^2)$ (b) $\frac{3}{40}M(2r^2+h^2)$

(c) $\frac{1}{20}M(23r^2+2h^2)$ (d) $\frac{1}{30}M(15r^2+4h^2)$

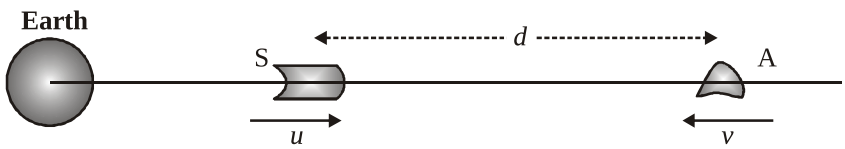
6. Two planets A and B move around the Sun in elliptic orbits with time periods T_A and T_B respectively. If the eccentricity of the orbit of B is ε and its distance of closest approach to the Sun is R, then the maximum possible distance between the planets is

[Eccentricity of an ellipse: $\varepsilon = \frac{r_{\max}-r_{\min}}{r_{\max}+r_{\min}}$]

(a) $\frac{1+\varepsilon^2}{1-\varepsilon^2}\left(1+\frac{T_A^{3/2}}{T_B^{3/2}}\right)R$ (b) $\frac{1+\varepsilon}{1-\varepsilon}\left(1+\frac{T_A^{2/3}}{T_B^{2/3}}\right)R$

(c) $\sqrt{\frac{1+\varepsilon}{1-\varepsilon}\left(1+\frac{T_A^3}{T_B^3}\right)}R$ (d) $\sqrt{\frac{1+\varepsilon^2}{1-\varepsilon^2}\left(1+\frac{T_A^{2/3}}{T_B^{2/3}}\right)}R$

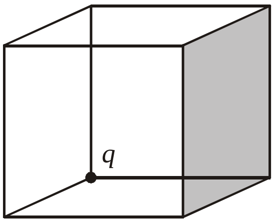
7. A spaceship S blasts off from the Earth. After some time, Earth station informs the crew that they have settled into a constant velocity $0.28c$ radially outward from the Earth, but unfortunately they are on a head-on collision course with an asteroid A at a distance of 15 light-minutes coming in towards the Earth along the same radius (see figure below).



Instruments on-board the spaceship immediately estimate the speed of the asteroid to have a constant value $0.24c$. It follows that the maximum time (in minutes) available to the crew to evacuate the ship before the collision is

(a) 60 (b) 30 (c) 29 (d) 63

8. A point charge q sits at a corner of a cube of side a, as shown in the figure on the right. The flux of the electric field vector through the shaded side is

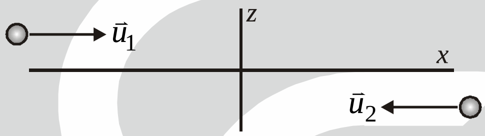


(a) $\dfrac{q}{8\varepsilon_0}$ (b) $\dfrac{q}{16\varepsilon_0}$ (c) $\dfrac{q}{24\varepsilon_0}$ (d) $\dfrac{q}{6\varepsilon_0}$

9. A parallel plate capacitor of circular cross section with radius $r \gg d$, where d is the spacing between the plates, is charged to a potential V and then disconnected from the charging circuit. If, now, the plates are slowly pulled apart (keeping them parallel) so that their separation is increased from d to d', the work done will be

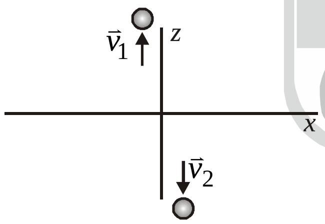
(a) $\dfrac{\pi\varepsilon_0 r^2 V^2}{2d}\left(1-\dfrac{d}{d'}\right)$ (b) $\dfrac{\pi\varepsilon_0 r^2 V^2}{2d}\left(\dfrac{d'}{d}-1\right)$ (c) $\dfrac{\pi\varepsilon_0 r^2 V^2}{2d}\dfrac{d'}{d}$ (d) $\dfrac{\pi\varepsilon_0 r^2 V^2}{2d}\dfrac{d}{d'}$

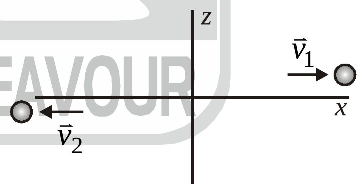
10. In the laboratory frame two electrons are shot at each other with equal and opposite velocities $\vec{u}_1$ and $\vec{u}_2$ respectively, but not along the same straight line, as shown below.

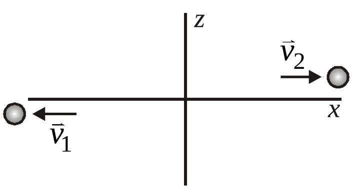


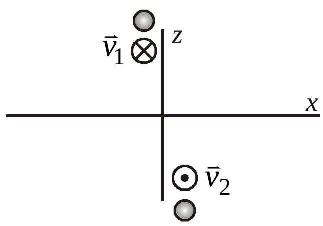
Each electron will be acted on by the Coulomb repulsion due to the other, as well as the Lorentz force due to its own motion in the magnetic field created by the other. Which of the diagrams given below best describes the final velocities $\vec{v}_1$ and $\vec{v}_2$ of these electrons?

[*You may assume that the electrons are distinguishable*]

(a)


(b)


(c)


(d)


11. A certain amount of fluid with heat capacity C_F Joules/ºC is initially at a temperature 0º C. It is then brought into contact with a heat bath at a temperature of 100º C, and the system is allowed to come into equilibrium. In this process, the entropy (in Joules / ºC) of the Universe changes by

(a) $100C_F$ (b) 0 (c) $0.055C_F$ (d) $0.044C_F$

12. A monatomic gas is described by the equation of state $p(V-bn)=nRT$ where b and R are constants and other quantities have their usual meanings. The maximum density (in moles per unit volume) to which this gas can be compressed is

(a) $\dfrac{1}{bn}$ (b) b (c) $\dfrac{1}{b}$ (d) infinity

13. In a quantum mechanical system, an observable A is represented by an operator $\hat{A}$. If $|\psi\rangle$ is a state of the system, but not an eigenstate of $\hat{A}$, then the quantity

$$r=\langle\psi|\hat{A}|\psi\rangle^2-\langle\psi|\hat{A}^2|\psi\rangle$$

satisfies the relation

(a) $r<0$ (b) $r=0$ (c) $r>0$ (d) $r\geq 0$

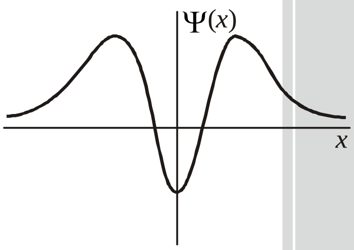
14. Consider a quantum mechanical system with three linear operators $\hat{A}$, $\hat{B}$ and $\hat{C}$, which are related by

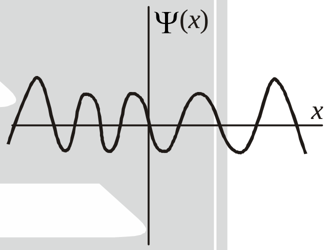
$$\hat{A}\hat{B}-\hat{C}=\hat{I}$$

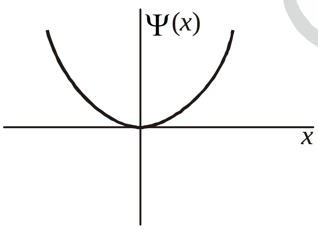
where $\hat{I}$ is the unit operator. If $\hat{A}=\dfrac{d}{dx}$ and $\hat{B}$ = x, then $\hat{C}$ must be

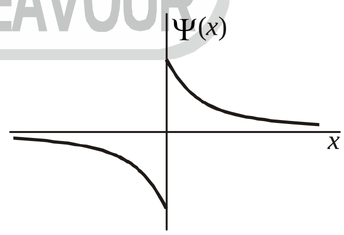
(a) zero (b) $\dfrac{d}{dx}$ (c) $-x\dfrac{d}{dx}$ (d) $x\dfrac{d}{dx}$

15. A particle of energy E moves in one dimension under the influence of a potentials $V(x)$. If $E>V(x)$ for some range of x, which of the following graphs can represent a bound state wave function of the particle?

(a)


(b)


(c)


(d)


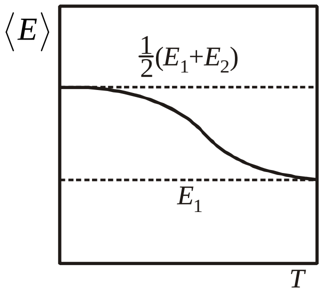
16. In a Davisson-Germer experiment, a collimated beam of electrons of energy 54eV, at normal incidence on a given crystal, shows a peak at a reflection angle of 40°. If the electron beam is replaced by a neutron beam, and the peak appears at the same value of reflection angle, then the energy of the neutrons must be

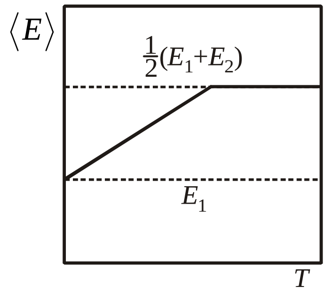
(a) 330 eV (b) 33 eV (c) 0.3 eV (d) 0.03 eV

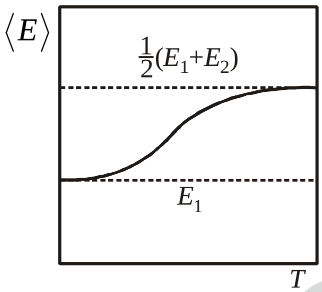
17. The velocity of an electron in the ground state of a hydrogen atom is v_H. If v_p be the velocity of an electron in the ground state of positronium, then

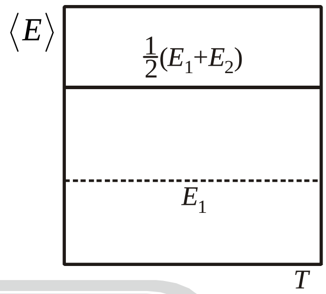
(a) $v_p=v_H$ (b) $v_p=2v_H$ (c) $v_p=\dfrac{v_H}{2}$ (d) $v_p=\sqrt{2}v_H$

18. Consider an ensemble of microscopic quantum mechanical systems with two energy levels E_1 and E_2, where $E_1 < E_2$. Which of the following graphs best describes the temperature dependence of the average energy $\langle E \rangle$ of the system?

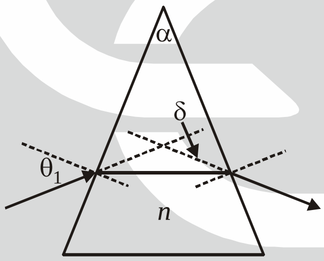
(a) 

(b) 

(c) 

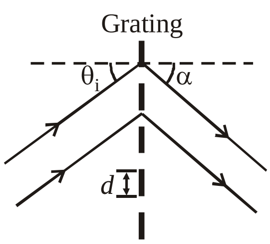
(d) 

19. A ray of light is incident on the surface of a thin prism at a small angle θ_1 with the normal, as shown in the figure on the right. The material of the prism has refractive index n and you may assume the outside refractive index to be unity. If the (small) apex angle of the prism is α, the deviation angle δ (angle between the incident and exited ray; see figure) is given by



(a) α (b) αn (c) $\alpha(n+1)$ (d) $\alpha(n-1)$

20. A parallel beam of light of wavelength λ is incident on a transmission grating with groove spacing d, at an angle θ_i, as shown in the figure on the left. The plane of incidence is normal to the grooves. After diffraction, the transmitted beam is seen to be at an angle α relative to the normal. Which of the following conditions must be satisfied for this to happen?



(a) $d(\sin\theta_i - \sin\alpha) = n\lambda$ (b) $d(\sin\theta_i + \sin\alpha) = n\lambda$

(c) $2d\sin(\theta_i - \alpha) = n\lambda$ (d) $2d\sin(\alpha + \theta_i) = n\lambda$

21. Let E_N be the energy released when one mole of pure ^{235}U undergoes controlled fission, and E_C be the energy released when one mole of pure carbon undergoes complete combustion. The ratio E_N/E_C will have the order of magnitude

(a) 10^4 (b) 10^8 (c) 10^9 (d) 10^6

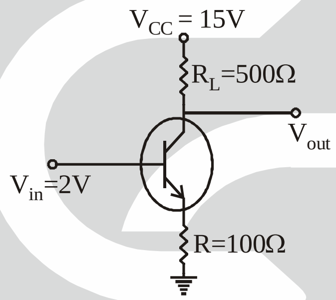
22. The entropy S of a black hole is known to be of the form

$$S = \alpha k_B A$$

where A is the surface area of the black hole and α is a constant, which can be written in terms of c (velocity of light in vacuum), $\hbar$ (reduced Planck's constant) and G_N (Newton's constant of gravitation). Taking the radius of the black hole as $R = \frac{2G_N M}{c^2}$ it follows that the entropy S is [λ is a numerical constant]

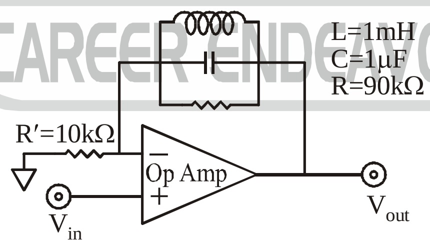
(a) $\frac{G_N^2 M^2 k_B}{\lambda(\hbar c)^4}$ (b) $\frac{\hbar c k_B}{\lambda G_N M}$ (c) $\frac{G_N^2 M^2 k_B}{\lambda \hbar c^4}$ (d) $\frac{G_N M^2 k_B}{\lambda \hbar c}$

23. The circuit depicted on the right has been made with a silicon n-p-n transistor. Assuming that there will be a 0.7V drop across a forward-biased silicon p-n junction, the power dissipated across the transistor will be, approximately,



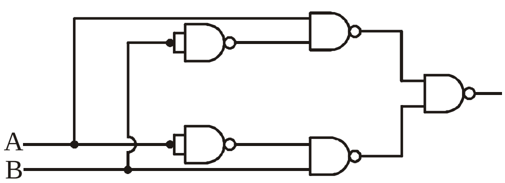
(a) 53 mW (b) 94 mW (c) 17 mW (d) 67 mW

24. An input of 1.0V DC is given to the ideal Op-Amp circuit depicted below. What will be the output voltage?



(a) 10.0 V (b) –9.0 V (c) 1.0 V (d) 0 V

25. The circuit shown below uses only NAND gates. Find the final output.



(a) A XOR B (b) A OR B (c) A AND B (d) A NOR B

Section-B

To be attempted only by candidates for Integrated Ph.D programme (Candidates for Ph.D programme will get no credit for atempting this section).

26. Consider the surface corresponding to the equation $4x^2 + y^2 + z = 0$. A possible unit tangent to this surface at the point (1, 2, –8) is

(a) $\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$

(b) $\frac{1}{5}\hat{j} - \frac{4}{5}\hat{k}$

(c) $\frac{4}{9}\hat{i} - \frac{8}{9}\hat{j} + \frac{1}{9}\hat{k}$

(d) $-\frac{1}{\sqrt{5}}\hat{i} + \frac{3}{\sqrt{5}}\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$

27. A particle with time-varying mass $m(t) = m_0\left(1-\frac{t}{\tau}\right)$, where m_0 and τ are positive constants, moves along the *x*-axis under the action of a constant positive force *F* for $0 \le t \le \tau$. If the particle is at rest at time $t = 0$, then at time *t* = *t*, its velocity *v* will be

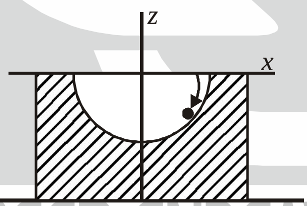
(a) $-\frac{\tau F}{m_0}\log\left(1-\frac{t}{\tau}\right)$

(b) $-\frac{Ft}{m_0}\log\frac{t}{\tau}$

(c) $\frac{Ft}{m_0}\left(1-\frac{t}{\tau}\right)^{-1}$

(d) $\frac{\tau F}{m_0}\left(1-\frac{t}{\tau}\right)$

28. A ball of mass *m* slides under gravity without friction inside a semicircular depression of radius *a* inside a fixed block placed on a horizontal surface, as shown in the figure. The equation of motion of the ball in the *x*-direction will be



(a) $\ddot{x} = \frac{g}{a}x\sqrt{1-\frac{x^2}{a^2}}$

(b) $\ddot{x} = \frac{g}{a}x$

(c) $\ddot{x} = -\frac{g}{a}x$

(d) $\ddot{x} = -\frac{g}{a}x\sqrt{1-\frac{x^2}{a^2}}$

29. A particle P, of rest mass *M* and energy *E*, suddenly decays into two particles A and B of rest masses m_A and m_B respectively, and both particles move along the straight line in which P was moving, A possible energy E_A of the particle A will be

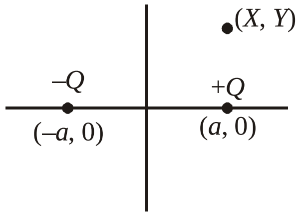
(a) $\frac{E}{2}\left\{1+\left(\frac{m_A - m_B}{M}\right)^2\right\}$

(b) $\frac{E}{2}\left\{1-\left(\frac{m_A^2 - m_B^2}{M^2}\right)\right\}$

(c) $\frac{E}{2}\left\{1+\left(\frac{m_A + m_B}{M}\right)^2\right\}$

(d) $\frac{E}{2}\left\{1+\left(\frac{m_A^2 - m_B^2}{M^2}\right)\right\}$

30. Consider two charges $+Q$ and $-Q$ placed at the points $(a, 0)$ and $(-a, 0)$ in a plane, as shown in the figure on the right. If the origin is moved to the point (X, Y), the magnitude of the dipole moment of the given charge distribution with respect to this origin will be



(a) $Q\sqrt{(a-X)^2+y^2}-Q\sqrt{(a+X)^2+y^2}$ (b) $2Qa$

(c) $Q(a-X)-Q(-a+X)$ (d) $2Qa\sqrt{X^2+Y^2}$

31. A plane electromagnetic wave travelling in a vacuum is characterised by the electric and magnetic fields

$$\vec{E}=\hat{i}(30\pi\ Vm^{-1})\exp i(\omega t+kz)$$

$$\vec{H}=\hat{j}(H_0\ Am^{-1})\exp i(\omega t+kz)$$

If $\omega, k>0$, the value of H_0 must be

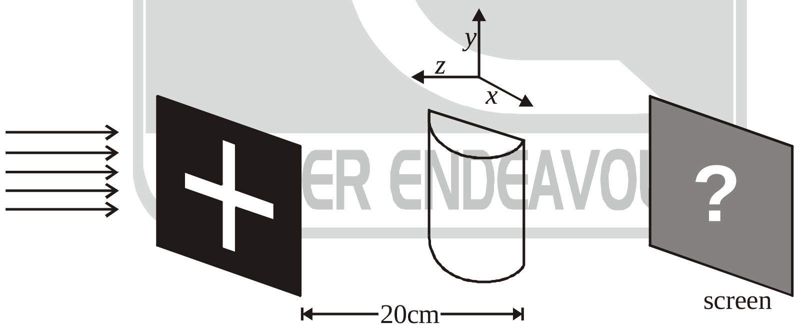
(a) 2π (b) 0.67 (c) 0.25 (d) 0.94

32. Which of the following classic experiments provides unambiguous proof that the Earth is a non-inertial frame of reference with respect to the fixed stars?

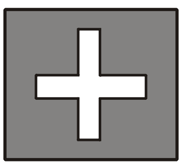
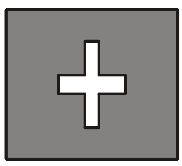
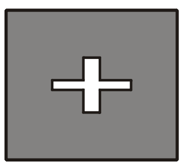
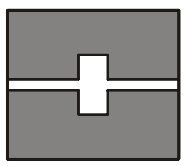
(a) Fizeau's rotating wheel experiment (b) Foucault's pendulum experiment

(c) Newton's coin-and-feather experiment (d) Michelson-Morley experiment

33. A cross-shaped opening is illuminated by a parallel beam of white light. A thin plano-convex cylindrical glass lens is placed 20cm in front of it, as shown in the figure below.



The radius of curvature of the curved surface of the lens is 5cm and 1.5 is the refractive index of glass. On a screen placed as shown at the plane where a real image forms on the other side of the lens, the image of the opening will appear as

(a)  (b)  (c)  (d) 

34. A classical ideal gas, consisting of N particles ($N \to \infty$) is confined in a box of volume V at temperature T and pressure p. The probability that, at any instant of time, a small sub-volume v_0 becomes totally void (i.e. no particles inside), due to a spontaneous statistical fluctuation, is

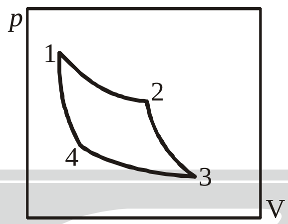
(a) $\exp\left(\frac{-v_0}{V}\right)$

(b) $\exp\left(\frac{-Nv_0}{V}\right)$

(c) $\frac{v_0}{V}\exp\left(\frac{-pV}{NT}\right)$

(d) $\frac{pv_0}{NT}$

35. The PV-diagram for a Carnot cycle executed by an ideal gas with $C_P / C_V = \gamma > 1$ is shown below. Note that 1, 2, 3 and 4 label the change-over points in the cycle. If, for this cycle, $\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^X$, then $X =$



(a) $1-\frac{1}{\gamma}$

(b) 0

(c) 1

(d) $-\frac{1}{\gamma}$

36. A harmonic oscillator has the wave function,

$$\psi(x,t) = \frac{1}{5}[3\varphi_0(x,t) - 2\sqrt{2}\varphi_1(x,t) + 2\sqrt{2}\varphi_2(x,t)]$$

where $\varphi_n(x,t)$ is the eigenfunction belonging to the n-th energy eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. The expectation value $\langle E \rangle$ of energy for the state $\psi(x,t)$ is

(a) $1.58\hbar\omega$

(b) $0.46\hbar\omega$

(c) $\hbar\omega$

(d) $1.46\hbar\omega$

37. An energy eigenstate of the Hydrogen atom has the wave function

$$\psi_{n\ell m}(r,\theta,\varphi) = \frac{1}{81\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} \sin\theta\cos\theta\exp\left[-\left(\frac{r}{3a_0}+i\varphi\right)\right]$$

where a_0 is the Bohr radius. The principal (n), azimuthal (ℓ) and magnetic (m) quantum numbers corresponding to this wave function are

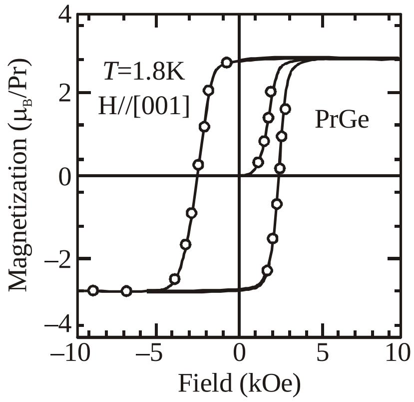
(a) $n=3, \ell=2, m=1$

(b) $n=2, \ell=1, m=1$

(c) $n=3, \ell=2, m=-1$

(d) $n=2, \ell=1, m=\pm 1$

38. The Curie temperature of a single crystal of PrGe is known to be 41K. The magnetization data of this sample is measured at 1.8K for the magnetic field applied parallel to the [001] direction is shown in the figure on the left. At a temperature of 38K, the hysteresis loop in the figure will

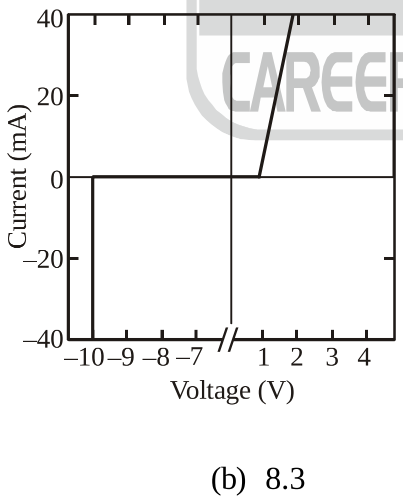


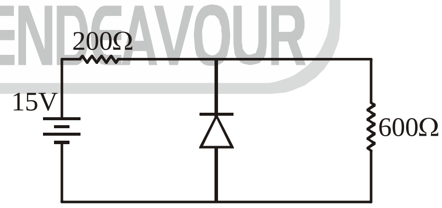
(a) have the same width
(b) increase in width
(c) decrease in width
(d) shrink to a line

39. Consider two energies of a free electron gas in a metal at an absolute temperature T, viz., $E_{\pm} = E_F \pm \Delta$ where E_F is the Fermi level. If the corresponding electron populations $n(E_{\pm})$ satisfy the relation $n(E_-)/n(E_+) = 2$, then Δ =

(a) $k_B T \ln 2$
(b) $2k_B T$
(c) $\frac{k_B T}{2}$
(d) $k_B T$

40. The figure on the right shows the current-voltage characteristics of a diode over a range of voltage and current where it is safe to operate the diode.
When this diode is used in the circuit on the extreme right, the approximate current, in mA, through the diode will be





(a) 0
(b) 8.3
(c) 16.7
(d) 25

Section-C

41. The integral $\int_{-\infty}^{\infty} dx\,\delta(x^2-\pi^2)\cos x$, evaluates to

(a) -1 (b) 0 (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

42. If $z = x+iy$ then the function $f(x,y)=(1+x+y)(1+x-y)+a(x^2-y^2)-1+2iy(1-x-ax)$ where a is a real parameter, is analytic in the complex z plane if a =

(a) -1 (b) $+1$ (c) 0 (d) i

43. A particle of mass m moves in one dimension under the influence of a potential energy

$$V(x) = -a\left(\frac{x}{\ell}\right)^2 + b\left(\frac{x}{\ell}\right)^4$$

where a and b are positive constants and ℓ is a characteristic length. The frequency of small oscillations about a point of stable equilibrium is

(a) $\frac{1}{2\pi\ell}\sqrt{\frac{b}{m}}$ (b) $\frac{1}{\pi\ell}\sqrt{\frac{a}{m}}$ (c) $\frac{1}{\pi\ell}\sqrt{\frac{a^2}{mb}}$ (d) $\frac{2b}{\pi\ell}\sqrt{\frac{1}{ma}}$

44. If a central force acting on a particle of mass m is given by

$$F(r) = -\frac{k}{r^2}$$

where r is the distance of the particle from the origin and k is a positive constant, the Hamiltonian for the system, in spherical polar coordinates, will have the form

(a) $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\varphi}^2) + \frac{k}{r}$ (b) $\frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_\varphi^2\text{cosec}^2\theta}{2mr^2} + \frac{k}{r}$

(c) $\frac{1}{2mr^2}(r^2 p_r^2 + p_\theta^2 + p_\varphi^2 + 2mkr)$ (d) $\frac{p_r^2}{2m} + \frac{p_\theta^2}{2m} + \frac{p_\varphi^2}{2m} + \frac{k}{r}$

45. The magnetic vector potential $\vec{A}(\vec{r})$ corresponding to a uniform magnetic field $\vec{B}$ is taken in the form

$$\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$$

where $\vec{r}$ is the position vector. If the electric field has the time-dependent form $\vec{E} = \vec{E}_0(\vec{r})e^{i\omega t}$, where ω is a constant, the gauge choice corresponding to this potential is a

(a) Lorenz gauge (b) non-linear gauge (c) Coulomb gauge (d) time-varying gauge

46. A binary star is observed to consist of a blue star B (peak wavelength 400 nm) and a red star R (peak wavelength 800 nm) orbiting each other. As observed from the Earth, B and R appear equally bright. Assuming that the stars radiate as perfect blackbodies, it follows that the ratio of volumes V_B/V_R of the two stars is

(a) $\frac{1}{64}$ (b) 64 (c) 16 (d) $\frac{1}{16}$

47. An system at temperature T has three energy states $0, \pm\varepsilon$. The entropy of the system in the low temperature $(T \to 0)$ and high temperature $(T \to \infty)$ limits are, respectively,

(a) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_B \exp(-3)$
(b) $S_{T\to 0} = S_{T\to\infty} = k_B \ln 3$
(c) $S_{T\to 0} = 0$ and $S_{T\to\infty} = k_B \ln 3$
(d) $S_{T\to 0} = 0$ and $S_{T\to\infty} = 3k_B / 2$

48. The state $|\psi\rangle$ of a quantum mechanical system, in a certain basis, is represented by the column vector

$$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}.$$

The operator $\hat{A}$ corresponding to a dynamical variable A, is given, in the same basis, by the matrix

$$\hat{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

If, now, a measurement of the variable A is made on the system in the state $|\psi\rangle$, the probability that the result will be +1 is

(a) $\frac{1}{\sqrt{2}}$
(b) 1
(c) $\frac{1}{2}$
(d) $\frac{1}{4}$

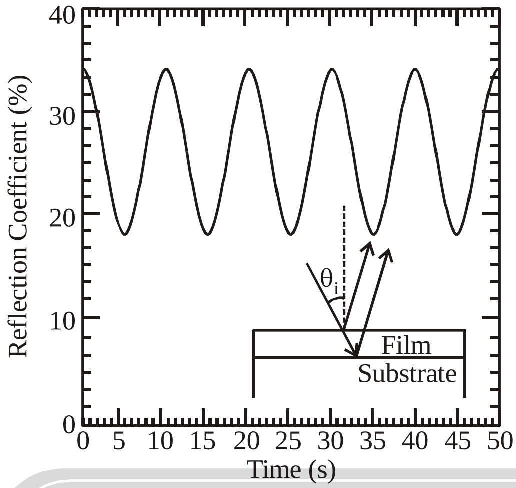
49. A spin-$\frac{1}{2}$ particle A decays to two other particles B and C. If B and C are of spin-$\frac{1}{2}$ and spin-1 respectively, then a complete list of the possible values of the orbital angular momentum of the final state (i.e. B + C) is

(a) 0, 1
(b) $\frac{1}{2}, \frac{3}{2}$
(c) 0, 1, 2
(d) 0, ± 1

50. When a pure element is vaporised and placed in a uniform magnetic field B_0, it is seen that a particular spectral line of wavelength λ, corresponding to a $J = 1 \to J = 0$ transition, gets split into three components $\lambda, \lambda \pm \Delta\lambda$. It follows that the Lande g-factor for the transition $J = 1 \to J = 0$ is given by

(a) $g = \frac{hc}{\mu_B B_0} \frac{\Delta\lambda^2}{\lambda}$
(b) $g = \frac{hc}{\mu_B B_0} \frac{\lambda}{\Delta\lambda^2}$
(c) $g = \frac{hc}{\mu_B B_0} \frac{\lambda^2}{\Delta\lambda}$
(d) $g = \frac{hc}{\mu_B B_0} \frac{\Delta\lambda}{\lambda^2}$

51. The rate of deposition of a dielectric thin film on a thick dielectric substrate was monitored by the following experiment: a laser beam of wavelength λ = 633 nm, at near-normal incidence θ_i, was reflected from the thin film (see inset figure on the right), and the reflection coefficient R was measured. As the film thickness increased R varied with time as shown on the right. The refractive index of the film is 3.07 and is less than that of the substrate. Using the graph, the approximate thickness of the film at the end of 25 seconds can be estimated to be

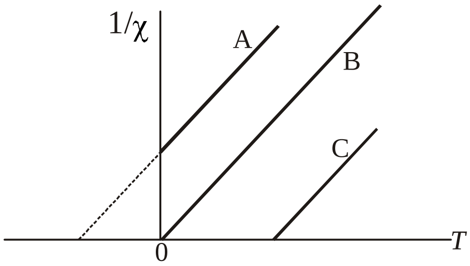


(a) 0.017 μm (b) 0.26 μm (c) 0.51 μm (d) 2.2 μm

52. The negative image on the right represents a very small portion of the night sky at a very high resolution. Notice the broken ring (s) around the central bright object in the middle of the picture. These are most likely to be due to

(a) debris from a smaller object torn apart by tidal forces
(b) gas clouds forming the remnant of a supernova explosion
(c) ice collected on the lens used for taking the picture
(d) gravitational lensing of a distant object by the central massive object

53. The magnetic susceptibility χ of three samples A, B and C, is measured as a function of their absolute temperature T, leading to the graphs shown below.



From these graphs, the magnetic nature of the samples can be inferred to be

(a) A : anti-ferromagnet B : paramagnet C : ferromagnet
(b) A : diamagnet B : paramagnet C : anti-ferromagnet
(c) A : paramagnet B : anti-ferromagnet C : ferromagnet
(d) A : anti-ferromagnet B : diamagnet C : paramagnet

54. A gold foil, having $N(0)$ number of ^{197}Au nuclides per cm^2, is irradiated by a beam of thermal neutrons with a flux of F neutrons-cm^{-2}-s^{-1}. As a result, the nuclide ^{198}Au, with a half-life τ of several years, is produced by the reaction

$$^{197}\text{Au} + n \rightarrow {}^{198}\text{Au} + \gamma$$

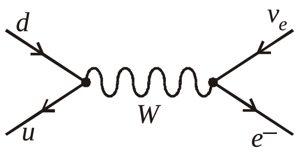
which has a cross section of σ cm^2. Assuming that the gold foil has 100% abundancy of ^{197}Au nuclides, the maximum number of ^{198}Au nuclides that can accumulate at any time in the foil is proportional to

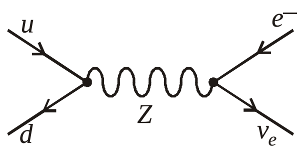
(a) $\sigma\tau F\, N(0)$ (b) $\dfrac{\tau}{\sigma F} N(0)$ (c) $\dfrac{1}{\sigma\tau F} N(0)$ (d) $\dfrac{\sigma F}{\tau} N(0)$

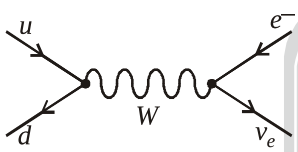
55. The process of electron capture

$$p + e^- \rightarrow n + \nu_e$$

takes place at the quark level through the Feynman diagram

(a)


(b)


(c)


(d)