

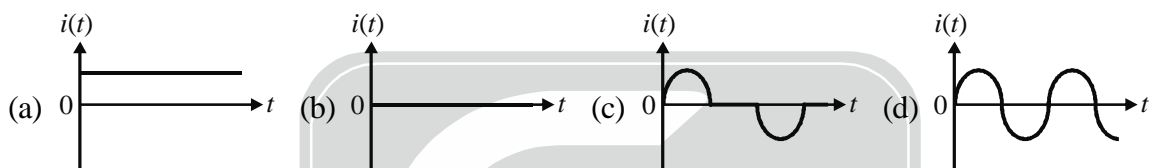


GS-2018-X (PHYSICS)

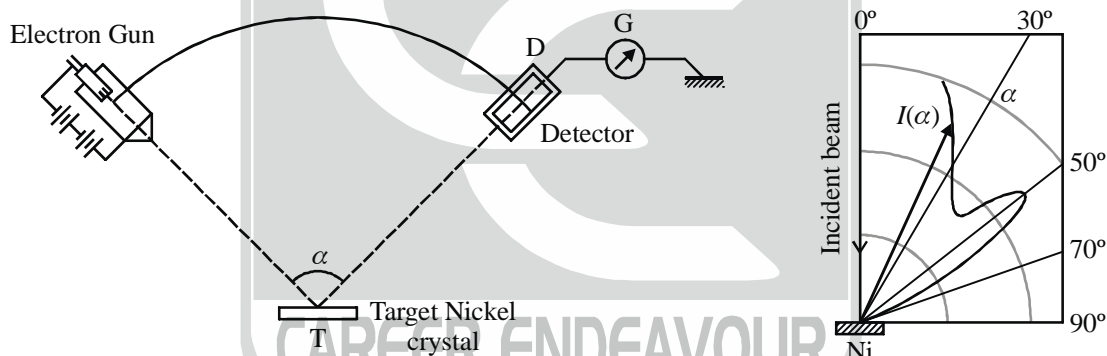
Tata Institute of Fundamental Research [T.I.F.R.]

SECTION – A

1. A circular coil of conducting wire, of radius a and n turns, is placed in a uniform magnetic field \vec{B} along the axis of the coil and is then made to undergo simple harmonic oscillations along the direction of the axis. The current through the coil will be best described by



2. The sketch shown illustrates the apparatus and results for a famous experiment. The graph on the right is a polar plot of the number of electrons received in the detector.



- (a) the energy levels of atoms in a metal are quantized.
 (b) there are magnetic domains inside a nickel sample.
 (c) electrons in a beam can behave as wave.
 (d) electrons have spin half.
3. A particle of mass m , moving in one dimension, satisfies the modified Schrödinger Equation:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + i\hbar u \frac{d\psi}{dx} = i\hbar \frac{d\psi}{dt}$$

where u is the velocity of the substrate. If, now, this particle is treated as a Gaussian wave packet peaked at wavenumber k , its group velocity will be $v_g =$

- (a) $\frac{\hbar k}{m} + u$ (b) $\frac{\hbar k}{2m} - u$ (c) $-\frac{\hbar k}{2m} + u$ (d) $\frac{\hbar k}{m} - u$
4. A set of polynomials of order n are given by the formula

$$P_n(x) = (-1)^n \exp\left(\frac{x^2}{2}\right) \frac{d^n}{dx^n} \exp\left(-\frac{x^2}{2}\right)$$

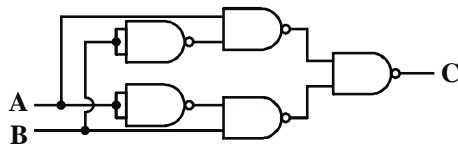
The polynomial $P_7(x)$ of order $n = 7$ is

- (a) $x^7 - 21x^5 + 105x^4 + 35x^3 - 105x$ (b) $x^6 - 21x^5 + 105x^4 - 105x^3 + 21x^2 + x$
 (c) $x^7 - 21x^5 + 105x^3 - 105x + 21$ (d) $x^7 - 21x^5 + 105x^3 - 105x$

5. Consider the surface defined by $ax^2 + by^2 + cz + d = 0$, where a, b, c and d are constants. If \hat{n}_1 and \hat{n}_2 are unit normal vectors to the surface at the points $(x, y, z) = (1, 1, 0)$ and $(0, 0, 1)$ respectively, and \hat{m} is a unit vector normal to both \hat{n}_1 and \hat{n}_2 , then $\hat{m} =$

- (a) $\frac{-a\hat{i} + b\hat{j}}{\sqrt{a^2 + b^2}}$ (b) $\frac{a\hat{i} - b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (c) $\frac{2a\hat{i} + 2b\hat{j} - c\hat{k}}{\sqrt{4a^2 + 4b^2 + c^2}}$ (d) $\frac{b\hat{i} - a\hat{j}}{\sqrt{a^2 + b^2}}$

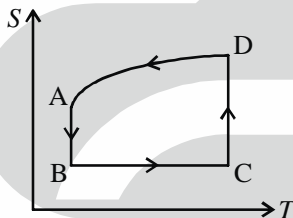
6. The circuit shown below uses only NAND gates.



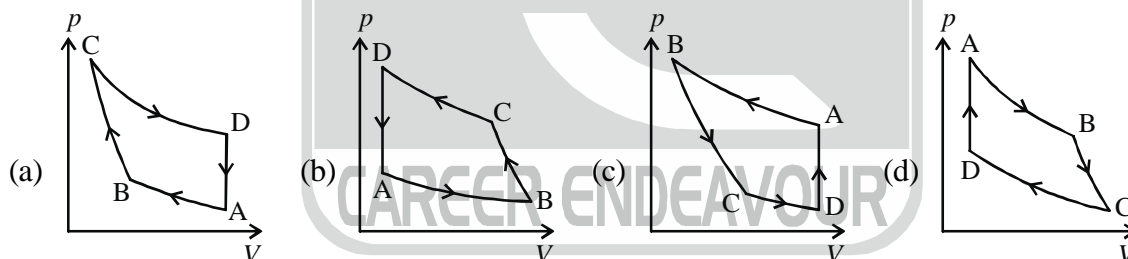
The final output at C is

- (a) **A AND B** (b) **A XOR B** (c) **A NOR B** (d) **A OR B**

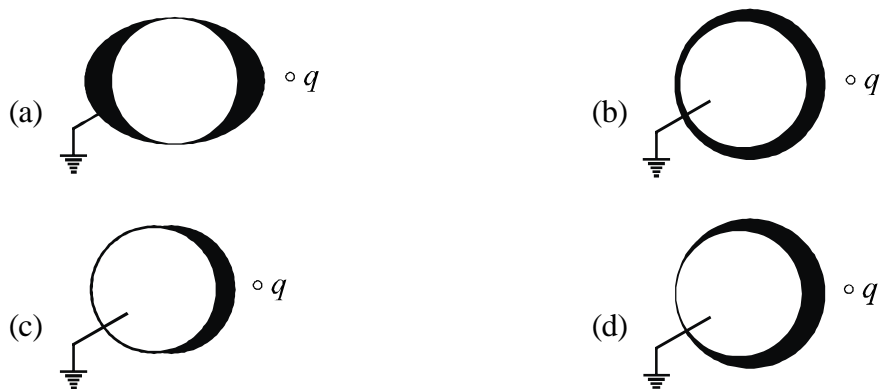
7. An ideal gas engine is run according to the shown in the s - T diagram below, where the process from D to A is known to be isochoric (i.e., maintaining $V = \text{constant}$).



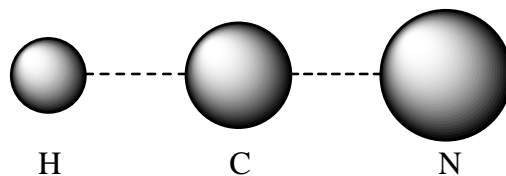
The corresponding cycle in the p - V diagram will most closely resemble



8. A point charge $q = 0$ is brought in front of a grounded conducting sphere. If the induced charge density on the sphere is plotted such that the thickness of the black shading is proportional to the charge density, the correct plot will most closely resemble

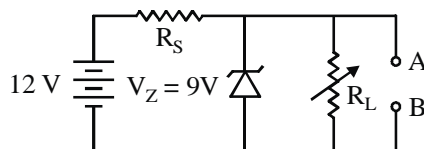


9. Consider the following linear model of a molecule of hydrogen cyanide (HCN) depicted below.



If follows that the molar specific heat of hydrogen cyanide gas at constant pressure must be

- (a) $5.5 R$ (b) $6 R$ (c) $5 R$ (d) $4.5 R$
10. Drawing power from a 12 V car battery, a 9 V stabilized DC voltage is required to power a car stereo system, attached to the terminals A and B, as shown in the figure.



If a Zener diode with ratings, $V_z = 9 \text{ V}$ and $P_{\text{max}} = 0.27 \text{ W}$, is connected as shown in the figure, for the above purpose, the minimum series resistance R_s must be

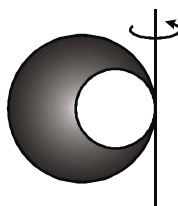
- (a) 103Ω (b) 111Ω (c) 97Ω (d) 100Ω
11. A beam of high energy neutrons is scattered from a metal lattice, where the spacing between nuclei is around 0.4 nm. In order to see quantum diffraction effects, the energy of the neutrons must be around
- (a) 78.5 eV (b) 5.11 meV (c) 511 keV (d) 7.85 MeV
12. Consider three straight, coplanar, parallel wires of infinite length where the distance between adjacent wires is d . Each wire carries a current I in the same direction. The perpendicular distance from the middle wire (on either side) where the magnetic field vanishes is
- (a) $d/3$ (b) $2d/3$ (c) $d/\sqrt{3}$ (d) $2d/\sqrt{3}$
13. Consider a thermal ensemble at temperature T which is composed to identical quantum harmonic oscillators of frequency ω_0 with non-overlapping wavefunctions. The probability that these will be an even number of energy quanta in the system is

- (a) $\tanh\left(\frac{\hbar\omega_0}{2k_B T}\right)$ (b) $\frac{1}{\exp\left(-\frac{\hbar\omega_0}{k_B T}\right) + 1}$ (c) $\frac{1}{\exp\left(\frac{\hbar\omega_0}{k_B T}\right) + 1}$ (d) $\frac{1}{\exp\left(-\frac{\hbar\omega_0}{k_B T}\right) - 1}$

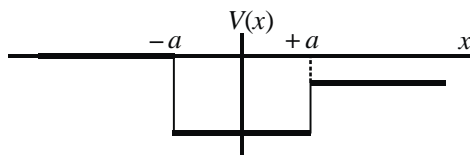
14. A thermally-insulated coffee mug contains 500 g of warm coffee at 80°C . Assuming that the heat capacity of this liquid is $1 \text{ cal g}^{-1}^\circ\text{C}^{-1}$ and the latent heat of fusion for ice is 80 cal g^{-1} , the amount of ice that must be dropped into the cup to convert it into cold coffee at 5°C is about
- (a) 441 g (b) 421 g (c) 469 g (d) 471 g

15. The three-dimensional object sketched on the right is made by taking a solid sphere of uniform density (shaded) with radius R , and scooping out a spherical cavity (unshaded) as shown, which has diameter R . If this object has mass M , its moment of inertia about the tangential axis passing through the point where the spheres touch (as shown in the figure) is

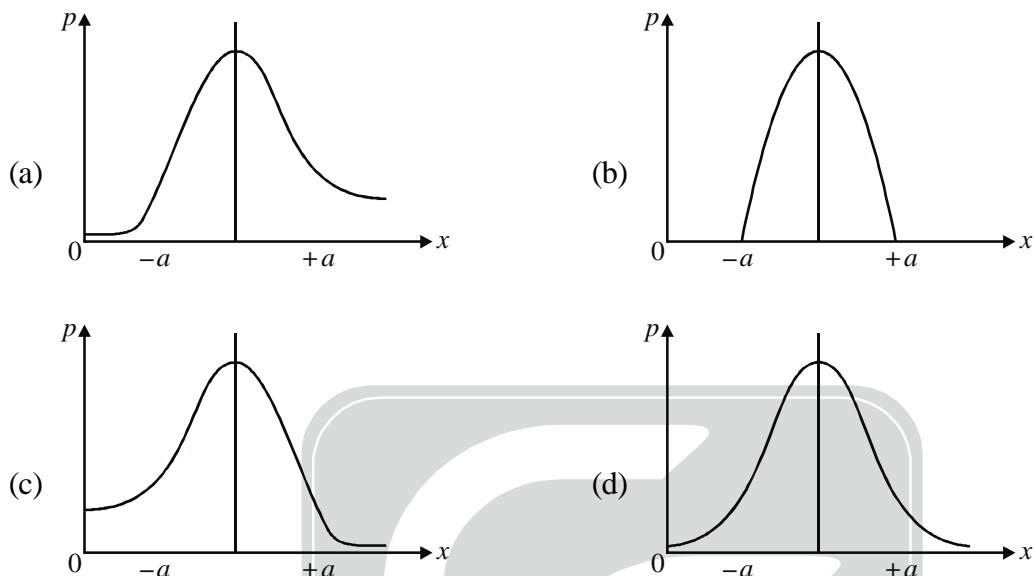
- (a) $\frac{62}{35} MR^2$ (b) $\frac{31}{70} MR^2$
 (c) $\frac{31}{20} MR^2$ (d) $\frac{3}{16} MR^2$



16. A particle moving in one dimension, is placed in an asymmetric square well potential $V(x)$ as sketched below.



The probability density $p(x)$ in the ground state will most closely resemble



17. A British coin has a portrait of Queen Elizabeth II on the 'heads' side the 'ONE POUND' written on the 'tails' side, while an Indian coin has a portrait of Mahatma Gandhi on the 'heads' side and '10 RUPEES' written on the 'tails' side (see below).



These two coins are tossed simultaneously twice in succession.

The result of the first toss was 'heads' for both the coins. The probability that the result of the second toss had a '10 RUPEES' side is

- (a) $4/7$ (b) $1/2$ (c) $3/5$ (d) $2/3$

18. A plane electromagnetic wave travelling through vacuum has electric field \vec{E} and magnetic field \vec{B} defined as

$$\vec{E} = (\hat{i} + \hat{j}) E_0 \exp i(\omega t - \vec{k} \cdot \vec{x})$$

$$\vec{B} = (\hat{i} - \hat{j} - \hat{k}) B_0 \exp i(\omega t - \vec{k} \cdot \vec{x})$$

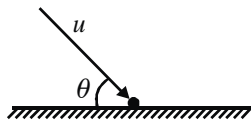
where E_0 and B_0 are real constants. The time-averaged Poynting vector will be given by

(a) $\vec{S} = \sqrt{\frac{\epsilon_0}{6\mu_0}} E_0^2 (-\hat{i} + \hat{j} - 2\hat{k})$ (b) $\vec{S} = -\frac{1}{2} \sqrt{\frac{3\epsilon_0}{\mu_0}} E_0^2 (\hat{i} - \hat{j} + 2\hat{k})$

(c) $\vec{S} = -\frac{2}{\sqrt{\epsilon_0 \mu_0}} E_0 B_0 (\hat{i} - \hat{j} + 2\hat{k})$ (d) $\vec{S} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} B_0^2 (\hat{i} - \hat{j} + 2\hat{k})$

19. A particle of mass m is bounced on the ground with a velocity u making an angle θ with the ground. The coefficient of restitution for collisions between the particle and the ground is ϵ and frictional effects are negli-

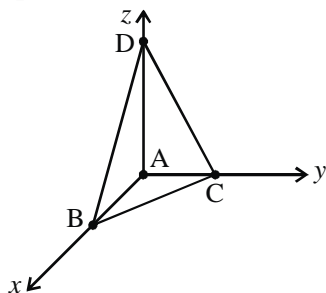
gible both on the ground and in the air. The horizontal distance travelled by the particle from the point of initial impact till it begins to slide along the ground is



- (a) $\frac{u^2}{g} \left(\frac{\varepsilon}{1-\varepsilon} \right) \tan 2\theta$ (b) $\frac{u^2}{2g} \left(\frac{\varepsilon}{1-\varepsilon} \right) \sin 2\theta$ (c) $\frac{u^2}{g} \left(\frac{1}{1-\varepsilon} \right) \sin \theta$ (d) $\frac{u^2}{g} \left(\frac{\varepsilon}{1-\varepsilon} \right) \sin 2\theta$

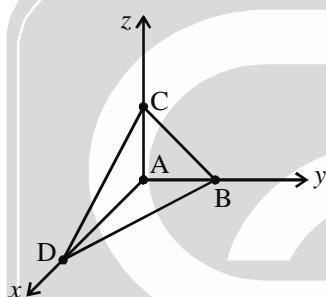
20. Which of the following operations will transform a tetrahedron ABCD with vertices as listed below:

| | x | y | z |
|---|---|---|---|
| A | 0 | 0 | 0 |
| B | 1 | 0 | 0 |
| C | 0 | 1 | 0 |
| D | 0 | 0 | 2 |



into a tetrahedron ABCD with vertices as listed below:

| | x | y | z |
|---|---|---|---|
| A | 0 | 0 | 0 |
| B | 0 | 1 | 0 |
| C | 0 | 0 | 1 |
| D | 2 | 0 | 0 |



up to suitable translation?

- (a) A rotation about y-axis by $\pi/2$, then a reflection in the xz -plane.
 (b) A rotation about x-axis by $\pi/2$, then a rotation about z-axis by $\pi/2$.
 (c) A reflection in the yz -plane, then a reflection in the xy -plane.
 (d) A reflection in the xy -plane, then a rotation about x-axis by $\pi/2$.

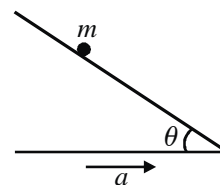
21. In a one-dimensional system, the boundary condition that the derivative of the wavefunction $\psi'(x)$ should be continuous at every point is applicable whenever

- (a) the wavefunction $\psi(x)$ is itself continuous everywhere.
 (b) there is a bound state and the potential is piecewise continuous.
 (c) there are bound or scattering states with definite momentum.
 (d) there is a bound state and the potential has no singularity anywhere.

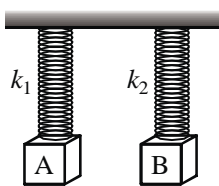
22. A particle of mass m is placed on an inclined plane making an adjustable angle θ with the horizontal, as shown in the figure. The coefficient of friction between the particle and the inclined plane is μ .

If the inclined plane is moving horizontally with a uniform acceleration $a < \frac{g}{\mu}$ (see figure), the value of θ for which the particle will remain at rest on the plane is

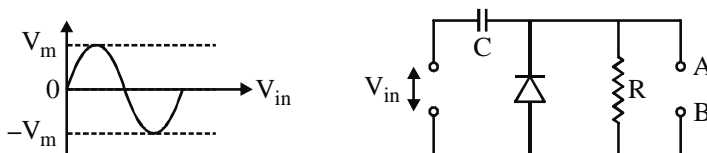
- (a) $\theta = \tan^{-1} \left(\frac{\mu g + a}{g - \mu a} \right)$ (b) $\theta = -\cot^{-1} \left(\frac{\mu a + g}{a + \mu g} \right)$
 (c) $\theta = \cot^{-1} \left(\frac{\mu a - g}{a + \mu g} \right)$ (d) $\theta = \tan^{-1} \left(\frac{\mu g + a}{g + \mu a} \right)$



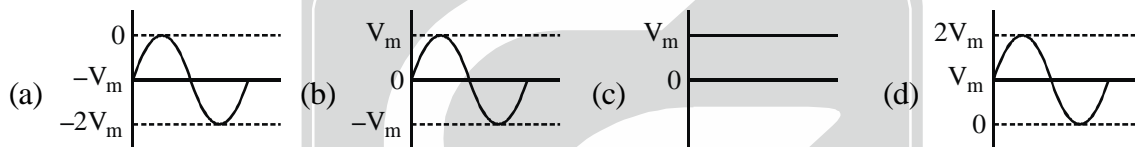
23. Two bodies A and B of equal mass are suspended from two rigid supports by separate massless springs having spring constants k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal the ratio of the amplitude of oscillations of A to that of B is



- (a) $\sqrt{k_2/k_1}$ (b) $\sqrt{k_1/k_2}$ (c) k_1/k_2 (d) k_2/k_1
24. The signal shown on the left side of the figure below is fed into the circuit shown on the right side.



If the signal has time period τ_S and the circuit has a natural frequency τ_{RC} , then, in the case when $\tau_S \ll \tau_{RC}$, the steady-state output will resemble



25. The eigenvalues of a 3×3 matrix M are $\lambda_1 = 2$, $\lambda_2 = -1$, $\lambda_3 = 1$, and the eigenvectors are

$$e_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, e_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, e_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

The matrix M is

- (a) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$

SECTION - B

- In an experiment to measure the Earth's mean albedo (i.e., fraction of solar energy reflected back into space), the solar constant (i.e., flux of solar energy incident upon the Earth), was measured as 1.37 kW m^{-2} . Assuming the Earth to behave as a perfect blackbody at a uniform surface temperature of -18°C , the albedo is about
 - 0.30
 - 0.18
 - 0.06
 - 0.46
- In a futuristic experiment, two rocket ships, each containing one astronaut Rakesh and Sunita respectively, blasted off from the Earth's surface simultaneously, and travelled into space in straight lines in opposite directions at uniform speeds of $0.3c$ and $0.5c$ respectively. They both travelled in straight lines for some time, then reversal direction smoothly and returned along the same paths with the same speeds. It was found that they returned simultaneously after exactly 9.0 years. Which of the following statements is correct?
 - The age of Rakesh increased by about 8.6 years and that of Sunita by about 7.8 years.
 - The ages of both Rakesh and Sunita increased by 9.0 years.
 - The age of Rakesh increased by about 9.4 years and that of Sunita by about 10.4 years.
 - The ages of both Rakesh and Sunita increased by about 3.06 years.

3. An array \mathbb{T} has elements T_{ijkl} where $i, j, k, l = 1, 2, 3, 4$. It is given that $T_{ijkl} = T_{jikl} = T_{ijlk} = -T_{klij}$ for all values of i, j, k, l . The number of independent components in this array is
 (a) 55 (b) 1 (c) 45 (d) 256
4. A dielectric interface is formed by two homogeneous and isotropic dielectrics 1 and 2 with dielectric constants $4/3$ and 1 respectively and it carries no residual free charge. A linearly polarized electromagnetic wave is incident on the interface from dielectric 1 at a point where the unit normal to the surface is

$$\hat{n} = \frac{1}{2}(\sqrt{3}\hat{i} + \hat{k})$$

pointing into the dielectric 1. The incident wave, which is incident from 1 into 2, just before it reaches the interface, has electric vector

$$\vec{E}_1 = \hat{i} E_0 \exp i\omega \left(t + \frac{\sqrt{\epsilon_1}}{c} z \right)$$

where E_0 is a real constant. The electric vector just after it crosses the interface is rotated from \vec{E}_1 by an angle

- (a) $\sin^{-1} \frac{1}{2\sqrt{19}}$ (b) $\tan^{-1} \frac{2}{5\sqrt{3}}$ (c) $\frac{\pi}{6}$ (d) $\csc^{-1} 3\sqrt{\frac{4}{19}}$

5. A perfectly straight tunnel is dug between any two points on the surface of the Earth, which can be treated as a static sphere of uniform density ρ . The tunnel does not necessarily pass through the centre of the Earth. If a particle is allowed to slide without friction in this tunnel under the action of the Earth's gravity, it will execute simple harmonic motion with time period

- (a) $\sqrt{\frac{3\pi}{\rho G}}$ (b) $\sqrt{\frac{3}{2\rho G}}$ (c) $\sqrt{\frac{3}{2\pi\rho G}}$ (d) $\sqrt{\frac{6\pi}{\rho G}}$

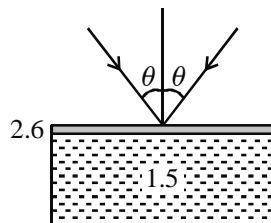
6. In a sample of germanium, at a temperature 77 K , optical excitations create an average density of 10^{12} conduction electrons per cm^3 . At this temperature, the electron and hole mobilities are equal, and given by

$$\mu = 0.5 \times 10^4 \text{ cm}^2 \text{ s}^{-1} \text{ V}^{-1}$$

The value of the Einstein diffusion coefficient for the electrons and holes is

- (a) $1.65 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ (b) $3.3 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}$ (c) $3.3 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ (d) $6.6 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$

7. In an experiment performed to determine the width of a thin transparent film of refractive index 2.6 deposited uniformly on the upper surface of a thick transparent glass block of refractive index 1.5 , a monochromatic laser beam of wavelength 550 nm was shone on the film at a variable angle θ with the normal to the surface. If the intensity of the reflected beam was maximum when $\theta = 5^\circ 24'$, the thickness of the film may be inferred to be



- (a) $2.92 \mu\text{m}$ (b) $13.2 \mu\text{m}$ (c) $1.12 \mu\text{m}$ (d) $0.48 \mu\text{m}$

8. The differential equation $x \frac{dy}{dx} - xy = \exp(x)$, where $y = e^2$ at $x = 1$, has the solution $y =$

- (a) $\exp(1+x)(1+\ln x)$ (b) $(1-x)\exp(x) + \exp(1+x)$
 (c) $\exp(x)\ln x + \exp(1+x)$ (d) $\exp(x^2+x)$

9. The molar equation of state of a gas at temperature T , pressure P and volume V is given by

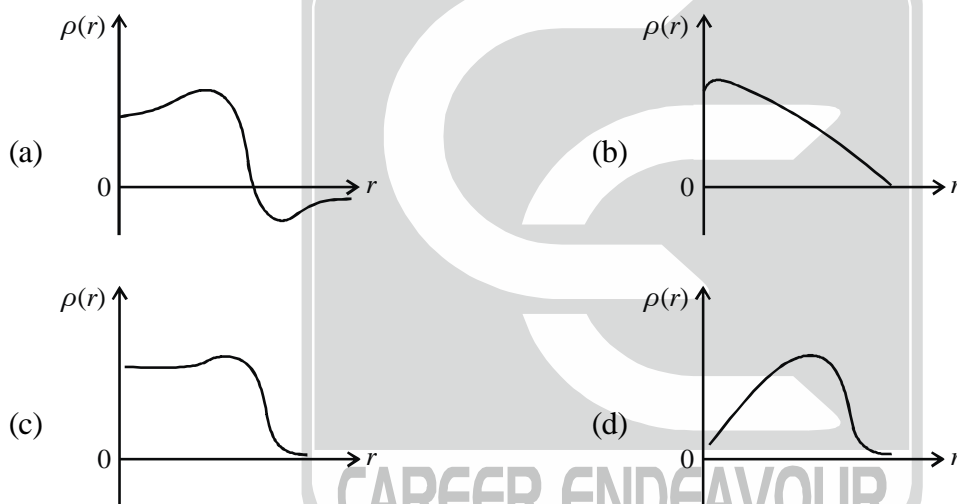
$$P = \frac{RT}{V-b} - \frac{a}{TV^2}$$

where a and b are two constants and R is the gas constant. The critical temperature and pressure for this gas will be

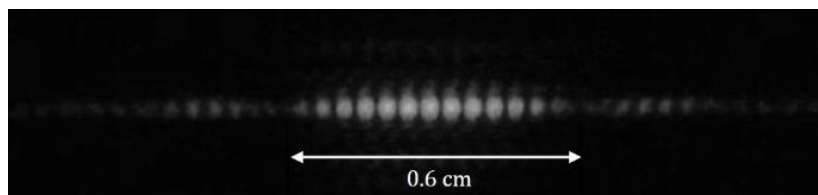
- (a) $T_c = \sqrt{\frac{8a}{3Rb}}$, $P_c = \frac{RT_c}{8b}$ (b) $T_c = \sqrt{\frac{a}{27Rb}}$, $P_c = \frac{RT_c}{b}$
 (c) $T_c = \sqrt{\frac{4a}{27Rb}}$, $P_c = \frac{RT_c}{4b}$ (d) $T_c = \sqrt{\frac{8a}{27Rb}}$, $P_c = \frac{RT_c}{8b}$

10. An excited gas, consisting of spinless charged particles, is confined in an infinite square well potential of width a , is found to radiate a spectrum whose α line (largest wavelength) has wavelength 816 nm. If the width a of the well is halved to $a/2$, the wavelength of the δ line (fourth-largest wavelength) will be
 (a) 1.088 μm (b) 0.306 μm (c) 1.224 μm (d) 26.122 μm

11. The electric charge density $\rho(r)$ inside a heavy spherical nucleus as a function of distance r from the centre may be approximated most closely by



12. The diffraction pattern due to a double slit experiment with two identical slits, is recorded in infra-red laser radiation of wavelength 1.2 μm on a specially-prepared photographic plate at a distance of 1.8 m from the centre of the slits. A photograph of the observed diffraction pattern is given below.



The distance between the slits can be calculated as

- (a) 5.04 mm (b) 4.32 mm (c) 5.76 mm (d) 9.36 mm
13. Consider a hydrogenic atom in its ground state as conceived in Bohr's theory, where an electron of charge $-e$ is rotating about a central nucleus of charge $+Ze$ in a circular orbit of radius

$$a = \frac{4\pi \epsilon_0 \hbar^2}{Ze^2 m}$$

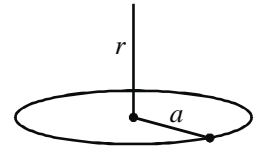
In this model, the magnetic field at a distance r from the nucleus, perpendicular to the orbit, will be

$$(a) \frac{Ze^3 \mu_0}{16\pi^2 \hbar \epsilon_0} \left(1 + \frac{r^2}{a^2}\right)^{-1/2}$$

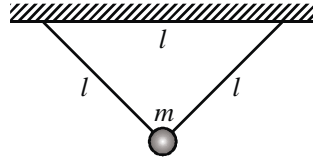
$$(b) \frac{Ze \mu_0}{\hbar \epsilon_0} \left(1 + \frac{r^2}{a^2}\right)^{-3/2}$$

$$(c) \frac{Ze^3 \mu_0}{8\pi \epsilon_0 \hbar} \left(1 + \frac{r^2}{a^2}\right)^{-3/2}$$

$$(d) \frac{Ze^2 \mu_0}{4\pi^2 \hbar \epsilon_0} \frac{1}{a} \left(1 + \frac{r^2}{a^2}\right)^{-1/2}$$



14. A pendulum is created by hanging bob of mass m from a rigid support (taken as zero level of potential) symmetrically using two massless inextensible strings, each of length l , making an equilateral triangle as shown in the figure below.



A correct Lagrangian for the angular oscillation of the bob in the plane perpendicular to the paper is

$$(a) L = \frac{3}{8} ml^2 \dot{\theta}^2 - \sqrt{3} mgl \sin^2 \frac{\theta}{2} \quad (b) L = \sqrt{3} ml^2 \dot{\theta}^2 + 4mgl \sin^2 \frac{\theta}{2}$$

$$(c) L = \frac{3}{4} m\dot{\theta}^2 - \sqrt{3} \frac{mg}{l} \sin^2 \frac{\theta}{2} \quad (d) L = \frac{3}{8} ml^2 \dot{\theta}^2 + mgl \dot{\theta} \sec^2 \theta + \sqrt{3} mgl \sin^2 \frac{\theta}{2}$$

15. The wavelength of a non-relativistic particle of mass m in a one-dimensional potential $V(x)$ has the form

$$\psi(x) = \sqrt{a} e^{-a|x|}$$

where $|x|$ denotes the absolute value of the coordinate x . The potential is $V(x) =$

$$(a) -\frac{\hbar^2}{m} \delta\left(\frac{x}{a}\right) \quad (b) \frac{\hbar^2 a}{m} \delta(x) \quad (c) 0 \quad (d) -\frac{\hbar^2 a}{m} \left\{a + \frac{1}{2} \delta(x)\right\}$$

SECTION - C

1. On a compact stellar object the gravity is so strong that a body falling from rest will soon acquire a velocity comparable with that of light. If the force on this body is $F = mg$ where m is the relativistic mass and g is a constant, the velocity of this falling body will vary with time as

$$(a) v = c \left\{1 - \exp\left(-\frac{gt}{c}\right)\right\}$$

$$(b) v = \frac{c}{1 - \frac{2c}{gt}}$$

$$(c) v = c \tanh \frac{gt}{c}$$

$$(d) v = \frac{2c}{\pi} \tan \frac{gt}{c}$$

2. An ideal monatomic gas at chemical potential $\mu = -1$ eV and a temperature given by $k_B T = 0.1$ eV is in equilibrium with an adsorbing metal surface, i.e., there are isolated sites distributed randomly on the metal surface where the gas atom can get bound. Each such binding site can adsorb 0, 1 or 2 atoms with the released energy being 0, -1 eV and -1.9 eV respectively. The average number of adsorbed molecules at each site would be

$$(a) \frac{2+e}{1+2e}$$

$$(b) \frac{1+2e}{1+e}$$

$$(c) \frac{1+e}{1+2e}$$

$$(d) \frac{1+e}{1-e}$$

3. Consider the complex function $f(x, y) = u(x, y) + iv(x, y)$, where

$$u(x, y) = x^2(2+x) - y^2(2+3x)$$

$$v(x, y) = y(\lambda x + 3x^2 - y^2)$$

and λ is real. If it is known that $f(x, y)$ is analytic in the complex plane of $z = x + iy$, then it can be written

- (a) $f = 2z\bar{z} + z^2 - \bar{z}^2$ (b) $f = z^2(2+z)$
 (c) $f = \bar{z}(2 + \bar{z}^2)$ (d) $f = z^2 + z^3$

4. An electron in a hydrogen atom is in a state described by the wavefunction:

$$\psi(\vec{r}) = \frac{1}{\sqrt{10}} \psi_{100}(\vec{x}) + \sqrt{\frac{2}{5}} \psi_{210}(\vec{x}) + \sqrt{\frac{2}{5}} \psi_{211}(\vec{x}) - \frac{1}{\sqrt{10}} \psi_{21,-1}(\vec{x})$$

where $\psi_{nlm}(\vec{x})$ denotes a normalized wavefunction of the hydrogen atom with the principal quantum number n , angular quantum number l and magnetic quantum number m .

Neglecting the spin-orbit interaction, the expectation values of \hat{L}_z and \hat{L}^2 for this state are

- (a) $\frac{3\hbar}{4}, \frac{9\hbar^2}{25}$ (b) $\frac{3\hbar}{10}, \frac{9\hbar^2}{5}$ (c) $\frac{3\hbar}{5}, \frac{9\hbar^2}{10}$ (d) $\frac{8\hbar}{10}, \frac{3\hbar^2}{5}$

5. A system of two spin- $\frac{1}{2}$ particles 1 and 2 has the Hamiltonian $\hat{H} = \varepsilon_0 \hat{h}_1 \otimes \hat{h}_2$, where

$$\hat{h}_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \hat{h}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and ε_0 is a constant with the dimension of energy. The ground state of this system has energy

- (a) $\sqrt{2} \varepsilon_0$ (b) $-4 \varepsilon_0$ (c) 0 (d) $-2 \varepsilon_0$

6. The table below gives the properties of four unstable particles $\mu^+, \pi^+, n^0, \Lambda^0$

Mass

| Particle | (MeV / c^2) | Spin | Principal decay mode |
|----------------------------|----------------|------|---|
| muon μ^+ | 105.66 | 1/2 | $\mu^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_e$ |
| pion π^+ | 139.57 | 0 | $\pi^+ \rightarrow \mu^+ + \nu_\mu$ |
| neutron n^0 | 939.56 | 1/2 | $n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$ |
| Lambda hyperon Λ^0 | 1,115.68 | 1/2 | $\Lambda^0 \rightarrow p^+ + \pi^-$ |

If arranged in order of DECREASING decay lifetime, the above list will read

- (a) $n^0, \mu^+, \pi^+, \Lambda^0$ (b) $\pi^+, n^0, \mu^+, \Lambda^0$ (c) $\mu^+, \Lambda^0, n^0, \pi^+$ (d) $n^0, \Lambda^0, \mu^+, \pi^+$

7. At time $t=0$, the wavefunction of a particle in a harmonic oscillator potential of natural frequency ω is given by

$$\psi(0) = \frac{1}{5} [3\varphi_0 - 2\sqrt{2}\varphi_1 + 2\sqrt{2}\varphi_2],$$

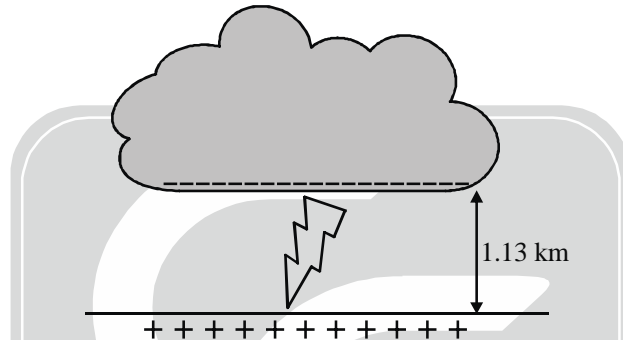
where $\varphi_n(x)$ denotes the eigenfunction belonging to the n^{th} eigenvalue of energy. At time $t = \tau$, the wavefunction is found to be

$$\psi(\tau) = -\frac{i}{5} [3\varphi_0 + 2\sqrt{2}\varphi_1 + 2\sqrt{2}\varphi_2]$$

The minimum value of τ is

- (a) $2\pi/3\omega$ (b) $\pi/2\omega$ (c) π/ω (d) $2\pi/\omega$

8. The integral $I = \int_0^{\infty} dx e^{-x} \delta(\sin x)$, where $\delta(x)$ denotes the Dirac delta function is
- (a) $\frac{\exp \pi}{\exp \pi + 1}$ (b) 1 (c) $\frac{1}{\exp \pi - 1}$ (d) $\frac{\exp \pi}{\exp \pi - 1}$
9. The magnetic vector corresponding to a uniform magnetic field \vec{B} is often taken as $\vec{A} = \frac{1}{2} \vec{B} \times \vec{x}$. This choice is
- (a) valid in the Coulomb gauge. (b) gauge invariant.
 (c) valid in the Lorentz gauge. (d) valid in the Weyl gauge.
10. A monsoon cloud has a flat bottom of surface area 125 km^2 . It floats horizontally over the ground at a level such that base of the cloud is 1.13 km above the ground (see figure). Due to friction with the air below, the base of the cloud acquires a uniform electric charge density. This keeps increasing slowly with time.



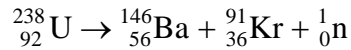
When the uniform electric field below the cloud reaches the value 2.4 MV m^{-1} a lightning discharge occurs, and the entire charge of the cloud passes to the Earth below – which, in this case, behaves like a grounded conductor.

Neglecting edge effects and inhomogeneities inside the cloud and the air below, the energy released in this lightning discharge can be estimated, in kilowatt-hours (kWh), as about

- (a) 10^5 (b) 10^{-1} (c) 10^9 (d) 10
11. At low temperature, the measured specific heat C_V of a sample is found to depend on temperature as $C_V = aT^{3/2} + bT^3$, where a and b are constants. This material has
- (a) one fermionic excitation with dispersion relation $\omega \propto k^4$;
 another bosonic excitation with dispersion relation $\omega \propto k$
- (b) one fermionic excitation with dispersion relation $\omega \propto k^2$;
 another fermionic excitation with dispersion relation $\omega \propto k$
- (c) one fermionic excitation with dispersion relation $\omega \propto k^2$;
 another bosonic excitation with dispersion relation $\omega \propto k^4$
- (d) one bosonic excitation with dispersion relation $\omega \propto k^2$;
 another bosonic excitation with dispersion relation $\omega \propto k$
12. The semi-empirical mass formula for a heavy nucleon (Z, A) can be written, to some approximation, as
- $$M(Z, A)c^2 = ZM_p c^2 + (A - Z)M_n c^2 - \lambda_1 A - \lambda_2 A^{2/3} - \lambda_3 \frac{Z(Z-1)}{A^{1/3}} - \lambda_4 \frac{(A-2Z)^2}{A} - \frac{\lambda_5}{A^{1/2}}$$
- where $M_p c^2 = 938 \text{ MeV}$, $M_n c^2 = 939 \text{ MeV}$, and $\lambda_1 = 16$, $\lambda_2 = 18$, $\lambda_3 = 0.7$, $\lambda_4 = 23$, all in MeV, where

$$\lambda_5 = \begin{cases} +12 & \text{MeV for even - even nuclei} \\ -12 & \text{MeV for odd - odd nuclei} \\ 0 & \text{for others} \end{cases}$$

Now, consider a spontaneous fission reaction



The energy released in this reaction will be close to

- (a) 190 MeV (b) 170 MeV (c) 17.92 keV (d) 19.2 MeV

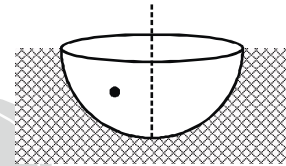
13. A bead of mass m slides under the influence of gravity along the frictionless interior of a hemispherical cup of radius a sunk vertically into the ground with its open side level with the ground (see sketch on the right). In terms of spherical polar coordinates (θ, φ) set up with the centre of the upper circle as origin, the Hamiltonian H for this system will be

(a) $H = \frac{m}{2}(a^2\dot{\theta}^2 + a^2\dot{\varphi}^2 \sin^2 \theta) + 2mga \sin^2 \frac{\theta}{2}$

(b) $H = \frac{1}{2ma^2}(p_\theta^2 + p_\varphi^2 \csc^2 \theta) + 2mga \sin^2 \frac{\theta}{2}$

(c) $H = \frac{m}{2}(a^2\dot{\theta}^2 + a^2\dot{\varphi}^2 \csc^2 \theta) + 2mga \sin^2 \frac{\theta}{2}$

(d) $H = \frac{1}{2ma^2}(p_\theta^2 + p_\varphi^2 \sin^2 \theta) + 2mga \sin^2 \frac{\theta}{2}$

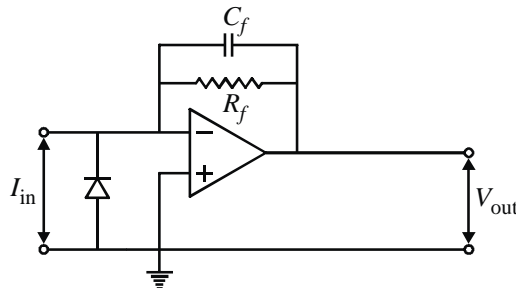


14. Consider N non-interacting distinguishable particle in equilibrium at an absolute temperature T . Each particle can only occupy one of two possible states of energy 0 and ε respectively ($\varepsilon > 0$). The entropy of the system, in terms of $\beta = \frac{\varepsilon}{k_B T}$ is

(a) $Nk_B \left\{ \ln(1 + e^{-\beta}) - \frac{e^{-\beta}}{1 + e^{-\beta}} \right\}$ (b) $Nk_B \left\{ \ln(1 + e^{-\beta}) - \frac{e^{-\beta}}{1 - e^{-\beta}} \right\}$

(c) $Nk_B \left\{ \ln(1 - e^{-\beta}) + \frac{\beta e^{-\beta}}{1 - e^{-\beta}} \right\}$ (d) $Nk_B \left\{ \ln(1 + e^{-\beta}) + \frac{\beta e^{-\beta}}{1 + e^{-\beta}} \right\}$

15. Consider the following circuit.



It is given that $C_f = 100$ pF, and for $I_{in} = 50$ nA D.C. $V_{out} = 1$ V D.C. Therefore, the bandwidth of the above circuit is

- (a) 145.3 Hz (b) 200.4 Hz (c) 15.8 Hz (d) 79.6 Hz