

DIELECTRICS

Dielectrics : Dielectrics are perfect insulators. In dielectrics electrons are very tightly bound to the atoms so that at ordinary temperatures they do not conduct any electric current.

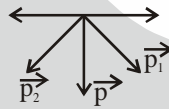
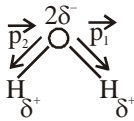
Examples: Solids: glass, porcelain; gases: H_2 , N_2 ; liquids: CH_3Cl , pure water

They are of two types.

- (i) Polar dielectrics
- (ii) Non-polar dielectrics

Polar dielectrics are those dielectrics whose molecules contains (are characterized by) the permanent dipole moments. Ex - H_2O , $NaCl$, KCl .

Non-polar dielectrics are those dielectrics whose molecules do not contain the permanent dipole moments. Ex - Solid Ar.



H_2O : Electronegativity of O is higher than H so it will have more negative charge, so it contain permanent dipole moment. It is the inherent property of molecules.

If we apply external electric field on non-polar dielectrics then it will be polarized and this polarization is called induced polarisation or induced dipole momentum.

Thus, $\vec{p}_{in} \propto \vec{E}$

$$\Rightarrow \boxed{\vec{p}_{in} = \alpha \vec{E}_{loc}} \quad \dots (1)$$

where, α is proportionality constant and known as polarizability of atom or molecule and \vec{E}_{loc} is local electric field.

Dipole moment : If we have two equal and opposite charges $+q$ and $-q$, separated by a distance r , then the dipole moment is defined as $p = qr$. The dipole moment is directed from the negative to positive charge.

Polarization: The polarization (\vec{P}) is defined as the dipole moment per unit volume i.e., $\vec{P} = \frac{\vec{p}}{\Delta V}$

$$\vec{P} = \chi_e \epsilon_0 \vec{E}_{loc} \text{ also } \vec{p}_{in} = \alpha \vec{E}_{loc} \text{ and } \vec{P} = N\vec{p}_{in} = N\alpha\vec{E}_{loc} \quad \dots (2)$$

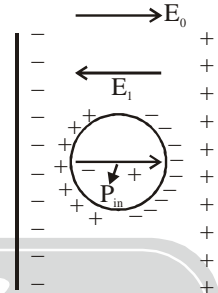
where χ_e is electric susceptibility and N is number of dipoles per unit volume.

$$\therefore \chi_e = \frac{N\alpha}{\epsilon_0}$$

$$\text{Since, } \epsilon_r = 1 + \chi_e \quad \therefore \epsilon_r = 1 + \frac{N\alpha}{\epsilon_0}$$

For non-polar dielectrics: If we have a large parallel plate capacitor and we fill a non-polar dielectric between its plates. Suppose external applied field is \vec{E}_0 . Due to this \vec{E}_0 , non-polar dielectrics will be polarized then plates occur some bound charge and electric field develop is \vec{E}_1 which is less than \vec{E}_0 .

$$\text{Thus, resultant electric field, } \vec{E} = \underbrace{\vec{E}_0 + \vec{E}_1}_{E_{mac}}$$



Macroscopic electric field is average electric field seen between the plates.

If there is a molecule between plates then it will behave as dipole and resultant field on this molecule is different from \vec{E}_{mac} and this electric field is called local field.

$$\vec{E}_{loc} = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

where \vec{E}_0 is externally applied electric field, \vec{E}_1 is depolarization field, $(\vec{E}_0 + \vec{E}_1)$ is macroscopic field, \vec{E}_2 is electric field due to the induce charges on the surface of the cavity (polarization field).

and \vec{E}_3 is electric field due to all the charges inside the cavity. Resultant of all these electric fields is the local field. Take a spherical cavity out from the surrounding of molecules. This cavity is called Lorentz cavity. If the dipole is at the centre of the cavity then charges on the cavity will be symmetric about centre.

$$\text{So, } \boxed{\vec{E}_3 = 0} \text{ (due to symmetry)}$$

$$\text{Now, } \vec{E}_{loc} = \vec{E}_0 + \vec{E}_1 + \vec{E}_2$$

$$\Rightarrow \vec{E}_{loc} = \vec{E}_{mac} + \vec{E}_2 \quad \left(\text{since } E_2 = \frac{\vec{P}}{3\epsilon_0} \right)$$

$$\Rightarrow \boxed{\vec{E}_{loc} = \vec{E}_{mac} + \frac{\vec{P}}{3\epsilon_0}}$$

If solid contains N atoms (or molecules) per unit volume, each atoms due to the application of electric field will become dipole, so there will be N dipoles.

$$\vec{P} = N\vec{p}_{in}$$

$$\Rightarrow \boxed{\vec{P} = N\alpha \vec{E}_{loc}} \quad \dots (3)$$

From equation (2) and equation (3), we get

$$\begin{aligned}
 N\alpha \vec{E}_{loc} &= \epsilon_0 \chi_e \vec{E}_{mac} \\
 \Rightarrow N\alpha \vec{E}_{loc} &= \epsilon_0 (\epsilon_r - 1) \vec{E}_{mac} && (\text{since } \chi_e = \epsilon_r - 1) \\
 \Rightarrow \frac{\vec{E}_{loc}}{\vec{E}_{mac}} &= \frac{\epsilon_0 (\epsilon_r - 1)}{N\alpha} \\
 \Rightarrow \boxed{\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha}{3\epsilon_0}} & \text{ where, } \alpha \text{ is electric polarizability and } \epsilon_r \text{ is dielectric constant.}
 \end{aligned}$$

This relation is known as Classius-Mossotie relation.

This is valid only for non-polar molecules.

Physical significance of Classius-Mossotie relation: Classius-Mossotie relation is a relationship between α and ϵ_r . α is a microscopic quantity and ϵ_r is macroscopic quantity (dielectric constant) because α corresponds to dipole moment and ϵ_r corresponds to polarisation.

For polar molecules, this equation becomes Langvein - Debye equation.

Types of polarisabilities: There are three types of polarisabilities

- (i) Electronic polarisability (α_e)
- (ii) Ionic polarisability (α_i)
- (iii) Dipolar polarisability (α_d)

Thus total polarisability α is sum of electronic, ionic and dipolar polarisabilities.

$$\text{i.e., } \alpha = \underbrace{\alpha_e + \alpha_i}_{\text{induced}} + \underbrace{\alpha_d}_{\text{Permanent}}$$

(i) **Electronic polarisability (α_e):** This type of polarisability arises due to displacement of electron cloud of an atom relative to its nucleus in the presence of applied electric field.

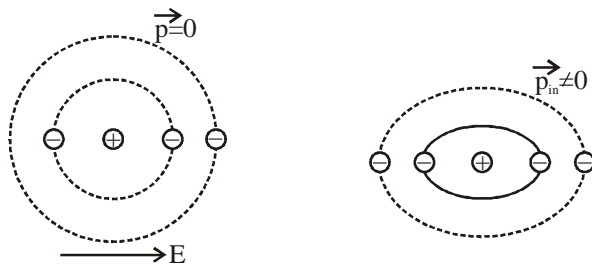
The Electronic polarisability is given by $\alpha_e = 4\pi\epsilon_0 R^3$, where R is radius of the atom.

This type of polarisability is independent of temperature but depends on the frequency of applied electric field. The frequency dependence of α_e is given by

$$\boxed{\alpha_e = \frac{e^2}{m(\omega_0^2 - \omega^2)}}$$

where m is mass of the electron, e is electronic charge, ω_0 is natural frequency of oscillation of atom and ω is frequency of applied electric field.

Unit of α : In C.G.S. units, α has unit of volume (cm^3), and in M.K.S. units, α is measured in $\text{F} - \text{m}^2$.



(ii) Ionic polarisability (α_i) :

This type of polarisability arises due to increase or decrease in the bond-length of ions.

This type of polarisability is independent of temperature but depends on the frequency of applied field. The ionic polarisability depends on frequency in following manner. For an ionic solid, the polarisability is given by

$$\alpha_i = \frac{e}{\omega_0^2} \left(\frac{1}{m} + \frac{1}{M} \right)$$

where m is mass of one ion (say Na^+),

M is mass of another ion (say Cl^-),

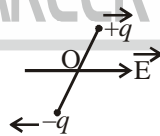
and ω_0 is natural frequency of the vibration of molecule (NaCl).

In general,
$$\alpha_i \approx \frac{\alpha_e}{10}$$

(iii) Dipolar polarisability (α_d) :

This type of polarisability arises due to the permanent dipole moment of the molecule. The dipolar polarisability is given by

$$\alpha_d = \frac{p_p^2}{3k_B T}$$



where, p_p is permanent dipole moment of the molecule, T is temperature and k_B is Boltzmann constant.

From above equation it is clear that α_d depends on both temperature and frequency of the field.

Hence total polarisability,

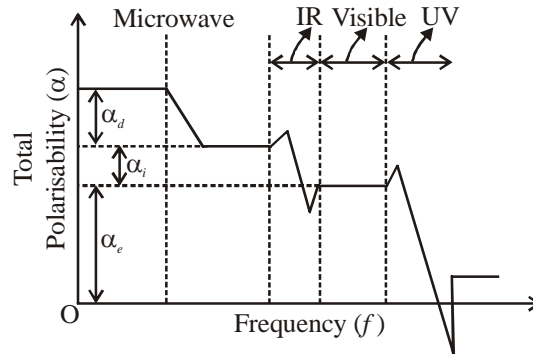
$$\alpha = \alpha_e + \alpha_i + \alpha_d$$

$$\Rightarrow \alpha = \alpha_0 + \frac{p_p^2}{3k_B T}$$

This relation is known as Langevin - Debye equation. This is valid for only polar molecules.

Frequency dependance of total polarisability: In visible region, total polarisation is electronic polarisation (α_e) i.e. $\alpha = \alpha_e$. Existence of $\alpha_i, \alpha_d, \alpha_e$ depends upon the relaxation time (τ). Lesser the τ , faster the

response i.e. $f \propto \frac{1}{\tau}$.



From this figure, $\tau_e < \tau_i < \tau_d$

At optical frequency or visible frequency: $\alpha = \alpha_e$

We have $\epsilon_r = n^2$

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_0}$$

This relation is known as Lorentz-Lorentz relation.

Static dielectric constant from Langevin-Debye Equation:

$$\text{Total polarisability } \alpha = \alpha_e + \alpha_i + \alpha_d = \alpha_e + \alpha_i + \frac{p_p^2}{3k_B T}$$

$$\text{Polarization : } \vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}_{mac} = N\alpha\vec{E}_{loc}$$

$$\text{If } \vec{E}_{loc} = \vec{E}_{mac}$$

This is possible if separation between atoms and molecules is so large that there is no interaction between the dipoles like in liquid or gases.

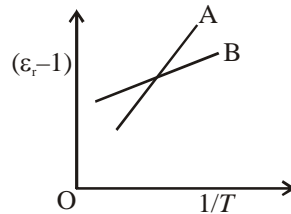
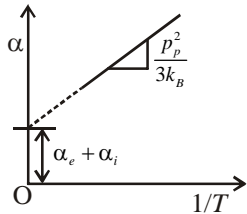
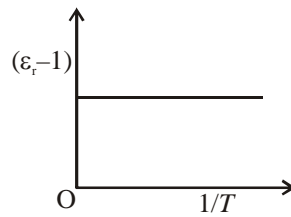
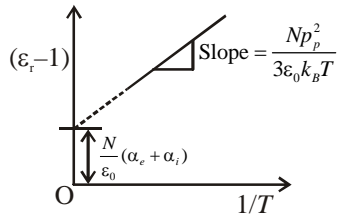
$$\text{So, } \vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}_{mac} = N\alpha\vec{E}_{loc}$$

$$\text{Thus, we get } (\epsilon_r - 1) = \frac{N}{\epsilon_0}(\alpha_e + \alpha_i) + \frac{Np_p^2}{3\epsilon_0 k_B T}$$

i.e., More slope \Rightarrow More polar and

Large intercept \Rightarrow More electronic and ionic polarizabilities.

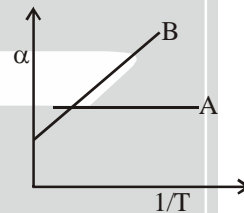
If graph is flat means molecule is non-polar.



From above figure it is clear that A is more polar because of more slope.

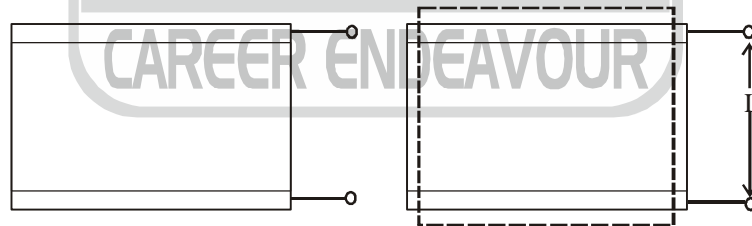
Those have permanent dipole moment are represented by B. Some of molecules that are characterized more and less polar are given below:

- (i) CH₄ → A
- (ii) CO₂ → A
- (iii) H₂O → B
- (iv) HCl → B
- (v) CO → B
- (vi) CS₂ → A
- (vii) SO₂ → B
- (viii) NO₂ → B



Piezoelectricity:

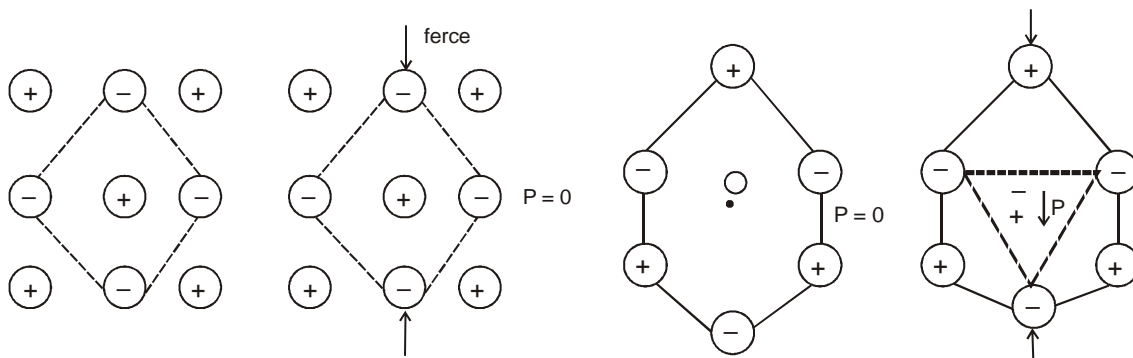
Certain crystals e.g. quartz (crystalline SiO₂) and BaTiO₃ become polarized when mechanically stressed. Appearance of surface charge leads to voltage difference between the two surfaces of the crystal. Same crystal also exhibit mechanical strain or distortion when they experience electric field.



Only certain crystal that exhibit no centre of symmetry, shows piezoelectricity.

Example: NaCl type has centre of symmetry.

Example: Hexagonal



Non-centro symmetric direction of polarization depends on the direction of applied stress. Generally, an applied stress in one direction can give rise to induced polarization in other crystal direction. Suppose, T_j is applied mechanical stress along some j direction, P_i is induced polarization along some i direction.

Two are linearly related as

$$P_i = d_{ij} T_j, \text{ where } d_{ij} \text{ is piezoelectric coefficient.}$$

The converse piezoelectric effect is that between an induced strain S_j along j direction and an applied electric field E_i along i direction.

$$S_j = d_{ij} E_i$$

They are called transducers because they convert electrical signal, electric field to mechanical signal, strain & vice-versa used in microphones, ultrasonic transducers.

Electro-mechanical coupling factor k is defined as

$$k^\Omega = \frac{\text{Electrical energy converted to mechanical energy}}{\text{Input of electrical energy}}$$

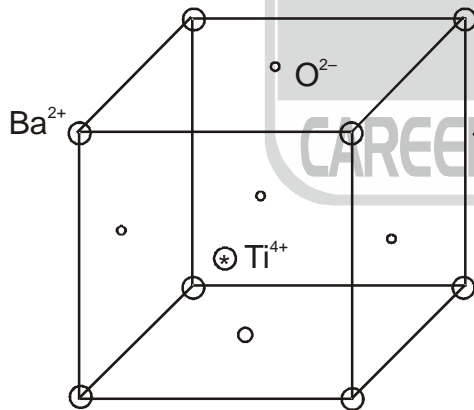
$$\text{or } k^L = \frac{\text{Mechanical energy converted to electrical energy}}{\text{Input of mechanical energy}}$$

Ferroelectric crystal:

Certain crystal are the occurrence of spontaneous polarization in certain crystal such as Barium Titanate (BaTiO_3). Ferroelectric crystal have a permanent polarization p as a result of spontaneous polarization. The direction of p can be defined by the application of an external field.

Pyroelectric crystal:

This material is a polar dielectric (such as BaTiO_3) in which a temperature change and T induces a proportional change Δp in the polarization that is $\Delta p = p \Delta T$, where p is pyroelectric coefficient of the crystal.



$$p = \frac{dp}{dT}, \text{ where } p = \text{pyroelectric coefficient}$$

$$\delta p = \epsilon_0 (\epsilon_r - 1) \delta E \text{ is used in infrared detectors}$$

$$\text{rather } \Delta p = \epsilon_0 (\epsilon_r - 1) \Delta E .$$

$$p \neq \epsilon_0 (\epsilon_r - 1) E$$

Δp normal to the plates changes the stored charge which is of relevance.

SOLVED PROBLEMS

1. The electronic polarizability of the Ar atom is $1.7 \times 10^{-40} \text{ Fm}^2$. What is the static dielectric constant of solid Ar (below 84 K) if its density is 1.8 g cm^{-3} .

Soln. $\epsilon_r = 1 + \frac{N\alpha_e}{\epsilon_0}$ where $N = \frac{N_A d}{M} = \frac{6.02 \times 10^{23} \times 1.8}{39.95} = 2.71 \times 10^{22}$

$$\therefore \epsilon_r = 1 + \frac{(2.71 \times 10^{22})(1.7 \times 10^{-40})}{(8.85 \times 10^{-12})} = 1.52$$

Using Clausius-Mosottie equation, $\epsilon_r = \frac{1 + \frac{2N\alpha_e}{3\epsilon_0}}{1 - \frac{N\alpha_e}{3\epsilon_0}} = \frac{1 + \left(\frac{2 \times (2.71 \times 10^{22}) \times 1.7 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}} \right)}{1 - \left(\frac{2.71 \times 10^{22} \times 1.7 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}} \right)} = 1.63$

2. Consider a pure Si Crystal that has $\epsilon_r = 11.9$
- (a) What is the electronic polarizability due to valence electron per Si atom (if one could portion the observed crystal polarization to individual atoms)?
- (b) Suppose that a Si crystal sample is electroded on opposite faces and has a voltage applied across it by how much is the local field greater than the applied field?
- (c) What is the Resonant frequency of corresponding $N/v = 5 \times 10^{28} \text{ m}^{-3}$.

Soln. (a) $\alpha_e = \frac{3\epsilon_0 (\epsilon_r - 1)}{N (\epsilon_r + 2)} = \frac{3(8.85 \times 10^{-12}) (11.9 - 1)}{5 \times 10^{28} (11.9 + 2)} = 4.17 \times 10^{-40} \text{ Fm}^2$

(b) Local field is $E_{loc} = E + \frac{P}{3\epsilon_0} \Rightarrow P = \chi_e \epsilon_0 E = (\epsilon_r - 1) \epsilon_0 E$

$$E_{loc} = E + \frac{1}{3}(\epsilon_r - 1)E \Rightarrow \frac{E_{loc}}{E} = \frac{1}{3}(\epsilon_r + 2) = 4.63$$

- (c) Since polarization is due to valence electrons and there are four for Si atom, we can

$$\text{Use } W_o = \left(\frac{Z_e^2}{m_e \alpha_e} \right)^{1/2} = \left[\frac{4(1.6 \times 10^{-19})^2}{(9.1 \times 10^{-31})(4.17 \times 10^{-40})} \right]^{1/2} = 1.65 \times 10^{16} \text{ rad s}^{-1} \quad (\text{For } \frac{\omega_o}{2\pi} = 2.6 \times 10^{15} \text{ Hz})$$

3. Consider the CsCl crystal which has one $\text{Cs}^+ - \text{Cl}^-$ pair per unit cell and a lattice parametre a of 0.412 nm. The electronic polarisability of Cs^+ and Cl^- ions is $3.45 \times 10^{-40} \text{ Fm}^2$ and $3.40 \times 10^{-40} \text{ Fm}^2$ respectively, and the mean ionic polarisability per ion pair is $6 \times 10^{-40} \text{ Fm}^2$. What is the dielectric constant at low frequencies and that at optical frequencies.

Soln. The CsCl structure has one cation Cs^+ and one anion (Cl^-) in the unit cell.

Give lattice parameters $a = 0.412 \times 10^{-9} \text{ m}$, Number of ions per unit volume in Ni is $1/a^3$.

$$\text{Number of ions per unit volume} = \frac{1}{(0.412 \times 10^{-9})^3} = 1.43 \times 10^{28} \text{ m}^{-3}$$

also concentration of Cations and Anions

$$\text{now } \frac{\epsilon_r - 1}{\epsilon_0 + 2} = \frac{1}{3\epsilon_0} \left[N_i \alpha_e (Cs^+) + N_i \alpha_e (Cl^-) + N_i \alpha_i \right]$$

(At low frequency) $\epsilon_r = 7.56$

(At high frequency optical frequency) ionic polarization in two slugs

$$\frac{\epsilon_r - 1}{\epsilon_0 + 2} = \frac{1}{3\epsilon_0} \left[N_i \alpha_e (Cs^+) + N_i \alpha_e (Cl^-) \right], \quad \epsilon_r = 2.71$$

4. Consider a Piezoelectric sample in the form of a cylinder. Suppose that the piezoelectric coefficient $d = 250 \times 10^{12} \text{ mV}^{-1}$ and $\epsilon_r = 1000$. The piezoelectric cylinder has a length of 10 mm and a diameter of 3 mm. The spark gap is in air and has a break down voltage of about 3.5 kV. What is the force required to spark the gap?

Soln. Induced polarization $T = \text{Stress}$, $P = dT = \frac{dF}{A}$

Induced polarization leads to induced surface polarization charges given by $Q = AP$. If C is the capacitance, then the induced voltage is

$$V = \frac{Q}{C} = \frac{AP}{\epsilon_0 \epsilon_r A} = \frac{LP}{\epsilon_0 \epsilon_r} = \frac{L(dF/A)}{\epsilon_0 \epsilon_r} = \frac{dLF}{\epsilon_0 \epsilon_r A}$$

$$\therefore F = \frac{\epsilon_0 \epsilon_r AV}{dL} = \frac{(1000 \times 8.85 \times 10^{-12}) \times (1.5 \times 10^{-3})^2 (3500)}{(250 \times 10^{-12})(10 \times 10^{-3})} = 87.6 \text{ N}$$

5. Determine the percentage of ionic polarisability in NaCl crystal which has optical index of refraction and the static dielectric constant as 1.5 and 1.6 respectively.

Soln. $\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0}$

$$\epsilon_r = 1.6 \text{ and } n = 1.5$$

$$\frac{1.6 - 1}{1.6 + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0} \quad \Rightarrow \quad \frac{1}{6} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0} \quad \dots (1)$$

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_0} \quad \Rightarrow \quad \frac{2.25 - 1}{2.25 + 2} = \frac{N\alpha_e}{3\epsilon_0} \quad \Rightarrow \quad \frac{5}{17} = \frac{N\alpha_e}{3\epsilon_0} \quad \dots (2)$$

Dividing equation (1) by equation (2), we get

$$\frac{1}{6} \times \frac{17}{5} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0} \times \frac{3\epsilon_0}{N\alpha_e} \quad \Rightarrow \quad \frac{17}{30} = 1 + \frac{\alpha_i}{\alpha_e}$$

$$\frac{17}{30} - 1 = \frac{\alpha_i}{\alpha_e} \quad \Rightarrow \quad \frac{(\alpha_e + \alpha_i)}{\alpha_e} = \frac{17}{30} \quad \Rightarrow \quad \frac{\alpha_e}{(\alpha_e + \alpha_i)} = \frac{30}{17} \quad \Rightarrow \quad 1 - \frac{\alpha_i}{(\alpha_e + \alpha_i)} = \frac{30}{17}$$

$$\Rightarrow \quad \frac{\alpha_i}{\alpha_e + \alpha_i} \times 100 = \left[1 - \left(\frac{n^2 - 1}{n^2 + 2} \right) \left(\frac{\epsilon_r + 2}{\epsilon_r - 1} \right) \right] \times 100$$

$$= \left[1 - \left[\frac{(1.5)^2 - 1}{(1.5)^2 + 2} \right] \left[\frac{5.6 + 2}{5.6 - 2} \right] \right] \times 100 = 51.4\%$$

PRACTICE SET

1. In a drop of water of radius 10^{-3} m, the molecular dipoles are pointed in the same direction. If the dipole moment of the water molecule is 6×10^{-30} C-m, its polarization will be
 (a) 8.4×10^{-10} C/m² (b) 8.4×10^{-12} C/m² (c) 8.4×10^{-14} C/m² (d) 8.4×10^{-16} C/m²
2. The relative permittivity of argon at 0°C and one atmosphere is 1.000435, the electronic polarizability of the argon atom will be
 (a) 1.43×10^{-38} Fm² (b) 1.43×10^{-39} Fm² (c) 1.43×10^{-40} Fm² (d) 1.43×10^{-41} Fm²
3. A photon of wavelength 1400 Å is absorbed by cold mercury vapour and two other photons are emitted. If the wavelength of one of them is 1850 Å, the wavelength of the other photon will be
 (a) 5755 Å (b) 5855 Å (c) 5955 Å (d) 6055 Å
4. Determine the % of ionic polarisability in NaCl crystal which has optical index of refraction as 1.5 and dielectric constant as 1.6.
5. Dielectric polarizability is a proportionality constant in the relation between
 (a) Electric dipole moment and local electric field
 (b) Displacement vector and local electric field
 (c) Electric dipole moment and charge density
 (d) Displacement vector and charge density
6. $\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$ is
 (a) Lorentz relation (b) Einstein relation
 (c) Clausius-Mosottie relation (d) None of these
7. For ferromagnetic materials, the permeability 'μ' is
 (a) < 1 (b) = 1 (c) > 1 (d) >> 1
8. Dipole moment of a current loop is given by
 (a) niA (b) nqA (c) $\frac{n}{q}$ A (d) $\frac{nA}{i}$
 where n = numbers of turns, i = current and A = area of cross section of the loop, q = charge.
9. Electrons are emitted with zero velocity from a certain metal surface when it is exposed to a radiation of wavelength 6850 Å. The work function of the metal is
 (a) 0.813 eV (b) 1.813 eV (c) 2.813 eV (d) 3.813 eV
10. When a surface of copper is irradiated by light of wavelength 1849 Å, the stopping potential is found to be 2.72 volts. The threshold frequency of the metal is
 (a) 9.658×10^{12} c/s (b) 9.658×10^{13} c/s (c) 9.658×10^{14} c/s (d) 9.658×10^{15} c/s

ANSWER KEY

- | | | | | | |
|---------------|---------------|---------------|-------------------|---------------|---------------|
| 1. (a) | 2. (c) | 3. (a) | 4. (51.4%) | 5. (a) | 6. (c) |
| 7. (d) | 8. (a) | 9. (b) | 10. (c) | | |