GS-2016 (PHYSICS)

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

SECTION-A

1. If x is a continuous variable which is uniformly distributed over the real line from $x = 0$ to $x \to \infty$ according to the distribution $f(x) = \exp(-4x)$ then the exception value of cos 4x is

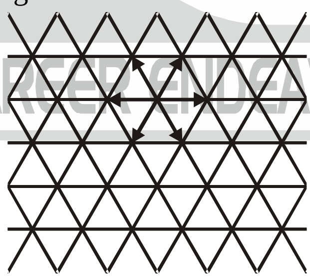
(a) zero (b) 1/2 (c) 1/4 (d) 1/16

2. If the eigenvalue of a symmetric 3 × 3 matrix **A** are 0, 1, 3 and the corresponding eigenvectors can be written as $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ respectively, then the matrix A^4 is

(a) $\begin{pmatrix} 41 & -81 & 40 \\ -81 & 0 & -81 \\ 40 & -81 & 41 \end{pmatrix}$ (b) $\begin{pmatrix} -82 & -81 & 79 \\ -81 & 81 & -81 \\ 79 & -81 & 83 \end{pmatrix}$

(c) $\begin{pmatrix} 14 & -27 & 13 \\ -27 & 54 & -27 \\ 13 & -27 & 14 \end{pmatrix}$ (d) $\begin{pmatrix} 14 & -13 & 27 \\ -13 & 54 & -13 \\ 27 & -13 & 14 \end{pmatrix}$

3. In a triangular lattice a particle moves from a lattice point to any of its 6 neighbouring points with equal probability, as shown in the figure on the right.



The probability that the particle is back at its starting point after 3 moves is

(a) 5/18 (b) 1/6 (c) 1/18 (d) 1/36

4. A ball is dropped vertically from a height H on to a plane surface and permitted to bounce repeatedly along a vertical line. After every bounce, its kinetic energy becomes a quarter of its kinetic energy before the bounce. The ball will come to rest after time

(a) infinity (b) $(2H/g)^{1/2}$ (c) $2(2H/g)^{1/2}$ (d) $3(2H/g)^{1/2}$

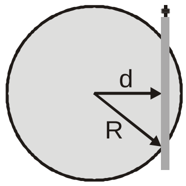
5. In a moving car, the wheels will skid if the brakes are applied too suddenly. This is because

(a) the inertia of the car will carry it forward
(b) the momentum of the car must be conserved
(c) the impulsive retarding force exceeds the limiting force of static friction
(d) the kinetic friction will suddenly get converted to static friction.

6. An aircraft, which weight 12,000 kg when unloaded, is on a relief mission, carrying 4,000 food packets weighing 1 kg each. The plane is gliding horizontally with its engines off at a uniform speed of 540 kmph when the first food packet is dropped. Assume that the horizontal air drag can be neglected and the aircraft keeps moving horizontally. If one food packet is dropped every second, then the distance between the last two packet drops will be

(a) 1.5 km (b) 200 m (c) 150 m (d) 100 m

7. Imagine that a narrow tunnel is excavated through the Earth as shown in the diagram on the left and that the mass excavated to create the tunnel is extremely small compared to Earth's mass M.



A person falls into the tunnel at one end, at time t = 0. Assuming that the tunnel is frictionless, the person will

(a) fall straight through, escaping Earth's gravity at time $2\pi\sqrt{R^3/GM}$

(b) describe simple harmonic motion with period $2\pi\,(d/R)\sqrt{R^3/GM}$

(c) describe simple harmonic motion with period $2\pi\sqrt{(R-d)^3/GM}$

(d) describe simple harmonic motion with period $2\pi\sqrt{R^3/GM}$

8. A grounded conducting sphere of radius a is placed with its centre at the origin. A point dipole moment $\vec{p} = p\hat{k}$ is placed at a distance d along the x-axis, where $\hat{i}, \hat{k}$ are the units vector along the x and z-axes respectively. This leads to the formation of an image dipole of strength $\vec{p}'$ at a distance d' from the centre along the x-axis. If $d' = a^2/d$, then $\vec{p}' =$

(a) $-\frac{a^4 p}{d^4}\hat{k}$ (b) $-\frac{a^3 p}{d^3}\hat{k}$ (c) $-\frac{a^2 p}{d^2}\hat{k}$ (d) $-\frac{ap}{d}\hat{k}$

9. A long, solid dielectric cylinder of radius a is permanentely polarized so that the polarization is everywhere radially outward, with a magnitude proportional to the distance from the axis of the cylinder, i.e., $\vec{P} = \frac{1}{2}P_0 r\hat{r}$. The bound charge density in the cylinder is given by

(a) $-P_0$ (b) P_0 (c) $\frac{-P_0}{2}$ (d) $\frac{P_0}{2}$

10. A circular loop of fine of radius R carrying a current I is placed in a uniform magnetic field B perpendicular to the plane of the loop. If the breaking tension of the wire is T_b, the wire break when the magnetic field exceeds

(a) $\frac{T_b}{IR}$ (b) $\frac{T_b}{2\pi IR}$ (c) $\frac{\mu_0 T_b}{2\pi IR}$ (d) $\frac{\mu_0 T_b}{4\pi IR}$

11. A gas of non-interacting particles, each of rest mass 1 MeV, is at a temperature T = 2.0×10^7 K and has an average particle density $n = 2.7 \times 10^{34}$ cm^{-3}. We can obtain a reasonably correct treatment of this system

(a) only by using special relativity as well as quantum mechanics
(b) by neglecting quantum mechanics but not special theory of relativity
(c) by neglecting special relativity but not quantum mechanics
(d) by neglecting both special relativity and quantum mechanics

12. A one-dimensional harmonic oscillator of mass m and natural frequency ω is in the quantum state

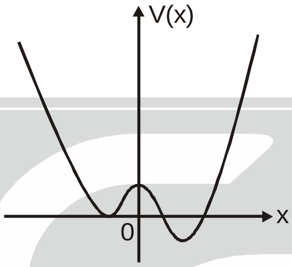
$$|\psi\rangle = \frac{1}{3}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle$$

at time $t = 0$, where $|n\rangle$ denotes the eigenstate with eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. It follows that the exception value $\langle x\rangle$ of the position operator $\hat{x}$ is

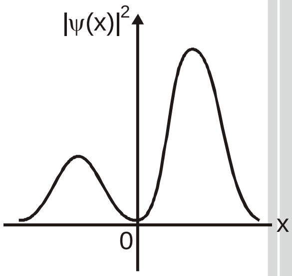
(a) $x(0)\left[\cos\omega t + \frac{1}{\sqrt{3}}\sin\omega t\right]$ (b) $x(0)\left[\cos\omega t - \sin\omega t\right]$

(c) $x(0)\left[\cos\omega t - \frac{1}{2}\sin\omega t\right]$ (d) $x(0)\left[\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right]$

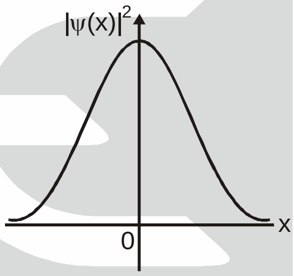
13. A particle is confined inside a one-dimensional potential well $V(x)$, as shown on the right. One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate for this particle is



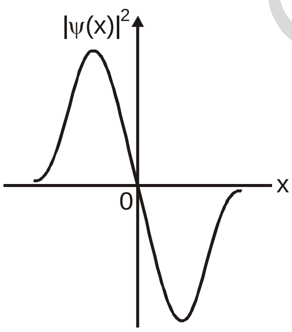
(a)



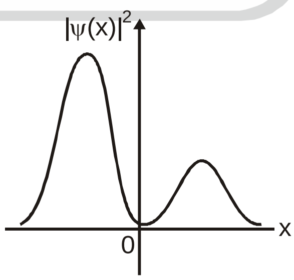
(b)



(c)



(d)



14. The energy per oscillator of an isolated system of a large number of identical non-interacting fermions in a one-dimensional harmonic oscillator potential is $5\hbar\omega/4$, where ω is the angular frequency of the harmonic oscillator. The entropy of the system per oscillator is given by

(a) 0.25 (b) 0.56 (c) 0.63 (d) 0.75

15. In the temperature range 100 – 1000 C, the molar specific heat of a metal varies with temperature T (measured in degrees Celsius) according to the formula $C_p = (1 + T/5)$ J-deg C^{-1} mol^{-1}. If 0.2 kg of the metal at 600 C is

brought in thermal contact with 0.1 kg of the same metal at 300 C, the final equilibrium temperature, in deg C, will be

[Assume that no heat is lost due to radiation and/or other effects].

(a) 466 (b) 567 (c) 383 (d) 519

16. In a fixed target experiment, a proton of total energy 200 GeV is bombarded on a proton at rest and produces a nucleus ${}^{Z}_{A}N$ and its anti-nucleus ${}^{Z}_{A}\bar{N}$

$$p + p \rightarrow {}^{Z}_{A}N + {}^{Z}_{A}\bar{N} + p + p$$

The heaviest nucleus ${}^{Z}_{A}N$ that can be created has atomic mass number A =

(a) 15 (b) 9 (c) 5 (d) 4

17. In a simple cubic lattice of lattice constant 0.287 nm, the number of atoms per mm^2 along the 111 plane is

(a) 2.11×10^{13} (b) 1.73×10^{13} (c) 1.29×10^{13} (d) 1.21×10^{13}

18. In a glass fibre, light propagates by total internal reflection from the inner surface. A very short pulse of light enters a perfectly uniform glass fibre at $t = 0$ and emerges from the other end of the fibre with negligible losses. If the refractive index of the glass used in the fibre is 1.5 and its length is exactly 1.0 km, the time t at which the output pulse will have completely exited the fibre will be

(a) 5.0 μs (b) 7.5 μs (c) 25 ns (d) 750 ns

19. In an ionization experiment conducted in the laboratory, different singly charged positive ions are produced and accelerated simultaneously using a uniform electric field along the x-axis. If we need to determine the masses of various ions produced, which of the following methods will NOT work

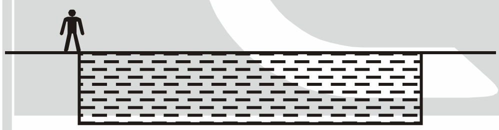
(a) Detect them at a fixed distance from the interaction point along x-axis and measure their time of arrival.

(b) Apply a uniform magnetic field along y-axis and measure the deviation.

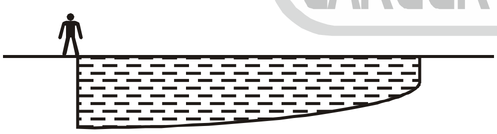
(c) Apply a uniform electric field along y-axis and measure the deviation

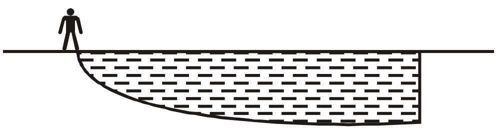
(d) Apply a uniform electric field along y-axis and a (variable) uniform magnetic field along z-axis simultaneously and note the zero deviation.

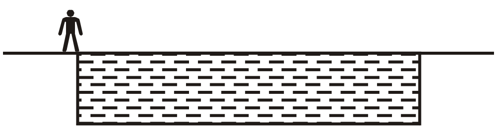
20. An observer stands at the edge of a swimming pool, as sketched in the figure below.

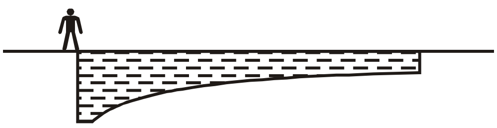


This observer will perceive the pool as

(a)


(b)


(c)


(d)


21. Consider a process in which atoms of Actinium-226 ($^{226}_{89}\text{Ac}$) get converted to atoms of Radium-226 ($^{226}_{88}\text{Ra}$) and the yield of energy is 0.64 MeV per atom. This occurs through

(a) Both $p \to n + e^+ + v_e$ and $p + e^- \to n + v_e$

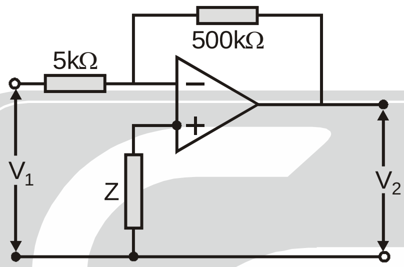
(b) Both $p \to n + e^+ + v_e$ and $n \to p + e^- + \bar{v}_e$

(c) Only $p \to n + e^+ + v_e$

(d) Only $p + e^- \to n + v_e$

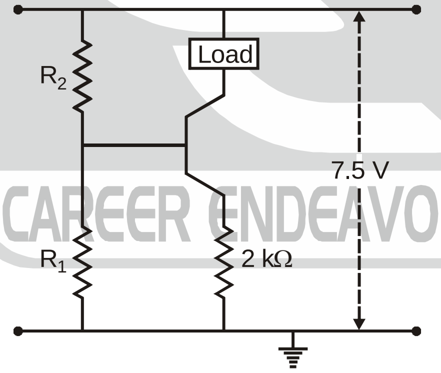
22. Two homonuclear diatomic molecules produce different rotational spectra, even through the atoms are known to have identical chemical properties. This leads to the conclusion that the atoms must be
(a) isotopes, i.e. with the same atomic number (b) isobars, i.e. with the same atomic weight
(c) isotones, i.e. with the same neutron number (d) isomers, i.e. with the same atomic number and weight

23. In the generalized operational amplifier circuit shown on the right, the amp. has a very high input impedance $(Z > 50\ M\Omega)$ and an open gain of 1000 for the frequency range under consideration. Assuming that the op. amp. draws negligible current, the voltage ratio V_2/V_1 is approximately



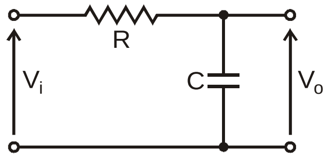
(a) –190 (b) –190 (c) –90 (d) 80

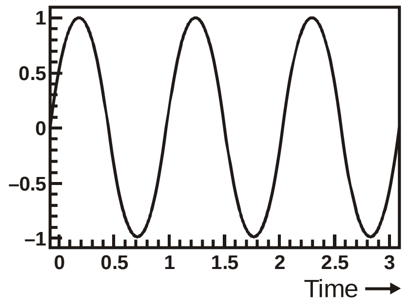
24. In the transistor circuit shown on the right, assume that the voltage drop between the base and the emitter is 0.5V.



What will be the ratio of the resistances R_2/R_1, in order to make this circuit function as a source of constant current, $I = 1$ mA ?
(a) 4.5 (b) 3.0 (c) 2.5 (d) 2.0

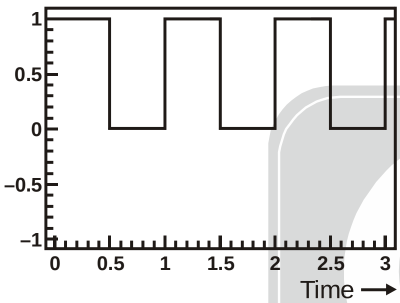
25. For the circuit depicted on the right, the input voltage V_i is a simple sinusoid as shown below, where the time period is much smaller compared to the time constant of this circuit.



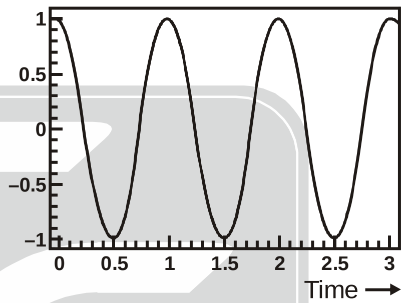


The voltage V_o across C is best represented by

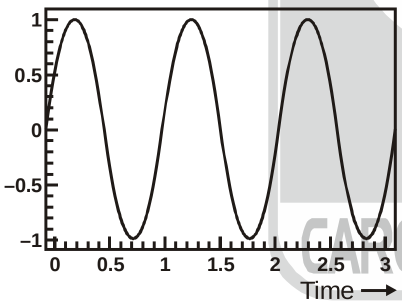
(a)



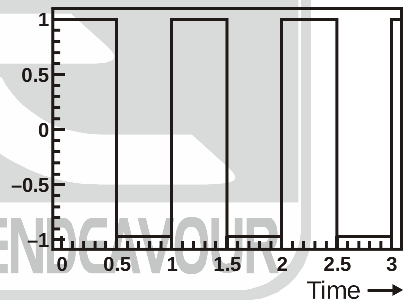
(b)



(c)



(d)

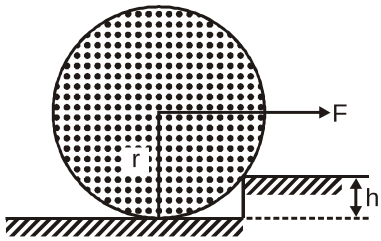


SECTION-B

26. The integral $\int_0^\infty \frac{dx}{x}\left[\exp\left(-\frac{x}{\sqrt{3}}\right)-\exp\left(-\frac{x}{\sqrt{2}}\right)\right]$ evaluates to

(a) zero (b) 2.03×10^{-2} (c) 2.03×10^{-1} (d) 2.03

27. A uniform solid wheel of mass M and radius r is halted at a step of height h as shown in the figure. The minimum force F, applied horizontally at the centre of the wheel, necessary to raise the wheel over this step is



(a) $Mg\dfrac{\sqrt{h(2r-h)}}{r+h}$ (b) $Mg\dfrac{\sqrt{h(2r+h)}}{r-h}$ (c) $Mg\dfrac{\sqrt{h(r+h)}}{r-h}$ (d) $Mg\dfrac{\sqrt{h(2r-h)}}{r-h}$

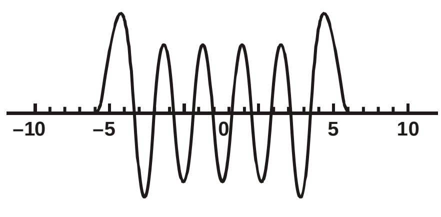
28. In a futuristic scenario, two spaceships, A and B, are running a race, where they start from the same point (marked START) but fly in opposite directions at constant speeds close to the speed of light. An observer fixed at the starting point observes that they both cross the points marked END, which are equidistant from the starting point, at the same time. Afterwards this observer receives messages from both spaceships.

Which of the following could be true?
(a) Both A and B agree that A won the race
(b) A and B both claim to have won the race
(c) Both A and B agree that they crossed the end point simultaneously
(d) A thinks B won the race while B thinks A won the race

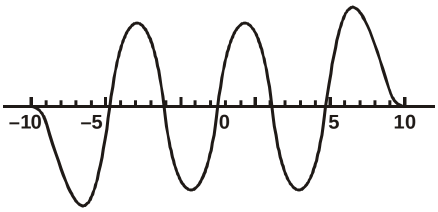
29. The equation of state for a gas is given by $\left[p+\left(\dfrac{\alpha N}{V}\right)^2\right](V-\beta N)=Nk_BT$ where P, V, T, N and k_B represent pressure, volume, temperature, number of atoms and the Boltzmann constant, respectively, while α and β are constants specific to the gas.
If the critical point C corresponds to a point of inflexion of the p-V curve, then the critical volume V_C and critical pressure p_c for this gas are given by
(a) $V_C=3\beta N,\ p_c=\alpha^2/3\beta^2$
(b) $V_C=3\beta N,\ p_c=\alpha/27\beta^2$
(c) $V_C=3\beta N,\ p_c=8\alpha^2/27\beta$
(d) $V_C=3\beta N,\ p_c=\alpha^2/27\beta^2$

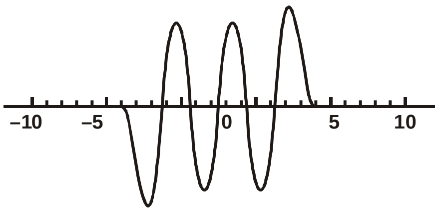
30. The lattice constant of a material is of the order of a μm, and its bond energies are of the order of an eV. The bulk modulus of such a material, in Pascals, is of the order of
(a) 10^{-1} (b) 10^{-3} (c) 10^{-6} (d) 10^{9}

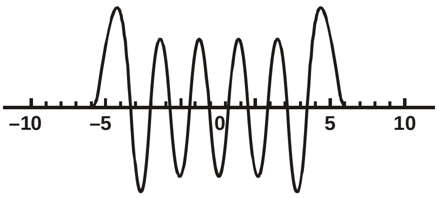
31. Two harmonic oscillators A and B are in excited eigenstates with the same excitation energy E, as measured from their respective ground state energies. The natural frequency of A is twice that of B.

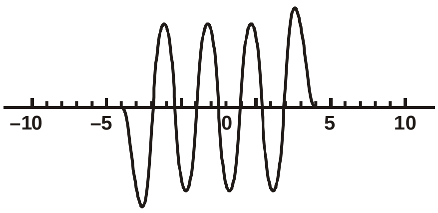


If the wavefunction of B is as sketched in the above picture, which of the following would best represent the wavefunction of A?

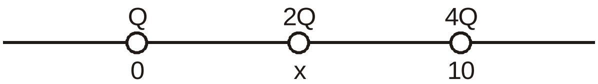
(a) 

(b) 

(c) 

(d) 

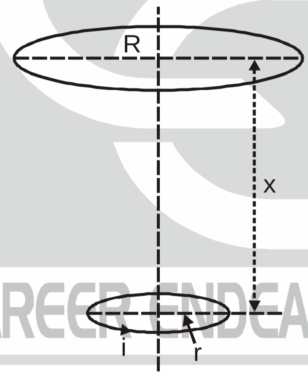
32. Three positively charged particles lie on a straight line at positions 0, x and 10 as indicated in the figure below. Their charges are Q, 2Q and 4Q cm respectively.



If the charges at $x = 0$ and $x = 10$ are fixed and the charge at x is movable, the system will be in equilibrium when $x =$

(a) 8 (b) 2 (c) 20/3 (d) 10/3

33. Consider the following system. Two circular loops of wire are placed horizontally, having a common axis passing vertically through the centre of each coil (see figure). The lower loop has radius r and carries a current i as shown in the figure. The upper loop has a radius R ($R >> r$) and is at a distance x ($x >> R$) above it.



If the lower loop is held fixed and the upper loop moves upwards with a uniform velocity $v = dx/dt$, then the induced e.m.f. and the direction of the induced current in this loop will be

(a) $3i\mu_0\pi^2r^2R^2v/2x^4$; anti-clockwise (b) $2i\mu_0\pi^2r^2R^2v/2x^4$; clockwise

(c) $3i\mu_0\pi^2r^2R^2v/2x^3$; anti-clockwise (d) $2i\mu_0\pi^2r^2R^2v/3x^3$; clockwise

34. A student in the laboratory is provided with a bunch of standard resistors as well as the following instruments

— Voltmeter accurate to 0.1 V
— Ammeter accurate to 0.01 A
— Constant current source (ideal)
— Thermometer accurate to 0.1 C
— Stop watch accurate to 0.05 s
— Constant voltage source (ideal)

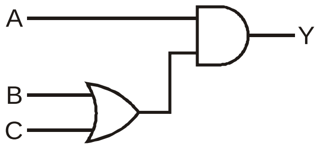
Using this equipment (and nothing else), the student is expected to measure the resistance R of one of the given resistors. The least accurate result would be obtained by

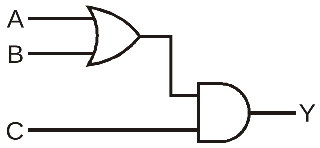
(a) measuring the Joule heating
(b) passing a constant current and measuring the voltage across it
(c) measuring the current on application of a constant voltage across it
(d) the Wheatstone bridge method

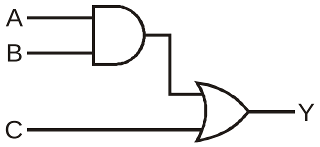
35. In a digital circuit for three input signals (A, B and C) the final output (Y) should be such that for inputs

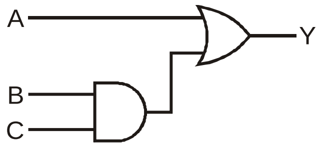
A	B	C
0	0	0
0	0	1
0	1	0

the output (Y) should be low and for all other cases it should be high. Which of the following digital circuits will give such output ?

(a)


(b)


(c)


(d)


PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

The following question (36 – 40) must be answered by integers of 3 digits each. You may round off, e.g. $123.0 \le x < 123.5$ as $x = 123$ and e.g. $123.5 \le x \le 124.0$ as $x = 124$ and so on. Answer these questions on the OMR by filling in bubbles as you did for your reference code. Use only values of constant given in the table 'USEFUL CONSTANT'.

Note that if the answer is, e.g. 25, you must fill in 025 and if it is, e.g. 5, you must fill in 005. If it is 0, you must fill in 000. If the zeros are not filled in (where required), the answer will be not be credited.

There are NO NEGATIVE MARKS for these questions.

36. On a planet having the same mass and diameter as the Earth, it is observed that objects become weightless at the equator. Find the time period of rotation of this planet in minutes (as defined on the Earth).

37. The function $y(x)$ satisfies the differential equation $x\dfrac{dy}{dx} = y(\ell n\, y - \ell n\, x + 1)$ with the initial condition $y(1) = 3$.

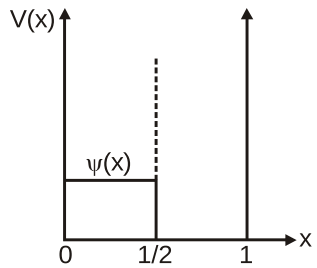
What will be the value of $y(3)$?

38. Two containers are maintained at the same temperature and are filled with ideal gases whose molecules have mass m_1 and m_2 respectively. The mean speed of molecules of the second gas is 10 times the r.m.s. speed of the molecules of the first gas. Find the ratio of m_1/m_2, to the nearest integer.

39. A particle is confined in a one-dimensional box of unit length, i.e. $L = 1$, i.e. in a potential

$$V(x) = \begin{cases} 0 & \text{if} \quad 0 < x < 1 \\ \infty & \text{otherwise} \end{cases}$$

The energy eigenvalues of this particle are denoted $E_0, E_1, E_2, E_3, \ldots$



In a particular experiment, the wavefunction of this particle, at $t = 0$, is given by

$$\psi(x) = \begin{cases} \sqrt{2} & \text{if} \quad 0 < x < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

If, simultaneouly, i.e. at $t = 0$, a measurement of the energy of the particle is made, find $100p_3$, where p_3 is the probability that the measurement will yield the energy E_3.

40. Consider a sawtooth waveform which rises linearly from 0 Volt to 1 Volt in 10 ns and then decays linearly to 0V over a period of 100 ns. Find the r.m.s. voltage in units of milliVolt?

SECTION-C

41. The value of the integral $\oint_C \frac{\sin z}{z^6} dz$ where C is the circle of centre $z = 0$ and radius = 1

(a) $i\pi$ (b) $i\pi/120$ (c) $i\pi/60$ (d) $-i\pi/6$

42. In a Rutherford scattering experiment, the number N of particles scattered in a direction θ, i.e. $dN/d\theta$, as a function of the scattering angle θ (in the laboratory frame) varies as

(a) $\csc^4 \frac{\theta}{2}$ (b) $\csc^2 \frac{\theta}{2} \cot \frac{\theta}{2}$ (c) $\csc^2 \frac{\theta}{2} \tan^2 \frac{\theta}{2}$ (d) $\sec^2 \frac{\theta}{2}$

43. In a simple stellar model, the density ρ of a spherical star of mass M varies according to the distance r from the centre according to $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$ where R is the radius of the star. The gravitational potential energy of this star (in terms of Newton's constant G_N) will be

(a) $-G_N M^2/4\pi R$ (b) $-3G_N M^2/5R$ (c) $-5G_N M^2/7R$ (d) $-3G_N M^2/7R$

44. A particle moving in one dimension is confined inside a rigid box located between $x = -a/2$ and $x = a/2$. If the particle is in its ground state $\psi(0) = \sqrt{2/a} \cos \frac{\pi x}{a}$ the quantum mechanical probability of its having a momentum p is given by

(a) $\frac{8\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2} \cos^2 \frac{pa}{2\hbar}$ (b) $\frac{\pi^2\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2} \sin^2 \frac{pa}{2\hbar}$

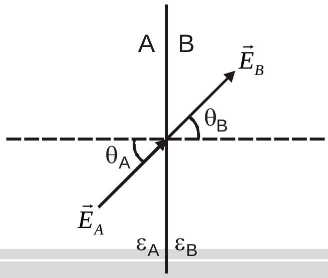
(c) $\frac{2\hbar^4}{(\pi^2\hbar^2 + p^2a^2)^2} \cos^2 \frac{pa}{2\hbar}$ (d) $\frac{16\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}$

45. Consider two spin-1/2 identical particles A and B, separated by a distance r, interacting through a potential $V(r) = \frac{V_0}{r}\vec{S}_A.\vec{S}_B$ where V_0 is a positive constant and the spins are $\vec{S}_{A,B} = \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ in terms of the Pauli spin matrices. The exception values of this potential in the spin-singlet and triplet states are

(a) Singlet : $-\frac{V_0}{3r}$, Triplet : $\frac{V_0}{r}$ (b) Singlet : $-\frac{3V_0}{r}$, Triplet : $\frac{V_0}{r}$

(c) Singlet : $\frac{3V_0}{r}$, Triplet : $-\frac{V_0}{r}$ (d) Singlet : $-\frac{V_0}{r}$, Triplet : $\frac{3V_0}{r}$

46. Two semi-infinite slabs A and B of dielectric constant ε_A and ε_B meet in a plane interface, as shown in the figure below.



If the electric field in slab A makes angle θ_A with the normal to the boundary and the electric field in slab B makes an angle θ_B with the same normal (see figure), then

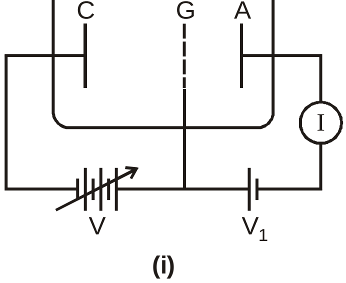
(a) $\cos\theta_A = \frac{\varepsilon_A}{\varepsilon_B}\cos\theta_B$ (b) $\sin\theta_A = \frac{\varepsilon_A}{\varepsilon_B}\sin\theta_B$

(c) $\tan\theta_A = \frac{\varepsilon_A}{\varepsilon_B}\tan\theta_B$ (d) $\sin\theta_A = \frac{\varepsilon_B}{\varepsilon_A}\sin\theta_B$

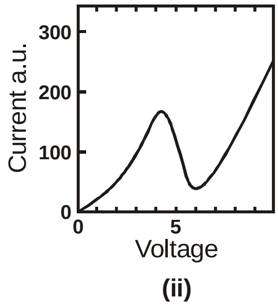
47. The Weizäcker semi-empirical mass formula for an odd nucleus with z protons and A nucleons may be written as $M(Z, A) = \alpha_1 A + \alpha_2 A^{2/3}/\alpha_3 A + \alpha_4 Z^2$ where the α_i are constant independent of Z, A. For a given A, if Z_A is the number of photons of the most stable isobar, the total energy released when an unstable nuclide undergoes a single β^- decay to (Z_A, A) is

(a) α_3 (b) α_4 (c) $\alpha_4 - \alpha_3$ (d) $\alpha_1 + \alpha_2$

48. In the experiment shown in figure (i) below, the emitted electrons from the cathode (C) are made to pass through the mercury vapor filled in the tube by accelerating them using a grid (G) at potential V, positive w.r.t. the cathode. The electrons are collected by the anode (A).



(i)



(ii)

The variation of electron current (I) as a function of V is given in figure (ii). The shape of this curve must be interpreted as due to

(a) ionization of mercury atoms (b) an emission line from mercury atoms

(c) attachment of electrons to mercury atoms (d) resonant backscattering of electrons to cathode from grid

49. The dispersion relation for electrons in the conduction band of a n-type semiconductor has the form $E(k) = ak^2 + b$ where a and b are constant. It was observed that the cyclotron resonance frequency of such electrons is $\omega_0 = 1.8 \times 10^{11}$ rad s^{-1}, when placed in a magnetic field $B = 0.1$ W m^{-2}. It follows that the constant a must be about

(a) 10^{-36} (b) 10^{-28} (c) 10^{-32} (d) 10^{-38}

50. Consider the hyperon decay (1) $\Lambda \rightarrow n + \pi^0$ followed by (2) $\pi^0 \rightarrow \gamma\gamma$. If the isospin component, baryon number and strangeness quantum numbers are denoted by I_Z, B and S respectively, then which of the following statements is completely correct ?

(a) In (1) I_Z is not conserved, B is conserved, S is not conserved;
In (2) I_Z is conserved, B is conserved, S is conserved.

(b) In (1) I_Z is conserved, B is not conserved, S is not conserved
In (2) I_Z is conserved, B is conserved, S is conserved

(c) In (1) I_Z is not conserved, B is conserved, S is not conserved
In (2) I_Z is not conserved, B is conserved, S is conserved

(d) In (1) I_Z is not conserved, B is conserved, S is conserved
In (2) I_Z is conserved, B is conserved, S is conserved

PLEASE READ CAREFULLY BEFORE PROCEEDING FURTHER

The answer to the following questions (51 – 55) must be answered by integers of 3 digits each. You may round off, e.g. $123.0 \le x < 123.5$ as $x = 123$ and e.g. $123.5 \le x \le 124.0$ as $x = 124$ and so on. Answer these questions on the OMR by filling in bubbles as you did for your reference code. Use only values of constant given in the table 'USEFUL CONSTANT'.

Note that if the answer is, e.g. 25, you must fill in 025 and if it is, e.g. 5, you must fill in 005. If it is 0, you must fill in 000. If the zeros are not filled in (where required), the answer will be not be credited

There are NO NAGATIVE MARKS for these questions.

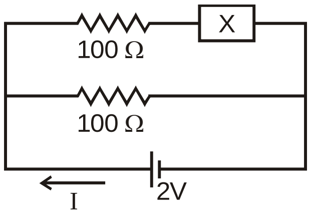
51. Given that infinite series $y(x) = 1 + 3x + 6x^2 + 10x^3 + \cdots + \frac{(n+1)(n+2)}{2} x^n + \cdots$, find the value of $y(x)$ for $x = 6/7$.

52. A quantum mechanical plane rotator consists to two rigidly connected particles of mass m and connected by a massless rod of length d is rotating in the xy-plane about their centre of mass. Suppose that the initial state of the rotor is given by $\psi(\varphi, t = 0) = A\cos^2\varphi$, where φ is the angle between one mass and the x-axis, while A is normalization constant. Find the exception value of $3\hat{L}_z^2$ in this state, in units of $\hbar^2$.

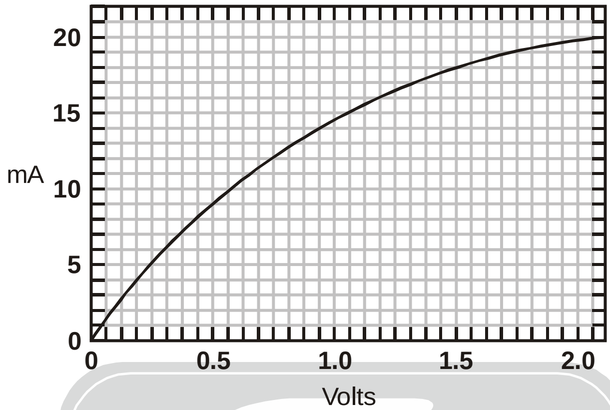
53. A continuous monochromatic ($\lambda = 600$ nm) laser beam is chopped into 0.1 ns pulses using some sort of shutter. Find the resultant linewidth $\Delta\lambda$ of the beam in units of 10^{-3} nm.

54. N distinguishable particles are distributed among three states having energies $E = 0$, k_BT and $2k_BT$ respectively. If the total equilibrium energy of the system is 138.06 k_BT, find the number N of particles.

55. The circuit shown below contains an unknown device X.



The current voltage characteristic of the device X were determined and are shown in the plot given below.



Determine the current *I* (in mA) flowing through the device X.