## GATE PHYSICS-PH 2019

### SECTION : GENERAL APTITUDE

1. The fishermen, _______ the flood victims owed lives, were rewarded by the government.
(a) whom (b) to which (c) to whom (d) that

2. Until Iran came along, India had never been _______ in kabaddi.
(a) defeated (b) defeating (c) defeat (d) defeatist

3. The radius as well as the height of a circular cone increase by 10%. The percentage increase in its volume is _______.
(a) 17.1 (b) 21.0 (c) 33.1 (d) 72.8

4. Some students were not involved in the strike.
If the above statement is true, which of the following conclusions is/are logically necessary ?
(1) Some who were involved in the strike were students.
(2) No student was involved in the strike.
(3) At least one student was involved in the strike.
(4) Some who were not involved in the strike were students.
(a) (1) and (2) (b) (3) only (c) (4) only (d) (2) and (3)

5. Five numbers 10, 7, 5, 4 and 2 are to be arranged in a sequence from left to right following the directions given below:
(1) No two odd or even numbers are next to each other.
(2) The second number from the left is exactly half of the left-most number.
(3) The middle number is exactly twice the right-most number.
Which is the second number from the right ?
(a) 2 (b) 4 (c) 7 (d) 10

6. Two trains started at 7 A.M. from the same point. The first train travelled north at a speed of 80 km/hr and the second train travelled south at a speed of 100 km/hr. The time at which they were 540 km apart is _______A.M.
(a) 9 (b) 10 (c) 11 (d) 11.30

7. The nomenclature of Hindustani music has changed over the centuries. Since the medieval period *dhrupad* styles were identified as *baanis*. Terms like *gayaki* and *baaj* were used to refer to vocal and instrumental styles, respectively. With the institutionalization of music education the term *gharana* became acceptable. *Gharana* originally referred to hereditary musicians from a particular lineage, including disciples and grand disciples.
Which one of the following is NOT correct ?
(a) *dhrupad*, *baani* (b) *gayaki*, vocal (c) *baaj*, institution (d) *gharana*, lineage

8. Since the last one year, after a 125 basis point reduction in repo rate by the Reserve Bank of India, banking institutions have been making a demand to reduce interest rates on small saving schemes. Finally, the government announced yesterday a reduction in interest rates on small saving schemes to bring them on par with fixed deposit interest rates.
Which one of the following statements can be inferred from the given passage ?
(a) Whenever the Reserve Bank of India reduces the repo, the interest rates on small saving schemes are also reduced.

(b) Interest rates on small saving schemes are always maintained on par with fixed deposit interest rates.
(c) The government sometimes takes into consideration the demands of banking institutions before reducing the interest rates on small saving schemes.
(d) A reduction in interest rates on small saving schemes follow only after a reduction in repo rate by the Reserve Bank of India.

9. In a country of 1400 million population, 70% own mobile phones. Among the mobile phone owners, only 294 million access the Internet. Among these Internet users, only half buy goods from e-commerce portals. What is the percentage of these buyers in the country ?
(a) 10.50 (b) 14.70 (c) 15.00 (d) 50.00

10. "I read somewhere that in ancient times the prestige of a kingdom depended upon the number of taxes that it was able to levy on its people. It was very much like the prestige of a head-hunter in his own community".
Based on the paragraph above, the prestige of a head-hunter depended upon ________.
(a) the prestige of the kingdom. (b) the prestige of the heads.
(c) the number of taxes he could levy. (d) the number of heads he could gather.

## SECTION : PHYSICS

1. During a rotation, vectors along the axis of rotation remain unchanged.

For the rotation matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$, the unit vector along the axis of rotation is

(a) $\frac{1}{3}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ (b) $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} - \hat{k}\right)$ (c) $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} - \hat{k}\right)$ (d) $\frac{1}{3}\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

2. For the differential equation $\frac{d^2y}{dx^2} - n(n+1)\frac{y}{x^2} = 0$, where $n$ is a constant, the product of its two independent solution is

(a) $\frac{1}{x}$ (b) $x$ (c) $x^n$ (d) $\frac{1}{x^{n+1}}$

3. At temperature $T$ Kelvin (K), the value of the Fermi function at an energy 0.5 eV above the Fermi energy is 0.01. Then $T$, to the nearest integer, is ________.
$(k_B = 8.62 \times 10^{-5}$ eV/K)

4. Consider a three-dimensional crystal of $N$ inert gas atoms. The total energy is given by

$$U(R) = 2N\varepsilon\left[p\left(\frac{\sigma}{R}\right)^{12} - q\left(\frac{\sigma}{R}\right)^{6}\right],$$

where $p = 12.13$, $q = 14.45$, and $R$ is the nearest neighbour distance between two atoms. The two constants, $\varepsilon$ and $R$, have the dimensions of energy and length, respectively. The equilibrium separation between two nearest neighbour atoms in units of $\sigma$ (rounded off to two decimal places) is ________.

5. The Hamiltonian for a quantum harmonic oscillator of mass $m$ in three dimensions is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2$$

where $\omega$ is the angular frequency. The expectation value of $r^2$ in the first excited state of the oscillator in units of $\hbar/m\omega$ (rounded off to one decimal place) is ________.

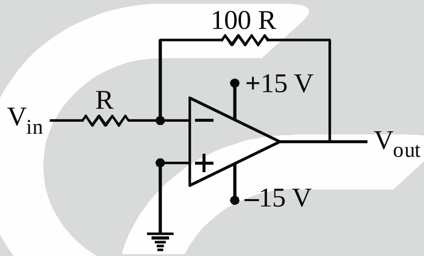
6. A massive particle X in free space decays spontaneously into two photons. Which of the following statements is true for X ?
(a) X is charged
(b) Spin of X must be greater than or equal to 2
(c) X is a boson
(d) X must be a baryon

7. The electric field of an electromagnetic wave in vacuum is given by $\vec{E} = E_0 \cos(3y + 4z - 1.5\times 10^9 t)\,\hat{x}$.
The wave is reflected from the $z = 0$ surface. If the pressure exerted on the surface is $\alpha\,\varepsilon_0 E_0^2$, the value of $\alpha$ (rounded off to one decimal place) is ________.

8. The pole of the function $f(z) = \cot z$ at $z = 0$ is
(a) a removable singularity
(b) an essential singularity
(c) a simple pole
(d) a second order pole

9. The relative magnetic permeability of a type-I superconductor is
(a) 0
(b) –1
(c) $2\pi$
(d) $1/4\pi$

10. For the following circuit, what is the magnitude of $V_{out}$ if $V_{in} = 1.5$ V ?




(a) 0.015 V
(b) 0.15 V
(c) 15 V
(d) 150 V

11. A large number *N* of ideal bosons, each of mass *m*, are trapped in a three-dimensional potential

$$V(r) = \frac{m\omega^2 r^2}{2}$$

The bosonic system is kept at temperature *T* which is much lower than the Bose-Einstein condensation temperature $T_c$. The chemical potential $(\mu)$ satisfies
(a) $\mu \le \frac{3}{2}\hbar\omega$
(b) $2\hbar\omega > \mu > \frac{3}{2}\hbar\omega$
(c) $3\hbar\omega > \mu > 2\hbar\omega$
(d) $\mu = 3\hbar\omega$

12. Considering baryon number and lepton number conservation laws, which of the following processes is/are allowed ?
(i) $p \to \pi^0 + e^+ + \nu_e$
(ii) $e^+ + \nu_e \to \mu^+ + \nu_\mu$
(a) Both (i) and (ii)
(b) Only (i)
(c) Only (ii)
(d) Neither (i) nor (ii)

13. An electric field $\vec{E} = E_0\hat{z}$ is applied to a hydrogen atom in $n = 2$ excited state. Ignoring spin, the $n = 2$ state is fourfold degenerate, which in the $|\ell, m\rangle$ basis are given by $|0,0\rangle, |1,1\rangle, |1,0\rangle$ and $|1,-1\rangle$. The $H'$ is the interaction Hamiltonain corresponding to the applied electric field, which of the following matrix elements is non-zero?
(a) $\langle 0,0|H'|0,0\rangle$
(b) $\langle 0,0|H'|1,1\rangle$
(c) $\langle 0,0|H'|1,0\rangle$
(d) $\langle 0,0|H'|1,-1\rangle$

14. An infinitely long thin cylindrical shell has its axis coinciding with the *z*-axis. It carries a surface charge density $\sigma_0 \cos\phi$, where $\phi$ is the polar angle and $\sigma_0$ is a constant. The magnitude of the electric field inside the cylinder is
(a) 0
(b) $\frac{\sigma_0}{2\varepsilon_0}$
(c) $\frac{\sigma_0}{3\varepsilon_0}$
(d) $\frac{\sigma_0}{4\varepsilon_0}$

15. The energy-wavevector ($E$-$k$) dispersion relation for a particle in two dimensions is $E = Ck$, where $C$ is a constant. If its density of states $D(E)$ is proportional to $E^p$ then the value of $p$ is __________.

16. Consider a transformation from one set of generalized coordinate and momentum $(q, p)$ to another set $(Q, P)$ denoted by,

$$Q = pq^s;\ P = q^r$$

where $s$ and $r$ are constants. The transformation is canonical if

(a) $s = 0$ and $r = 1$ (b) $s = 2$ and $r = -1$ (c) $s = 0$ and $r = -1$ (d) $s = 2$ and $r = 1$

17. A conventional type-I superconductor has a critical temperature of 4.7 K at zero magnetic field and a critical field in Tesla at 2 K (rounded off to three decimal place) _________.

18. The Hamiltonian for a particle of mass $m$ is $H = \frac{p^2}{2m} + kqt$, where $q$ and $p$ are the generalized coordinate and momentum, respectively, $t$ is time and $k$ is a constant. For the initial condition, $q = 0$ and $p = 0$ at $t = 0$, $q(t) \propto t^\alpha$. The value of $\alpha$ is __________.

19. A circular loop made of a thin wire has radius 2 cm and resistance 2 Ω. It is placed perpendicular to a uniform magnetic field of magnitude $|\vec{B}_0| = 0.01$ Tesla. At time $t = 0$ the field starts decaying as $\vec{B} = \vec{B}_0 e^{-t/t_0}$, where $t_0 = 1\,\text{s}$. The total charge that passes through a cross section of the wire during the decay is $Q$. The value of $Q$ in μC (rounded off to two decimal places) is _________.

20. For a spin-$\frac{1}{2}$ particle, let $|\uparrow\rangle$ and $|\downarrow\rangle$ denote its spin up and spin down states, respectively. If $|a\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle\right)$ and $|b\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$ are composite states of two such particles, which of the following statements is true for their total spin S?

(a) $S = 1$ for $|a\rangle$ and $|b\rangle$ is not an eigenstate of the operator $\hat{S}^2$

(b) $|a\rangle$ is not an eigenstate of the operator, $\hat{S}^2$ and $S = 0$ for $|b\rangle$

(c) $S = 0$ for $|a\rangle$ and $S = 1$ for $|b\rangle$

(d) $S = 1$ for $|a\rangle$, and $S = 0$ for $|b\rangle$

21. The nuclear spin and parity of $^{40}_{20}\text{Ca}$ in its ground state is

(a) $0^+$ (b) $0^-$ (c) $1^+$ (d) $1^-$

22. In order to estimate the specific heat of phonons, the appropriate method to apply would be
(a) Einstein model for acoustic phonons and Debye model for optical phonons.
(b) Einstein model for optical phonons and Debye model for acoustic phonons.
(c) Einstein model for both optical and acoustic phonons.
(d) Debye model for both and acoustic phonons.

23. The electric field of an electromagnetic wave is given by $\vec{E} = 3\sin(kz - \omega t)\hat{x} + 4\cos(kz - \omega t)\hat{y}$. The wave is
(a) linearly polarized at an angle $\tan^{-1}(4/3)$ from the $x$-axis.
(b) linearly polarized at an angle $\tan^{-1}(3/4)$ from the $x$-axis.
(c) elliptically polarized in clockwise direction when seen travelling towards the observer.
(d) elliptically polarized in counter-clockwise direction when seen travelling towards the observer.

24. Let $|\psi_1\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |\psi_2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ represent two possible states of a two-level quantum system. The state obtained by the incoherent superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$ is given by a density matrix that is defined as $\rho \equiv c_1|\psi_2\rangle\langle\psi_1| + c_2|\psi_2\rangle\langle\psi_2|$. If $c_1 = 0.4$ and $c_2 = 0.6$, the matrix element $\rho_{22}$ (rounded off to one decimal place) is _______________

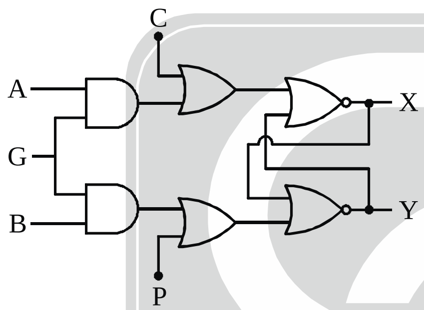
25. Consider a one-dimensional gas of $N$ non-interacting particles of mass $m$ with the Hamiltonian of a single particle given by,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2(x^2 + 2x)$$

The high temperature specific heat in units of $R = Nk_B$ ($k_B$ is the Boltzmann constant) is

(a) 1 (b) 1.5 (c) 2 (d) 2.5

26. For the following circuit, the correct logic values for the entries $X_2$ and $Y_2$ in the truth table are:




| G | A | B | P | C | X | Y |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | $X_2$ | $Y_2$ |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 |

(a) 1 and 0 (b) 0 and 0 (c) 0 and 1 (d) 1 and 1

27. Low energy collision ($s$-wave scattering) of pion $(\pi^+)$ with deuteron ($d$) results in the production of two protons $(\pi^+ + d \to p + p)$. The relative orbital angular momentum (in units of $\hbar$) of the resulting two-proton system for this reaction is

(a) 0 (b) 1 (c) 2 (d) 3



28. In a set of $N$ successive polarizers, the $m^{th}$ polarizer makes an angle $(m\pi/2N)$ with the vertical. A vertically polarized light beam of intensity $I_0$ is incident on two such sets with $N = N_1$ and $N = N_2$, where $N_2 > N_1$. Let the intensity of light beams coming out be $I(N_1)$ and $I(N_2)$, respectively. Which of the following statements is correct about the two outgoing beams ?

(a) $I(N_2) > I(N_1)$; the polarization in each case is vertical.

(b) $I(N_2) < I(N_1)$; the polarization in each case is vertical.

(c) $I(N_2) > I(N_1)$; the polarization in each case is horizontal.

(d) $I(N_2) < I(N_1)$; the polarization in each case is horizontal.

29. The ground state electronic configuration of the rare-earth ion $(Nd^{3+})$ is $[Pd]\,4f^3\,5s^2\,5p^6$. Assuming L-S coupling, the Lande $g$-factor of this ion is 8/11. The effective magnetic moment in units of Bohr magneton $\mu_B$ (rounded off to two decimal places) is ________.

30. The spin-orbit interaction term of an electron moving in a central field is written as $f(r)\vec{l}\cdot\vec{s}$, where $r$ is the radial distance of the electron from the origin. If an electron moves inside a uniformly charged sphere, then

(a) $f(r) = \text{constant}$ (b) $f(r) \propto r^{-1}$ (c) $f(r) \propto r^{-2}$ (d) $f(r) \propto r^{-3}$

31. Consider the motion of a particle along the $x$-axis in a potential $V(x) = F|x|$. Its ground state energy $E_0$ is estimated using the uncertainty principle. Then $E_0$ is proportional to

(a) $F^{1/3}$ (b) $F^{1/2}$ (c) $F^{2/5}$ (d) $F^{2/3}$

32. A particle of mass $m$ moves in a lattice along the $x$-axis in a periodic potential $V(x) = V(x+d)$ with periodicity $d$. The corresponding Brillouin zone extends from $-k_0$ to $k_0$ with these two $k$-points being equivalent. If a weak force $F$ in the $x$-direction is applied to the particle, it starts a periodic motion with time period $T$. Using the equation of motion $F = \frac{dp_{\text{crystal}}}{dt}$ for a particle moving in a band, where $p_{\text{crystal}}$ is the crystal momentum of the particle, the period $T$ is found to be ($h$ is Planck constant)

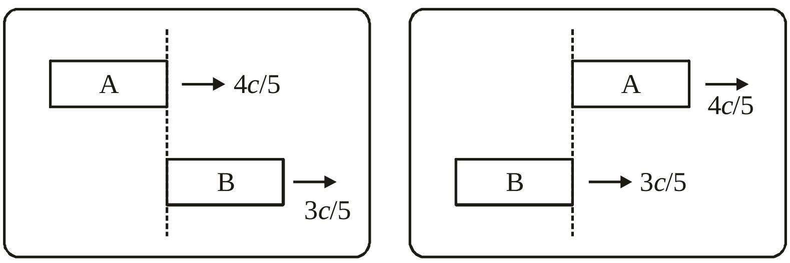
(a) $\sqrt{\frac{2md}{F}}$ (b) $2\sqrt{\frac{2md}{F}}$ (c) $\frac{2h}{Fd}$ (d) $\frac{h}{Fd}$

33. A radioactive element X has a half-life of 30 hours. It decays via alpha, beta and gamma emissions with the branching ratio for beta decay being 0.75. The particle half-life for beta decay in unit of hours is ________.

34. In a thermally insulated container, 0.01 kg of ice at 273 K is mixed with 0.1 kg of water at 300 K. Neglecting the specific heat of the container, the change in the entropy of the system in J/K on attaining thermal equilibrium (rounded off to two decimal places) is __________.
(Specific heat of water is 4.2 kJ/kg-K and the latent heat of ice is 335 kJ/kg).

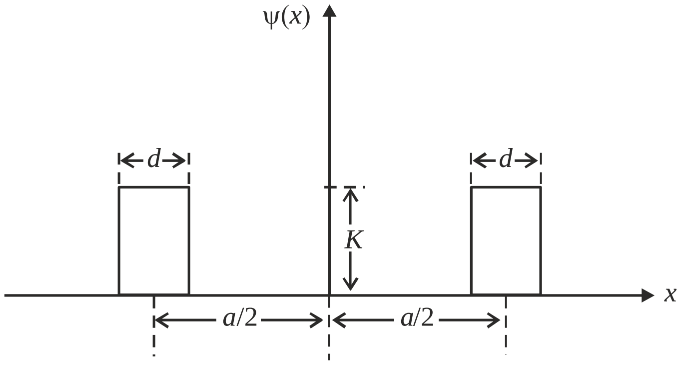
35. The Hamiltonian of a system is $H = \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & -1 \end{pmatrix}$ with $\varepsilon << 1$. The fourth order contribution to the ground state energy of $H$ is $\gamma\varepsilon^4$. The value of $\gamma$ (rounded off to three decimal places) is __________.

36. Two spaceships A and B, each of the same rest length $L$, are moving in the same direction with speeds $\frac{4c}{5}$ and $\frac{3c}{5}$, respectively, where $c$ is the speed of light. As measured by B, the time taken by A to completely overtake B [see figure below] in units of $L/c$ (to the nearest integer) is __________.



(i) (ii)

37. The wave function $\psi(x)$ of a particle is as shown below :




Here, $K$ is a constant, and $a > d$. The position uncertainty $(\Delta x)$ of the particle is

(a) $\sqrt{\frac{a^2+3d^2}{12}}$ (b) $\sqrt{\frac{3a^2+d^2}{12}}$ (c) $\sqrt{\frac{d^2}{6}}$ (d) $\sqrt{\frac{d^2}{24}}$

38. A 3-bit analog-to-digital converter is designed to digitize analog signals ranging from 0 V to 10 V. For this converter, the binary output corresponding to an input of 6 V is

(a) 011 (b) 101 (c) 100 (d) 010

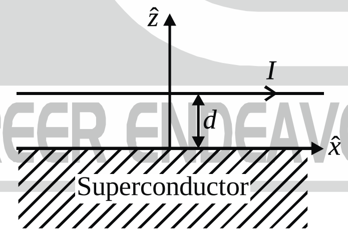
39. Consider the following Boolean expression:

$$(\bar{A}+\bar{B})\left[\overline{A(B+C)}\right]+A(\bar{B}+\bar{C})$$

It can be represented by a single three-input logic gate. Identify the gate

(a) AND (b) OR (c) XOR (d) NAND

40. An infinitely long wire parallel to the $x$-axis is kept at $z = d$ and carries a current $I$ in the positive $x$-direction above a superconductor filling the region $z \le 0$ (see figure). The magnetic field $\vec{B}$ inside the superconductor is zero so that the field just outside the superconductor is parallel to its surface. The magnetic field due to this configuration at a point $(x, y, z > 0)$ is




(a) $\left(\frac{\mu_0 I}{2\pi}\right)\frac{-(z-d)\hat{j}+y\hat{k}}{\left[y^2+(z-d)^2\right]}$ (b) $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{-(z-d)\hat{j}+y\hat{k}}{y^2+(z-d)^2}+\frac{(z+d)\hat{j}-y\hat{k}}{y^2+(z+d)^2}\right]$

(c) $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{-(z-d)\hat{j}+y\hat{k}}{y^2+(z-d)^2}-\frac{(z+d)\hat{j}-y\hat{k}}{y^2+(z+d)^2}\right]$ (d) $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{y\hat{j}+(z-d)\hat{k}}{y^2+(z-d)^2}+\frac{y\hat{j}-(z+d)\hat{k}}{y^2+(z+d)^2}\right]$

41. In a certain two-dimensional lattice, the energy dispersion of the electron is

$$\varepsilon(\vec{k})=-2t\left[\cos k_x a+2\cos\frac{1}{2}k_x a\cos\frac{\sqrt{3}}{2}k_y a\right],$$

where $\vec{k}=(k_x, k_y)$ denotes the wave vector, $a$ is the lattice constant and $t$ is a constant in units of eV. In this lattice the effective mass energy tensor $m_{ij}$ of electrons calculated at the centre of the Brillouin zone has the form $m_{ij}=\frac{\hbar^2}{ta^2}\begin{pmatrix}\alpha & 0\\ 0 & \alpha\end{pmatrix}$. The value of $\alpha$ (rounded off to three decimal places) is ________.

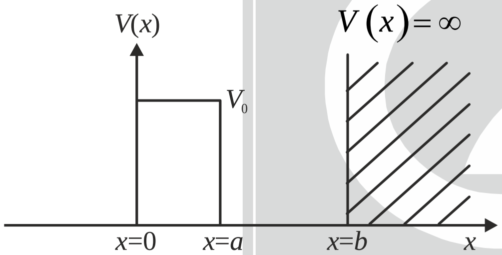
42. Two events, one of the earth and the other one on the Sun, occur simultaneously in the earth's frame. The time difference between the two events as seen by an observer in a spaceship moving with velocity $0.5c$ in the earth's frame along the line joining the earth to the Sun is $\Delta t$, where $c$ is the speed of light. Given that light travels from the Sun to the earth in 8.3 minutes in the earth's frame, the value of $|\Delta t|$ in minutes (rounded off to two decimal place) is _________.
(Take the earth's frame to be inertial and neglect the relative motion between the earth and the Sun).

43. The Hamiltonian operator for a two-level quantum system is $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$. If the state of the system at $t = 0$ is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ then $|\langle\psi(0)|\psi(t)\rangle|^2$ at a later time $t$ is

(a) $\frac{1}{2}\left(1+e^{-(E_1-E_2)t/\hbar}\right)$ (b) $\frac{1}{2}\left(1-e^{-(E_1-E_2)t/\hbar}\right)$

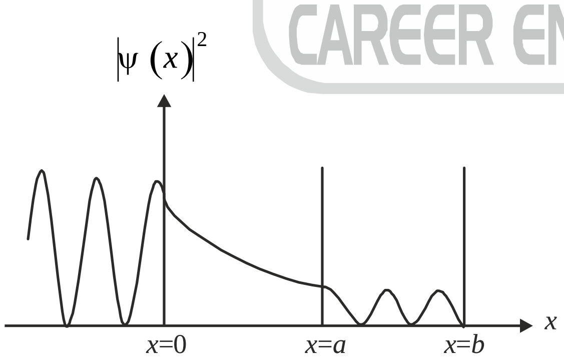
(c) $\frac{1}{2}\left(1+\cos\left[(E_1-E_2)t/\hbar\right]\right)$ (d) $\frac{1}{2}\left(1-\cos\left[(E_1-E_2)t/\hbar\right]\right)$

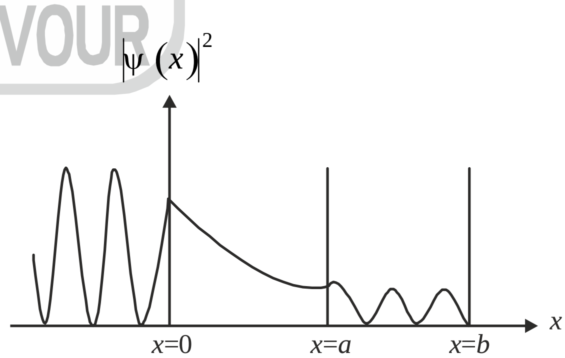
44. Consider a potential barrier $V(x)$ of the form:




where $V_0$ is a constant. For particles of energy $E < V_0$ incident on this barrier from the left which of the following schematic diagrams best represents the probability density $|\psi(x)|^2$ as a function of $x$.



(a)



(b)
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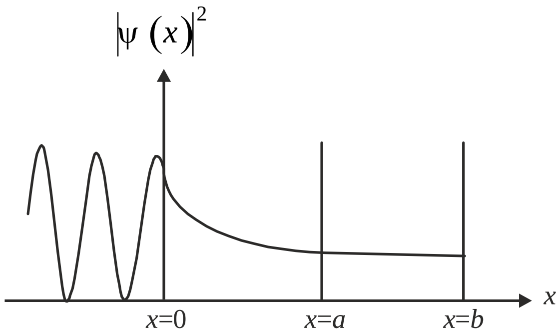


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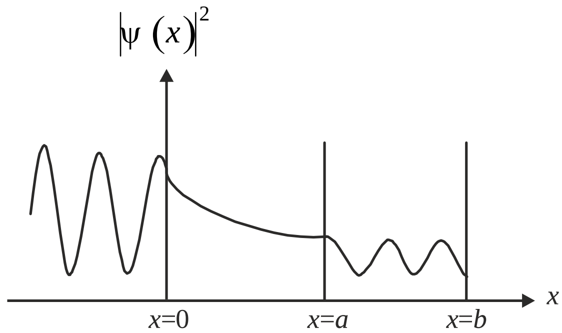


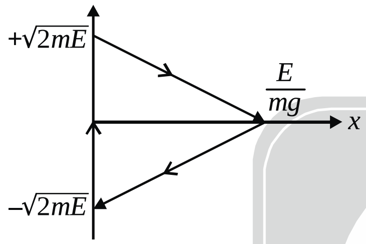
45. A projectile of mass 1 kg is launched at an angle of 30° from the horizontal direction at $t = 0$ and takes time $T$ before hitting the ground. If its initial speed is 10 $ms^{-1}$, the value of the action integral for the entire flight in the units of kg $m^2$ $s^{-1}$ (rounded off to one decimal place) is __________.

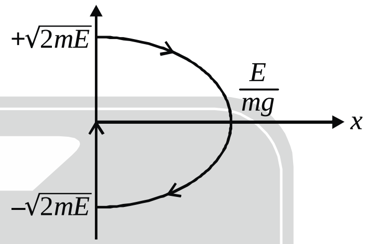
46. Consider two systems A and B each having two distinguishable particles. In both the systems, each particle can exist in states with energies 0, 1, 2 and 3 units with equal probability. The total energy of the combined system is 5 units. Assuming that the system A has energy 3 units and the system B has energy 2 units, the entropy of the combined system is $k_B \ln \lambda$. The value of $\lambda$ is __________.

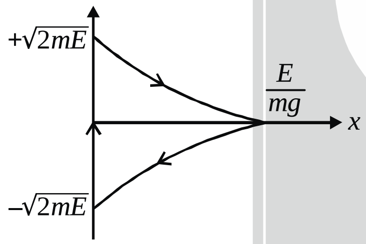
47. A ball bouncing off a rigid floor is described by the potential energy function:

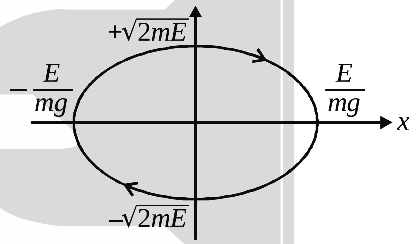
$$V(x) = \begin{cases} mgx & \text{for} \quad x > 0 \\ \infty & \text{for} \quad x \le 0 \end{cases}$$

Which of the following schematic diagram best represents the phase space plot of the ball ?

(a)



(b)


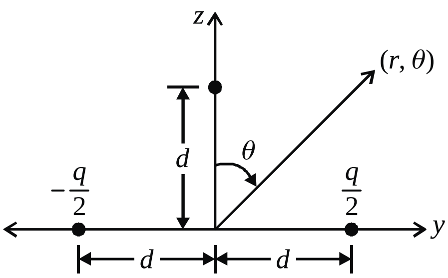

(c)



(d)





48. For a given load resistance $R_L = 4.7$ ohm, the power transfer efficiencies $\left(\eta = \dfrac{P_{\text{load}}}{P_{\text{total}}}\right)$ of a dc voltage source and a dc current source with internal resistances $R_1$ and $R_2$, respectively, are equal. The product $R_1 R_2$ in units of $\text{ohm}^2$ (rounded off to one decimal place) is __________.

49. Consider a system of three charges as shown in the figure below:




For $r = 10\,\text{m}$; $\theta = 60°$; $q = 10^{-6}$ Coulomb, and $d = 10^{-3}\,\text{m}$, the electric dipole potential in volts (rounded off to three decimal places) at a point $(r, \theta)$ is ____________.

$$\left[\text{Use}: \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right]$$

50. Consider the Hamiltonian $H(q,\,p)=\dfrac{\alpha p^2 q^4}{2}+\dfrac{\beta}{q^2}$, where $\alpha$ and $\beta$ are parameters with appropriate dimensions, and $q$ and $p$ are the generalized coordinate and momentum, respectively. The corresponding Lagrangian $L(q,\dot{q})$ is

(a) $\dfrac{1}{2\alpha}\dfrac{\dot{q}^2}{q^4}-\dfrac{\beta}{q^2}$ (b) $\dfrac{2}{2\alpha}\dfrac{\dot{q}^2}{q^4}+\dfrac{\beta}{q^2}$ (c) $\dfrac{1}{2\alpha}\dfrac{\dot{q}^2}{q^4}+\dfrac{\beta}{q^2}$ (d) $-\dfrac{1}{2\alpha}\dfrac{\dot{q}^2}{q^4}+\dfrac{\beta}{q^2}$

51. Let $\theta$ be a variable in the range $-\pi \le \theta < \pi$. Now consider a function:

$$\psi(\theta)\begin{cases}1 & \text{for } -\frac{\pi}{2}\le\theta<\frac{\pi}{2}\\ 0 & \text{otherwise}\end{cases}$$

If its Fourier series is written as $\psi(\theta)=\sum_{m=-\infty}^{\infty} C_m e^{-im\theta}$, then the value of $|C_3|^2$ (rounded off to three decimal place) is __________.

52. Electrons with spin in the $z$-direction $(\hat{z})$ are passed through a Stern-Gerlach (SG) set up with the magnetic field at $\theta=60°$ from $\hat{z}$. The fraction of electrons that will emerge with their spin parallel to the magnetic field in the SG set up (rounded off to two decimal places) __________________

$$\left[\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right]$$

53. The value of the integral $\displaystyle\int_{-\infty}^{\infty}\frac{\cos(kx)}{x^2+a^2}dx$, where $k>0$ and $a>0$, is

(a) $\dfrac{\pi}{a}e^{-ka}$ (b) $\dfrac{2\pi}{a}e^{-ka}$ (c) $\dfrac{\pi}{2a}e^{-ka}$ (d) $\dfrac{3\pi}{2a}e^{-ka}$

54. The vector potential inside a long solenoid, with $n$ turns per unit length and carrying current $I$, written in cylindrical coordinates is $\vec{A}(s,\phi,z)=\dfrac{\mu_0 nI}{2}s\hat{\phi}$. If the term $\dfrac{\mu_0 nI}{2}s(\alpha\cos\phi\,\hat{\phi}+\beta\sin\phi\,\hat{s})$, where $\alpha\neq 0$, $\beta\neq 0$, is added to $\vec{A}(s,\phi,z)$, the magnetic field remains the same if

(a) $\alpha=\beta$ (b) $\alpha=-\beta$ (c) $\alpha=2\beta$ (d) $\alpha=\dfrac{\beta}{2}$

$$\left[\begin{array}{c}\text{Useful formulae}: \vec{\nabla}t=\dfrac{\partial t}{\partial s}\hat{s}+\dfrac{1}{s}\dfrac{\partial t}{\partial\phi}\hat{\phi}+\dfrac{\partial t}{\partial z}\hat{z};\\ \vec{\nabla}\times\vec{v}=\left(\dfrac{1}{s}\dfrac{\partial v_z}{\partial\phi}-\dfrac{\partial v_\phi}{\partial z}\right)\hat{s}+\left(\dfrac{\partial v_s}{\partial z}-\dfrac{\partial v_z}{\partial s}\right)\hat{\phi}+\dfrac{1}{s}\left(\dfrac{\partial(sv_\phi)}{\partial s}-\dfrac{\partial v_s}{\partial\phi}\right)\hat{z}\end{array}\right]$$

55. A solid cylinder of radius $R$ has total charge $Q$ distributed uniformly over its volume. It is rotating about its axis with angular speed $\omega$. The magnitude of the total magnetic moment of the cylinder is

(a) $QR^2\omega$ (b) $\dfrac{1}{2}QR^2\omega$ (c) $\dfrac{1}{4}QR^2\omega$ (d) $\dfrac{1}{8}QR^2\omega$