Chapter 6

2-D Transformation

Two Dimensional Transformation

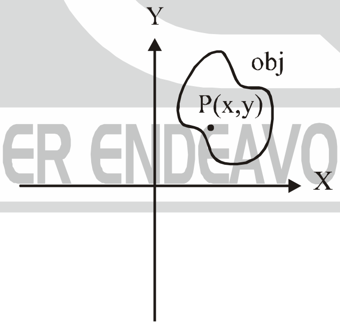
Fundamental to all computer graphics system is the ability to simulate the manipulation of objects in space. This simulated spatial manipulation is referred to as transformation. The need for transformation arises when several objects, each of which is independently defined in its own coordinate system, need to be properly positioned in to a common scene in a master coordinate system. Transformation is also useful in other areas of the image synthesis process.

There are two complementary points of view for describing object transformation:

1. The object itself is transformed relative to a stationary coordinate system or background.
2. The second point of view holds that the object is held stationary while the coordinate system is transformed relative to the object.

GEOMETRIC TRANSFORMATION:

Let us impose a coordinate system on a plane. An object 'obj' in the plane can be considered as a set of points. Every object points P has coordinate (x, y) and so the object is the sum total of all its coordinate point.



If the object is moved to a new position it can be regarded as a new object "obj", all of whose coordiante point P' can be obtained from the original point P by the application of geometric transformation.

Translation:

In translation, an object is displayed a given distance and direction from its original position. If the displacement is given by the vector $V = t_x I + t_y J$, the new object points $P'(x', y')$ can be found by applying the transformation $t_v t_0 \;\; P(x, y)$

$$\boxed{P' = Tv(P)}$$

where $x' = x + t_x$

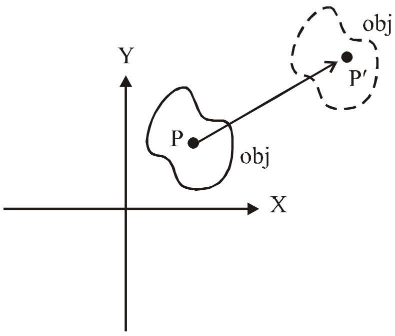
$y' = y + t_y$

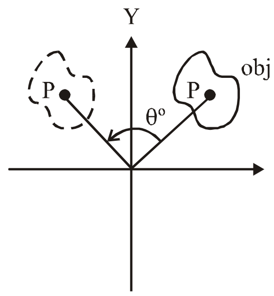
We can express the translation as a single matrix equation by using column vectors to represent coordinate positions and the translation vector.

$$P = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad P' = \begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix}, \qquad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

This allows us to write the two-dimensional translation equations in the matrix form :

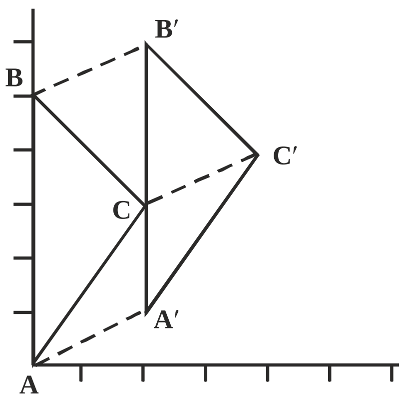
$$P' = P + T$$





Problem: Let us consider a triangle ABC with A(0, 0), B(0, 5) and C(2, 3). We apply the translation on the triangle ABC with $t_x = 2$ and $t_y = 1$. What is the updated coordinate of the triangle ABC.

Soln.
$$x' = x + t_x$$
$$y' = y + t_y$$



Rotation about the origin:

In rotation, the object is rotated θ^0 about the origin. The convention is the direction of rotation is counter clockwise if θ is positive angle, and clockwise if θ is a negative angle. The transformation of rotation R_θ is:

$$\boxed{P' = R_\theta(P)}$$

Where $x' = x\cos(\theta) - y\sin(\theta)$

$$y' = x\sin(\theta) + y\cos(\theta)$$

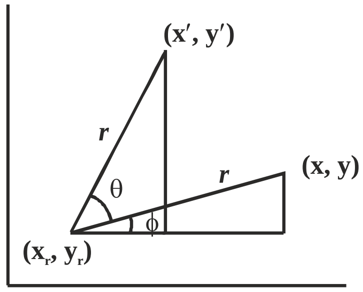
We can write the rotation equations in the matrix form:

$$P' = R \cdot P$$

where the rotation matrix is

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Rotation of a point about an arbitrary pivot position is illustrated in figure shown below. Using the trigonometric relationships in this figure, we can generalize to obtain the transformation equations for rotation of a point about any specified rotation position (x_r, y_r):



$$x' = x_r + (x - x_r)\cos\theta - (y - y_r)\sin\theta$$
$$y' = y_r + (x - x_r)\sin\theta + (y - y_r)\cos\theta$$

Example: Perform a 45º rotation of a triangle A(0, 0), B(1,1), C(5, 2) about the origin.

Soln. We can represent the given triangle, in matrix form, using homogeneous coordinates of the vertices

$$[ABC] = \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}$$

The matrix of rotation is: $R_\theta = R_{45^\circ} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So, the new coordinates $A'B'C'$ of the rotated triangle ABC can be found as

$$[A'B'C'] = [ABC]\cdot R_{45^\circ} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ 3\sqrt{2}/2 & 7\sqrt{2}/2 & 1 \end{bmatrix}$$

Thus, $A' = (0,0),\ B' = (0, \sqrt{2}),\ C' = (3\sqrt{2}/2, 7\sqrt{2}/2)$

Rotation around a fixed point (h, k):

(1) Translate (h, k) to (0, 0) by translating factor $t_x = -h$, $t_y = -k$

(2) Apply the rotation by θ

(3) Translate (0, 0) to (h, k) by translating factor as $t_x = h$, $t_y = k$

Using $v = xI + yJ$ as the translation vector, we have the following sequence of three transformations:

$$R_{\theta,A} = T_{-v}\cdot R_\theta \cdot T_v$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x & -y & 1 \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x & y & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ (1-\cos\theta)\cdot h + k\cdot\sin\theta & (1-\cos\theta)\cdot k - h\cdot\sin\theta & 1 \end{pmatrix} \qquad \text{... (i)}$$

Example: Performa a 45º rotation of a triangle A(0, 0), B(1, 1), C(5, 2) about an arbitrary point P(–1, –1).

Soln. Given triangle ABC, as shown in figure, can be represented in homogeneous coordinates of vertices as

$$[ABC] = \begin{matrix} A \\ B \\ C \end{matrix}\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix}$$

From equation (i), a rotation matrix R_Q, A about a arbitrary point A(x, y) is:

$$R_q,\, A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ (1-\cos\theta)\cdot h + k\cdot\sin\theta & (1-\cos\theta)\cdot k - h\cdot\sin\theta & 1 \end{pmatrix}$$

Thus, $R_{45^\circ},\, A = \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -1 & \left(\sqrt{2}-1\right) & 1 \end{pmatrix}$

So, the new coordinates

$$[A'B'C'] = [ABC]\cdot R_{45^\circ,A} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -1 & \left(\sqrt{2}-1\right) & 1 \end{pmatrix} = \begin{matrix} A' \\ B' \\ C' \end{matrix}\begin{bmatrix} -1 & \left(\sqrt{2}-1\right) & 1 \\ -1 & 2\sqrt{2}-1 & 1 \\ \left(\frac{3}{2}\sqrt{2}-1\right) & \left(\frac{9}{2}\cdot\sqrt{2}-1\right) & 1 \end{bmatrix}$$

Thus, $A' = \left(-1, \sqrt{2}-1\right)$, $B' = \left(-1, 2\sqrt{2}-1\right)$, and $C' = \left(\frac{3}{2}\sqrt{2}-1, \frac{9}{2}\sqrt{2}-1\right)$

The following figure (a) and (b) shows a given triangle, before and after the rotation.

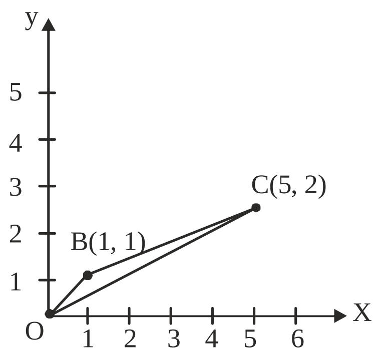


Figure (a)

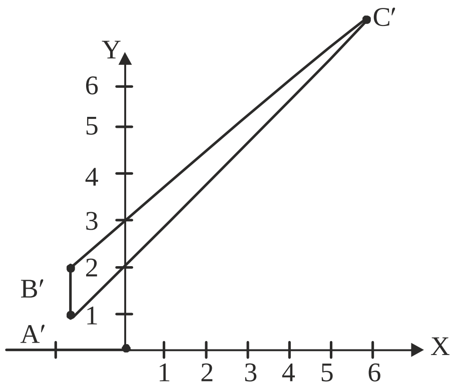


Figure (a)

Scaling with respect to the origin:

Scaling is the process of expanding or compressing the dimension of an object. Positive scaling constant s_x and s_y are used to describe changes in length with respect to x direction and y direction, respectively. A scaling constant greater than one indicates an expansion of length and less than one, compression of length. The scaling transformation s_{sx}, s_y.

$$(x', y') = S_{s_x}, s_y (x, y) \qquad \text{where } x' = x \cdot S_x,\ y' = y \cdot S_y$$

The transformation equations can also be written in the matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

where, S is the 2 by 2 scaling matrix.

When S_x and S_y are assigned the same value, a uniform scaling is produced that maintains relative object proportions.

Note:

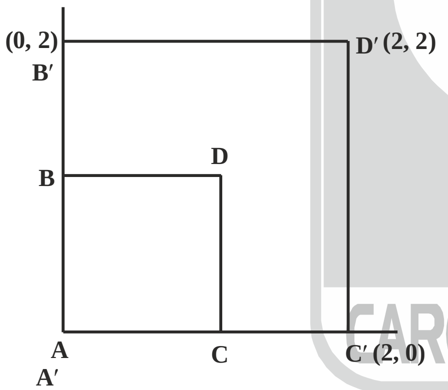
(i) $S_x = S_y > 1$: Enlarge object size

(ii) $S_x = S_y < 1$: Reduce/shrink object size

(iii) $S_x = S_y = 1$: No change

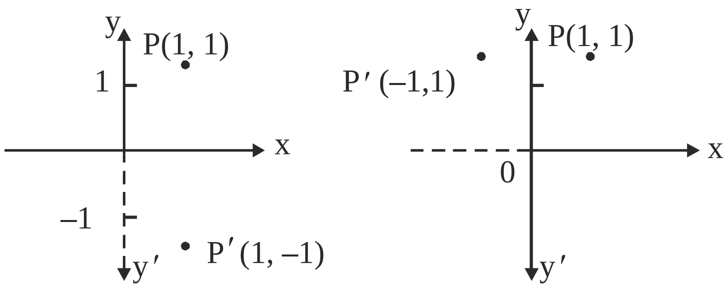
Problem: Let us consider a square ABCD with A(0, 0), B(0, 1) and D(1, 1). It is scaled by $S_x = 2$, $S_y = 2$. What is the new co-ordinate of the square.

Soln.



Mirror Reflection About Axis:

If the new coordinate system is obtained by reflecting the old system about either x or y axis, the relationship between coordinate is given by the coordinate transformation $\bar{M}_x$ and $\bar{M}_y$.



(a) (b)

For reflection about the x axis

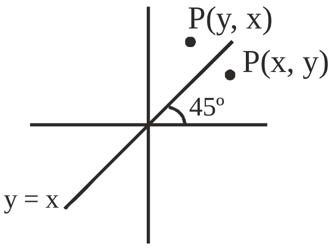
Where, $x' = x \ ; \ y' = -y$

For reflection about the y-axis

$$(x', y') = \bar{M}_y (x, y)$$

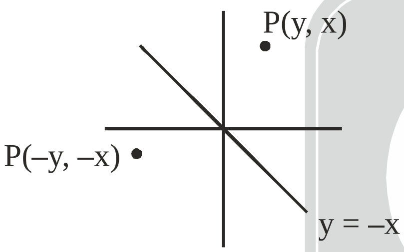
Where $x' = -x$, $y' = y$

Reflection around y = x :



The matrix for the transformation is $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Reflection around y = – x:



The matrix for the transformation is $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Reflection around any arbitrary line, y = mx + C

Translate (0, C) to (0, 0). Here, $t_x = 0$ and $t_y = -C$. After that rotate line by θ^o clockwise. Take the reflection around x-axis. Now, rotating θ^o anticlockwise and finally translating (0, 0) to (0, C) by $t_x = 0$, $t_y = C$.

Inverse coordinate transformtion:

Each coordinate transformation has an inverse which can be found by applying the opposite transformation.

Translation in opposite direction $\bar{T}_v^{-1} = \bar{T}_{-v}$

Rotation in the opposite direction $\bar{R}_\theta^{-1} = \bar{R}_{-\theta}$

Scaling: $\bar{s}_{s_x, s_y}^{-1} = \bar{s}_{1/s_x, 1/s_y}$

Mirror reflection : $\bar{M}_x^{-1} = \bar{M}_x$; $\bar{M}_y^{-1} = \bar{M}_y$

Composite Tranformation:

More complex geometric and coordinate transformation can be built from the basic transformations described above by using the process of composition of function. For example such operation as rotation about a point other than the origin or reflection about lines other than the axis can be constructed from the basic transformation.

Example:

Magnification of an object while keeping its centre fixed. Let the geometric centre at c(h,k) figure (a). Choosing a magnification factor $s > 1$, we construct the transformation by performing the following sequence of basic transformation.

1. Translate the object so that its centre coincides with the origin figure (b)
2. Scale the object with respect to the origin figure (c)
3. Translate the scaled object back to the original position figure (d)

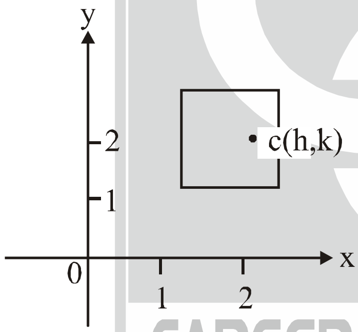
The required transformation $S_{s,C}$ can be formed by composition $S_{s,C} = T_v^{-1}, S_{s,s} . T_v$ where $v = hI + kJ$. By using composition, we can build more general and reflection transformation. We shall use the following notation:

(1) $S_{s_x, s_y} p$ scaling with respect to a fixed point P.

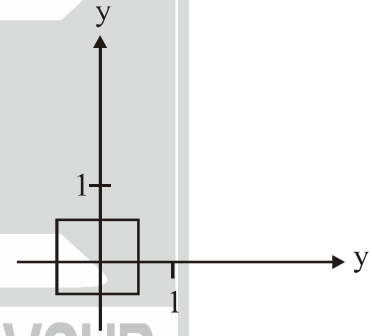
(2) R_θ, p - rotation about a point P, and

(3) M_L – reflection abut a line L.

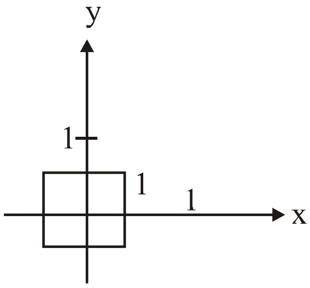
The matrix description of these transformation can be found in problem.



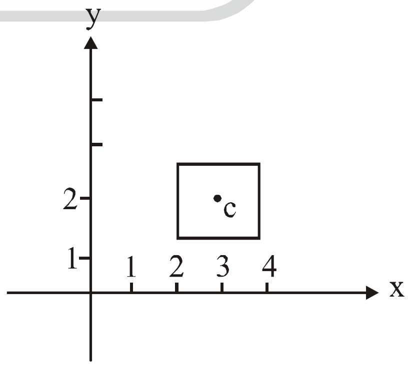
(a)



(b)



(c)



(d)

Homogeneous coordinate:

Homogeneous coordinate is a technique based on projective geometry. Any point (a, b) in two dimensional cartesian system is represented as (a, b, 1) in homogeneous coordinate system. Further any point (a, b, w) when $w \neq 0$ in homogeneous coordinates corresponds the point $\left(\frac{a}{w}, \frac{b}{w}\right)$ in two dimensional cartesian system. This corresponds is logical because the point in (a, b) in two dimensional is equivalent (a, b, 0) in three dimensional is equivalent to (a, b, 0) in three dimensional on z = 0 plane which is projected to z = 1 plane to be equivalent to (a, b, 1). Similarly a point (a, b, c) in 3D cartesian system is modelled as (a, b, c, 1) in homogeneous coordinate and a point (a, b, c, w) in homogeneous. The advantages of the representation in homogeneous coordinate are as follows.

1. The two dimensional transforamtion can be represented as 3×3 matrices so that usage of the operation become relatively easy.

2. The floating point arithematic can be avoided sometime by transforming them into integer arithematic.

We represent the coordinate pair (x, y) of a point P by the tripple (x, y, 1) in homogenous coordinate. Then translation in the direction $v = t_x I + t_y J$ can be expressed by the matrix function.

$$T_v = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{pmatrix}$$

$$(x, y, 1)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{pmatrix} = \begin{pmatrix} x + t_x & y + t_y & 1 \end{pmatrix}$$

From this we extract the coordinate pair $(x + t_x, y + t_y)$.

The rotation matrix after introducing homogenous coordinate will be

$$R\theta = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and $$(x, y, 1)\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} x\cos\theta - y\sin\theta & x\sin\theta + y\cos\theta + 1 \end{pmatrix}$$

The scaling transformation in homogeneous coordinate become

$$s_{s_x}, s_y = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{as} \quad (x, y, 1)\begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} xs_x & ys_y & 1 \end{pmatrix}$$

Concatentation of Matrices:

The advantages of introducing a matrix from for translation is that we can now build complex transformation by multiplying the basic matrix transformation. This process is sometime called conatenation of matrices and the resulting matrix is often referred to as the composite transformation matrix (CTM). Here, we are using the fact that the composition of matrix functions is equivalent to matrix multiplication. We must able to represent the basic transformations as 3×3 homogeneous coordinate matrices so as to be compatible (from the point of view of matrix multiplication) with the matrix of translation. This is accomplished by augmenting the 2×2 matrix with a third column $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and a thrid row $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$. The homogenous matrix therefore becomes

$$\begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example: Show that the order in which transformations are performed is important by applying the transformation of the triangle ABC by
(i) Rotating by 45º about the origin and then translating in the direction of the vector (1, 0) and
(ii) Translating first in direction of the vector (1, 0) and then rotating by 45º about the origin, where
$A=(1,0)$, $B=(0,1)$ and $C=(1,1)$

Soln. We can represent the given triangle, as shown in figure (a), in terms of Homogeneous coordinates are

$$[ABC]=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

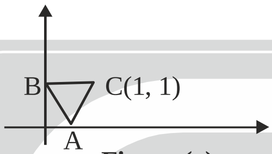


Figure (a)

Suppose the rotation is made in the counter clockwise direction. Then, the transformation matrix for rotation, R_{45^o}, in terms of homogeneous coordinate sytem is given by

$$R_{45^o}=\begin{bmatrix} \cos 45^o & \sin 45^o & 0 \\ -\sin 45^o & \cos 45^o & 0 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the transflation matrix, T_v, where V = 1I + 0J is

$$T_v=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

where t_x and t_y is the translation factors in the x and y directions respectively.
(i) Now the rotation followed by translation can be computed as

$$R_{45^o}\cdot T_v=\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}=\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

So, the new coordinates $A'B'C'$ of a given triangle ABC can be found as

$$[A'B'C']=[ABC]\cdot R_{45^o}\cdot T_v$$

$$=\begin{bmatrix}1&0&1\\0&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}&0\\-1/\sqrt{2}&1/\sqrt{2}&0\\1&0&1\end{bmatrix}=\begin{bmatrix}\left(1/\sqrt{2}+1\right)&1/\sqrt{2}&1\\\left(-1/\sqrt{2}+1\right)&1/\sqrt{2}&1\\1&\sqrt{2}&1\end{bmatrix} \qquad \text{... (I)}$$

implies that the given triangle A(1, 0), B(0, 1), C(1, 1) be transformed into

$A'\left(\frac{1}{\sqrt{2}}+1,\frac{1}{\sqrt{2}}\right)$, $B'\left(\frac{-1}{\sqrt{2}}+1,\frac{1}{\sqrt{2}}\right)$ and $C'\left(1,\sqrt{2}\right)$, respectively, as shown in figure (b)

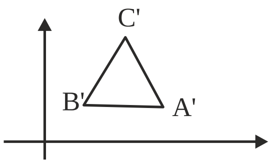


Figure (b)

Similarly, we can obtain the translation followed by rotation transformation as

$$T_v\cdot R_{45^\circ}=\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}\cdot\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}&0\\-1/\sqrt{2}&1/\sqrt{2}&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}&0\\-1/\sqrt{2}&1/\sqrt{2}&0\\1/\sqrt{2}&1/\sqrt{2}&1\end{bmatrix}$$

And hence, the new coordinates $A'B'C'$ can be computed as

$$\left[A'B'C'\right]=\left[ABC\right]\cdot T_v\ R_{45^\circ}$$

$$=\begin{bmatrix}1&0&1\\0&1&1\\1&1&1\end{bmatrix}\cdot\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}&0\\-1/\sqrt{2}&1/\sqrt{2}&0\\1/\sqrt{2}&1/\sqrt{2}&1\end{bmatrix}=\begin{bmatrix}2/\sqrt{2}&2/\sqrt{2}&1\\0&2/\sqrt{2}&1\\1/\sqrt{2}&3/\sqrt{2}&1\end{bmatrix} \qquad \text{... (II)}$$

Thus, in this case, the given triangle A(1, 0), B(0, 1) and C(1, 1) are transformed into

$A''\left(2/\sqrt{2},2/\sqrt{2}\right)$, $B''\left(0,2/\sqrt{2}\right)$ and $C''\left(\frac{1}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$, as shown in the figure (c)

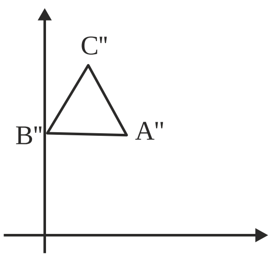


Figure (c)

By (I) and (II), we see that the two transformations do not commute.

Shear Transformation:

There are two more transformation which are useful in graphics application namely the x shear and y-shear called shear transformation. The shear transformation cause the image to slant. X-shear maintain the y-coordinates but changes the x value which cuase the vertical lines to tilt left or right. The y-shear preserves all the x-coordinates value the shifts the y coordinates values but shifts the y-coordinate. This causes horizontal lines to transformation using the transformation of rotation and scaling. However it is easy to use the matrix form directly. Similar, it is also possible to built rotation and scaling transformation out of shear transformation.

The x-shear transformation in the homogeneous matrix form is given by

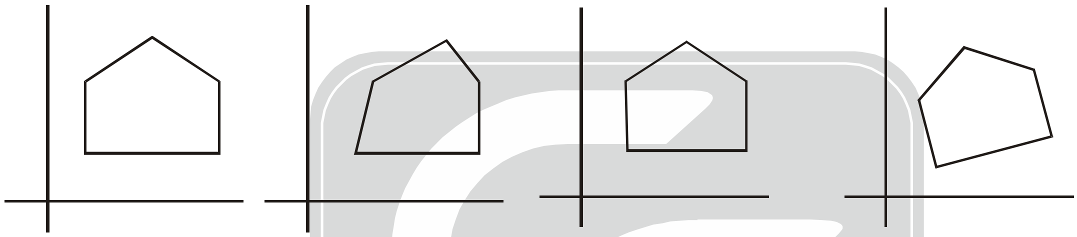
$$Sh_x = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Where a is the x-shear factor.

When b is the y-shear factor, the homogeneous matrix transformation of y shear is

$$Sh_y = \begin{pmatrix} 1 & b & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The effects of there are shown in figure



Before x-shear   After x-shear   Before Y-shear   After y-shear

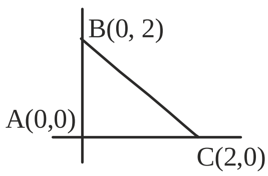
Affine transformation: A transformation which preserves the parallel lines and number of points in the objects.

NUMERICAL PROBLEMS

1. Rotate a triangle A(0, 0), B(0, 2), C(2, 0) by an angle 45º in anti-clockwise direction. Find the new coordinate of triangle ABC.

Soln. $R_{45^\circ} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$

$$R_{45^\circ} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

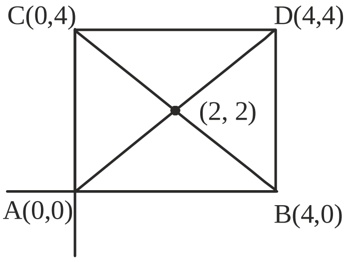


$$\begin{bmatrix} 0 & 0 \\ 0 & 2 \\ 2 & 0 \end{bmatrix}_{3\times2} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}_{2\times2} = \begin{bmatrix} 0 & 0 \\ -2/\sqrt{2} & 2/\sqrt{2} \\ 2/\sqrt{2} & 2/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ -\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$$

2. Consider a square A(0, 0), B(4, 0), C(0, 4), D(4, 4). Perform the rotation by 45º clockwise of the square ABCD by fixing the centre of the square

Soln.



$$R_\theta = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\text{Translating factor} = (t_x, t_y) \begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{bmatrix} -2 & -2 \\ 2 & -2 \\ 2 & 2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} + [2, 2]$$

$$= \begin{bmatrix} -2/\sqrt{2} - 2/\sqrt{2} & 2/\sqrt{2} - 2/\sqrt{2} \\ 2/\sqrt{2} - 2/\sqrt{2} & -2/\sqrt{2} - 2/\sqrt{2} \\ 2/\sqrt{2} + 2/\sqrt{2} & -2/\sqrt{2} - 2/\sqrt{2} \\ -2/\sqrt{2} + 2/\sqrt{2} & 2/\sqrt{2} + 2/\sqrt{2} \end{bmatrix} + [2,2] \Rightarrow \begin{bmatrix} -(4/\sqrt{2}) + 2 & 2 \\ 2 & -(4/\sqrt{2}) + 2 \\ (4/\sqrt{2}) + 2 & -(4/\sqrt{2}) + 2 \\ 2 & (4/\sqrt{2}) + 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -(2\sqrt{2}) + 2 & 2 \\ 2 & -(2\sqrt{2}) + 2 \\ (2\sqrt{2}) + 2 & -(2\sqrt{2}) + 2 \\ 2 & (2\sqrt{2}) + 2 \end{bmatrix}$$

Clipping Algorithm

Window-To-Viewport Mapping:

Window: The box/tool which helps to take the picture of our interest to be drawn on the display surface i.e. what we want to see.

A window is specified by four world coordinats.

$w_{x_{min}}$, $w_{x_{max}}$

$w_{y_{min}}$, $w_{y_{max}}$

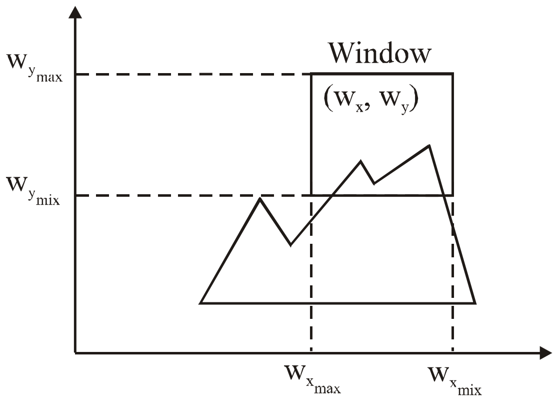


Figure 1

View-port: The box/tool which helps to draw the pixture on the different portion of the screen is called as view port. If we fix the view port and move the window, its shows different portion of the object/pixture. But when we fix the window and move the view port it shows some parts of the picture on the screen.

Similarly, a viewpoint is described by four normalized device coordinate.

$$w_{x_{min}}, \quad w_{x_{max}}$$

$$w_{y_{min}}, \quad w_{y_{max}}$$

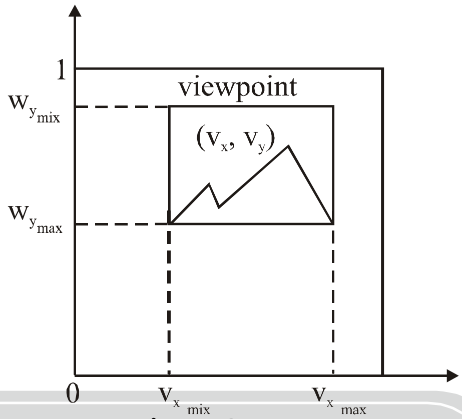


Figure 2

The objective of window to viewpoint mapping is to convert the world coordinate (w_x, w_y) of an arbitrary point to its coordinate (v_x, v_y). In order to maintain the same relative placement of the point in the viewport as in window, we require:

$$\frac{W_x - W_{x_{min}}}{W_{x_{max}} - W_{x_{min}}} = \frac{V_x - V_{x_{min}}}{v_{x_{max}} - V_{x_{min}}}$$

and

$$\frac{W_y - W_{y_{min}}}{W_{y_{max}} - W_{y_{min}}} = \frac{V_y - V_{y_{min}}}{v_{y_{max}} - V_{y_{min}}}$$

Thus,

$$V_x = \frac{V_{x_{max}} - V_{x_{min}}}{W_{x_{max}} - W_{x_{min}}}\left(W_x - W_{x_{min}}\right) + V_{x_{min}}$$

$$V_y = \frac{V_{y_{max}} - V_{y_{min}}}{W_{y_{max}} - W_{y_{min}}}\left(W_y - W_{y_{min}}\right) + V_{y_{min}}$$

Since the eight coordinate value, the define the window and viewport are just constraints, we can express these two formulas for computing (V_x, V_y) from (W_x, W_y) in terms of a translate scale translate transform N.

$$\left(V_x, V_y\ 1\right) = \left(W_x, W_y\ 1\right).N$$

Where

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ V_{x_{min}} & V_{y_{min}} & 1 \end{bmatrix} \begin{bmatrix} \frac{V_{x_{max}} - V_{x_{min}}}{W_{x_{max}} - W_{x_{min}}} & 0 & 0 \\ 0 & \frac{V_{y_{max}} - V_{y_{min}}}{W_{y_{max}} - W_{y_{min}}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -W_{x_{min}} & -W_{y_{min}} & 1 \end{bmatrix}$$

Note that geometric distortions occur (eg. squares in the window become rectangles in the viewport) whenever the two scaling constant differ.

Windowing: The process of selecting the portion of the picture of our interest and that portion of the picture need to be drawn on the screen is known as windowing.

Clipping : The process of selecting the portion of the picture which is not of our interest and then to eliminate is known as Clipping.

Point Clipping:
Point clipping is essentially the evaluation of the following inequalities.

$$x_{min} \le x \le x_{max} \text{ and } y_{min} \le y \le y_{max}$$

where $x_{min}, x_{max}, y_{min}$ and y_{max} define the clipping window. A point (x, y) is considered inside the window when the inequalities all evaluate to true.
Lines that do not intersect the clipping window are either completely inside the window or completely outside the window. On the otherhand, a line that intersects the clipping window are either completely inside the window or completely outside the window. The following algorithm provide efficient ways to decide the spacial relationship between an arbitrary lines and the clipping window and to find intersecting point(s).

1. Cohen–Sutherland Algorithm.
2. Liany - Barsky Algorithm
3. Nicholl-Lee-Nicholl Algorithm
4. Mid point Subdivision Algorithm

1. Cohen-Sutherland Algorithm:
In this algorithm we divide the line clipping process into two phases.
1. Identify those lines which intersect the clipping window and so need to be clipped.
2. Perform the clipping:
All lines fall into one of the following clipping categories.
(a) **Visible:** Both end points of the line lie within the window.
(b) **Not visible:** The definitely lie outside the window. This will occur if the linefrom (x_1, y_1) to (x_2, y_2) satisfies any one of the following four inequalities.

$$x_1, x_2 > x_{max} \qquad y_1, y_2 > y_{max}$$
$$x_1, x_2 < x_{min} \qquad y_1, y_2 < y_{min}$$

(c) **Clipping candidate:** The line is in neither category 1 nor 2

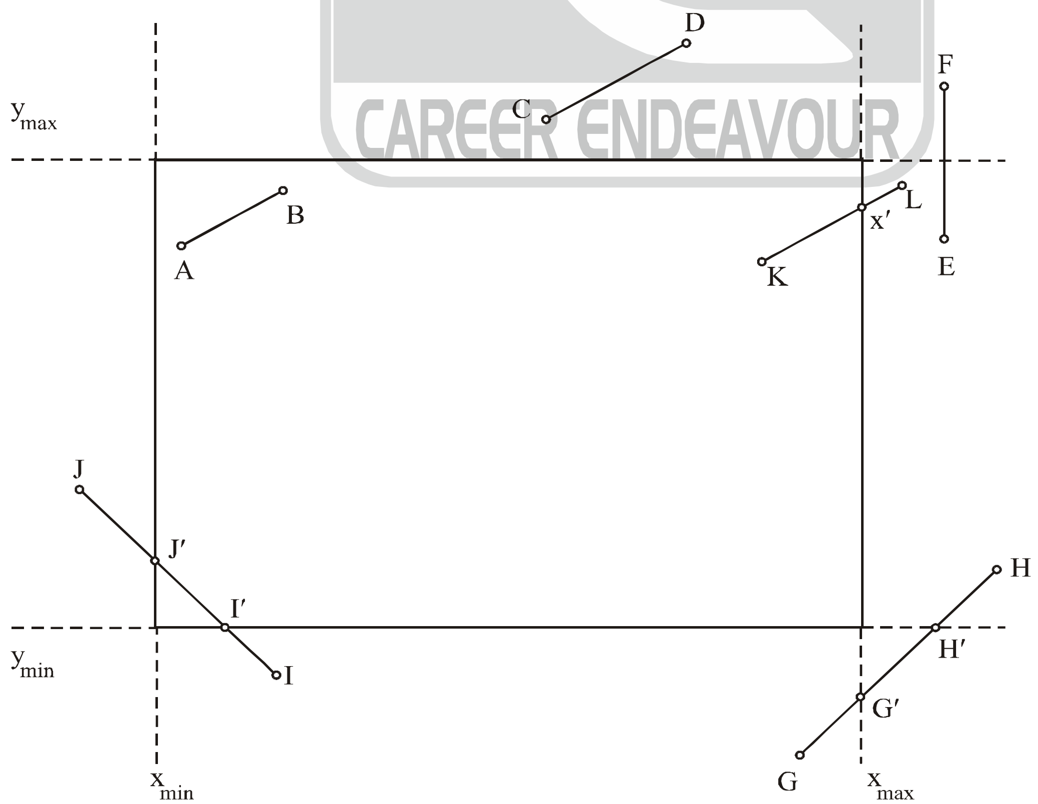


Figure 3