CHAPTER

9

Mensuration & area of plane figures

1. Plane Figure:

Any figure bounded by three or more straight lines or bounded by a closed line is a **Plane Figure**. The space closed within that figure is called its **Area.** The measurement of the length of the lines enclosing the space called its **Perimeter.**

Parallel lines: Two straight lines are parallel if they lie on the same plane and do not intersect.
Transversal: It is a straight line that intersects two parallel lines.

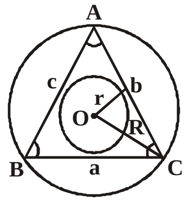
2. Polygons: Polygons are figures formed by a closed series of rectilinear (straight) segments.
Example: Triangle, Rectangle, Pentagonal (5 sides), Hexagonal (6 sides) …………..
Polygons can broadly be divided into two types:
(i) **Regular polygons:** Polygons with all the sides and angles equal.
(ii) **Irregular polygons:** Polygons in which all the sides or angles are not of the same measure.

Properties:
(i) Sum of the angles of a polygon with n sides = (2n -4)p/2 Radians = (n–2) 180º degrees
(ii) Sum of the exterior angles = 360º i.e. in the figure below: $\theta_1 + \theta_2 + ... + \theta_6 = 360^o$
In general, $\theta_1 + \theta_2 + ... + \theta_n = 360^o$
(iii) Number of sides = 360º/exterior angle.
(iv) Perimeter = n×s.

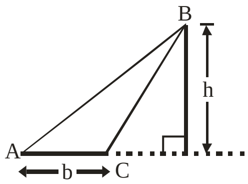
(a) Triangles (Δ):
A triangle is a polygon having three sides. Sum of all the angles of a triangle = 180º.
Types of Triangle:
(i) Acute angle triangle: Triangles with all angles acute (less than 90º)
(ii) Obtuse angle triangle: Triangles with one of the angles obtuse (more than 90º).
(iii) Right angle triangle: Triangle with one of the angles equal to 90º.

• **Important points about triangles (Δ):**
(i) Sum of the length of any two sides of a triangle has to be always greater than the third side.
(ii) Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
(iii) Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
(iv) The sine rule: a/sin A = b/sin B = c/sin C = 2R (where R = circum radius).
(v) The cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

This is true for all sides and respective angles.



• Formulas for area of a triangle:



(i) Area = $\frac{1}{2}$ base × height . Height = Perpendicular distance between the base and vertex opposite to it.

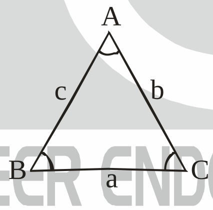
(ii) Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$ where, s is half perimeter. (a, b and c being the length of the sides of the triangle).

(iii) Area = rs (where r is inradius).

(iv) Area = $\frac{1}{2}$ × product of two sides × sine of the included angle.

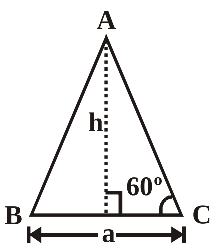
$= \frac{1}{2}$ ac sin B= $\frac{1}{2}$ ab sin C= $\frac{1}{2}$ bc sin A.



(v) Δ Area $= \frac{abc}{4R}$, where R = circumradius.

• Equilateral triangles (of side a): Triangle having all the angles equal to 60º.

(i) $h = \frac{a\sqrt{3}}{2}$ $\left(\therefore \sin 60^\circ = \sqrt{3}/2 = h/\text{side}\right)$ (ii) Area = $\frac{1}{2}$ (base) ×(height) = $\frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$



(iii) $R\left(\text{circum radius}\right) = \frac{2h}{3} = \frac{a}{\sqrt{3}}$ (iv) $r\left(\text{in radius}\right) = \frac{h}{3} = \frac{a}{2\sqrt{3}}$

• Properties of equilateral triangle:

(i) The incenter and circumcentre lies at a point that divides the height in the ratio 2:1.

(ii) The circum radius is always twice the in radius [R=2r].

(iii) Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.

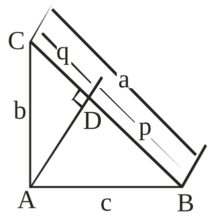
(iv) An equilateral triangle in a circle will have the maximum area compared to other triangles inside the same circle.

• Right angled triangle: Triangle with one of the angles equal to 90º.

Pythagoras Theorem: In the case of a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In the given figure , for triangle $ABC, a^2 = b^2 + c^2$

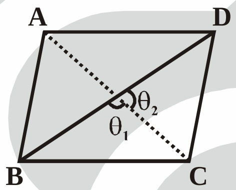
Area = $\frac{1}{2}$ (product of perpendicular sides) = $\frac{1}{2}$ (bc)



3. Quadrilateral:

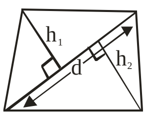
Formulas for area of quadrilaterals:

(i) Area = $\frac{1}{2}$ (product of diagonals) × (sine of the angle between, them)



If θ_1 and θ_2 are the two angles made between themselves by the two diagonals, we have by the property of intersecting lines $\rightarrow \theta_1 + \theta_2 = 180°$. Then the area of quadrilateral = $\frac{1}{2} d_1 d_2 \sin\theta_1 = \frac{1}{2} d_1 d_2 \sin\theta_2$.

(ii) Area = $\frac{1}{2}$ × diagonal × sum of the perpendiculars to it from opposite vertices = $\frac{d(h_1 + h_2)}{2}$



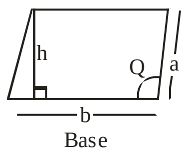
(iii) Area of a circumscribed quadrilateral $A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$ Where, $S = \frac{a+b+c+d}{2}$

(Where a, b, c and d are the lengths of the sides.).

Types of Quadrilaterals:

(i) Parallelogram:

A parallelogram is a quadrilateral with opposite sides parallel



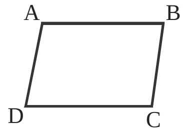
Formulas for area of parallelogram:

(i) Area = Product of any two adjacent sides × sine of the included angle.= ab sin Q

$$\boxed{\text{Area of parallelogram} = \frac{1}{2} h \cdot (a+b)}$$

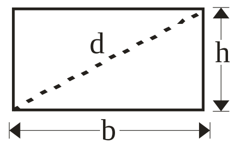
(ii) Perimeter = 2(a + b) where a and b are any two adjacent sides.

Properties of parallelogram:



(i) Diagonals of a parallelogram bisect each other.
(ii) Bisectors of the angles of a parallelogram form a rectangle.
(iii) A parallelogram inscribed in a circle is a rectangle.
(iv) A parallelogram circumscribed about a circle is a rhombus.
(v) The opposite angles in a parallelogram are equal.
(vi) The sum of the squares of the diagonals is equal to the sum of the squares of the four sides in the figure.

(ii) Rectangles: A rectangle is a parallelogram with all angles 90°.



(i) Area = Base × Height = b× h.
(ii) Diagonal $(d) = \sqrt{b^2 + h^2} \rightarrow$ by Pythagoras theorem.

Properties of a rectangle:
(i) Diagonals are equal and bisect each other.
(ii) Bisectors of the angels of a rectangle form another rectangle.
(iii) All rectangles are parallelogram but the reverse is not true.

(c) Rhombus: A parallelogram having all the sides equal is a rhombus.

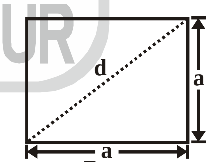
(i) Area = $\frac{1}{2}$ × product of diagonals × sine of the angle between them.

$$= 1/2 \times d_1 \times d_2 \sin 90^\circ \text{ (Diagonals in rhombus intersect at right angles)} = 1/2 \times d_1 d_2$$

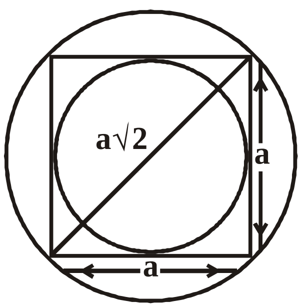
(ii) Area = product of adjacent sides × sine of the angle between them.

Properties of Rhombus:
(i) Diagonals bisect each other at right angles.
(ii) All rhombuses are parallelogram but the reverse is not true.
(iii) A rhombus may or may not be square but all squares are rhombus.

(d) Squares:
A square is a rectangle with sides are equal or a rhombus with each angle 90°.
(i) Area = base × height = a^2.



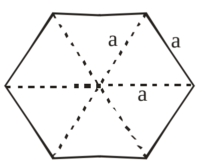
(ii) Area = $\frac{1}{2}$ $(\text{diagonal})^2 = \frac{1}{2}$ d^2 (square is a rhombus too).
(iii) Perimeter = 4a(a = side of the square)
(iv) Diagonal $= a\sqrt{2}$
(v) In radius = $\frac{a}{2}$



Properties of Square:
(i) Diagonals are equal and bisect each other at right angles.
(ii) Side is the diameter of the inscribed circle.
(iii) Diagonal is the diameter of the circumscribing circle.

$$\Rightarrow \quad \text{Diameter} = a\sqrt{2} \qquad \text{Circumradius} = \frac{a}{\sqrt{2}}$$

(e) Regular Hexagon:



(i) Area $= \left[\left(3\sqrt{3}\right)/2\right](\text{side})^2 = \frac{3\sqrt{3} \times a^2}{2}$

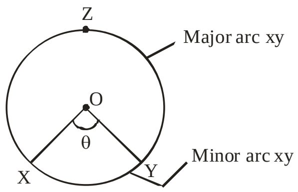
(ii) A regular hexagon is actually a combination of 6 equilateral triangles all of side 'a'.
(iii) If you look at the figure closely it will not be difficult to realize that circumradius (R) = a; i.e. the side of the hexagon is equal to the circumradius of the same.

4. Circle:

(i) Area = πr^2. (ii) Circumference = $2\pi r$ (r = radius). (iii) Area = ½ × circumference × r.

Arc of circle: It is a part of the circumference of the circle. The bigger one is called the major arc and the smaller one the minor arc.

(iv) Length $\left(\text{Arc XY}\right) = \dfrac{\theta}{360} \times 2\pi r$



(v) Sector of a circle is a part of the area of a circle between two radii.

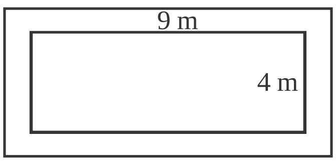
(vi) Area of a sector = $\dfrac{\theta}{360} \times \pi r^2$ (where θ is the angle between two radii)

$$= \left(\frac{1}{2}\right) r \times \text{length}\left(\text{arc xy}\right) \quad \left(\because \quad \pi r\theta / 180 = \text{length arc xy}\right)$$

$$= \frac{1}{2} \times r \times \frac{2\pi r\theta}{360}$$

SOLVED EXAMPLES

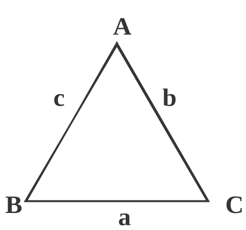
1. Pradeep wishes to make a gravel path around his rectangular pond. The path must be the same width all the way round, as shown in the diagram. The pond measures 4m by 9m and he has enough gravel to cover an area of $48m^2$. How wide around the path be?



(a) 2 metre (b) 3 metre (c) 1.5 metre (d) 8 metre

2. What is the minimum value of the perimeter of a triangle, if two of its sides are 5cm and 7cm respectively? (the sides have integer volumes)

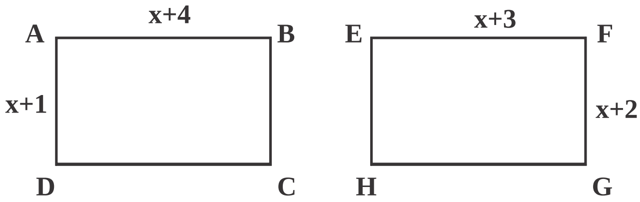
Soln. In a triangle, c > b–a



Therefore, c > 7–5 or c > 2

The minimum length of the third side is 3 cm
Hence, the minimum value of the perimeter = 5 + 7 + 3 = 15 cm

3. What is the area of rectangle EFGH of given figure, if the area of rectangle ABCD is 100?



Soln. Area of rectangle ABCD = (x + 1) × (x + 4) = $x^2 + 5x + 4$

Area of rectangle EFGH = (x + 2) × (x + 3) = $x^2 + 5x + 6$

If we compare the area of rectangle ABCD and EFGH, the area of rectangle EFGH is 2 more than the area of rectangle ABCD.

Therefore, Area of rectangle EFGH = 100 + 2 = 102

4. Euclid has a triangle in mind. Its longest side 20 units and another of its sides has length 10 units. Its area is 80 square units. What is the exact length of its third side?

(a) $\sqrt{270}$ units (b) $\sqrt{240}$ units (c) $\sqrt{250}$ units (d) $\sqrt{260}$ units

5. If a square and an equilateral triangle have equal perimeters, what is the ratio of the area of the triangle to the area of the square?

(a) $\frac{4\sqrt{3}}{9}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) None of these

EXERCISE

1. In two triangles, ratio of the areas in 4:3 and the ratio of their heights is 3:4. Find the ratio of their bases.

(a) $\frac{16}{9}$ (b) $\frac{9}{16}$ (c) $\frac{12}{9}$ (d) $\frac{9}{12}$

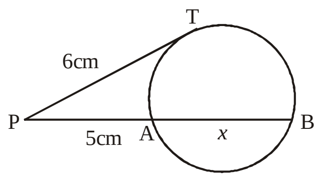
2. A rectangular gram plot 100m by 65m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per square metre.

(a) Rs 620 (b) Rs 660 (c) Rs 680 (d) Rs 640

3. AB is a chord of a circle of radius 14 cm. The chord subtends a angle of 90° at the centre of the circle. Find the area of minor segment.

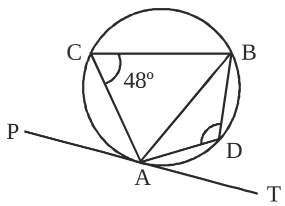
(a) 98 sqcm (b) 56 sqcm (c) 112 sqcm (d) 100 sqcm.

4. Find the value of 'x' in the given figure



(a) 2.2 cm (b) 1.6 cm (c) 3 cm (d) 2.5 cm

5. In the given figure, find angle ADB



(a) 130° (b) 132° (c) 122° (d) 120°

6. In a triangle ABC, point D is an side AB and point E is on side AC, such that BCED is a trapezium. DE:BC = 3:5. Calculate ratio of area of triangle ADE and the trapezium BCED.
(a) 3:4 (b) 9:16 (c) 3:5 (d) 9:25

7. A motocycle stuntman, belonging to a fair, rides over the vertical walls of a circular well at an average speed of 54 kmph for 5 minutes. If the radius of the well is 5 meters, then the distance travelled is:
(a) 2.5 km (b) 3.5 km (c) 4.5 km (d) 5.5 km

8. The least perimeter of an equilateral triangle in which a circle of radius 'r' can be inscribed is:
(a) $3r\sqrt{3}$ (b) $\sqrt{3}\,r$ (c) $\frac{6r}{\sqrt{3}}$ (d) $6r\sqrt{3}$

9. Consider a square of side 6 cm. A circle is inscribed inside the square. Another circle circumscribes the square. The ratio of the areas of the inscribed circle to the circumscribed circle is
(a) $1:\frac{\pi}{4}$ (b) $1:\pi$ (c) 1:1.5 (d) 1 : 2

10. A carpenter is designing a box. The floor of the box is a rectangular whose length is two feet more than its width. How long should the box be (in feet) if the area of the floor of the box were to be 15 square feet?
(a) 3 (b) 5 (c) 10 (d) 15

11. A wire of some length is bent in circular form and has an area of 308 sq. cm. If the same length of wire is straightended out and bent in the form of a square, the approximate area of the square in sq. cm may be
(a) 242 (b) 121 (c) 308 (d) 69.29

12. Consider a sqaqure circumscribed by a circle with a radius of 4 units. The area of the square in square units is
(a) $16\sqrt{2}$ (b) 16π (c) 32 (d) 64

13. Four cows are tethered at four corners of a square plot of side 14 meters so that the adjacent cows can just reach one another. There is a small circular pond of area 20 m^2 at the centre. The area left ungrazed is:
(a) 22 m^2 (b) 42 m^2 (c) 84 m^2 (d) 168 m^2.

14. Instead of walking along two adjacent sides of a rectangluar field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

15. There are two cubes on a table in which the volume of the second is half that of the first. If the first cube occupies a certain area (Y) on the table, how much area (approximately) does the second occupy?
(a) $\frac{Y}{\sqrt[3]{2}}$ (b) $\frac{Y}{2}$ (c) $\frac{Y}{\sqrt[3]{4}}$ (d) $\frac{Y}{\sqrt{2}}$

16. A paper of zie $\ell \times w\left(\ell > w\right)$ is folded in half along the longer dimension. It is then folded in half along the other dimension and a third time, along the direction of the first fold. What are the dimension of the folded paper?
(a) $\frac{\ell}{4}\times\frac{w}{4}$ (b) $\frac{\ell}{8}\times\frac{w}{4}$ (c) $\frac{\ell}{8}\times\frac{w}{8}$ (d) $\frac{\ell}{4}\times\frac{w}{2}$

17. Paper sizes are given by A_0, A_1, A_2 etc. such that A_0 is two times larger (in area) than A_1, A_1 is two times larger than A_2 and so on. The longer dimension of each smaller size is equal to the shorter dimension of the larger size. For example, the longer dimension of A_2 is the same as the shorter dimension of A_1. In this scheme if A_4 is 210 × 297 mm in size, what are the dimensions of A_0 in mm?
(a) 840×594 (b) 420×594 (c) 840×1188 (d) None of the above.

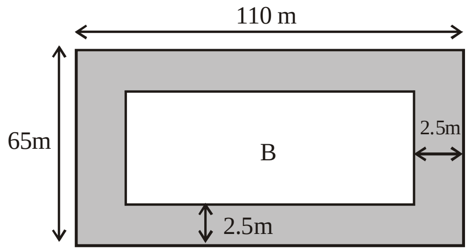
HINTS & SOLUTIONS

1. Let the bases of the two triangles be x and y and their heights be 3h and 4h respectively.

Then, $\dfrac{\frac{1}{2}(x)(3h)}{\frac{1}{2}(y)(4h)} = \dfrac{4}{3} \quad \Rightarrow \dfrac{x}{y} = \left(\dfrac{4}{3}\right)\left(\dfrac{4}{3}\right) = \dfrac{16}{9}$

Answer is (a).

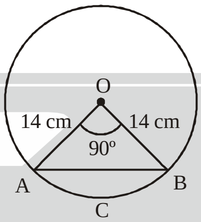
2. Area of plot = 110 × 65 = 7150 m^2.
Area excluding the path = (100–5) (65–5) = 6300 m^2.
Area of path = (7150 – 6300) m^2 = 850 m^2.



Cost of gravelling the path = $850\left(\dfrac{80}{100}\right)$ Rs = Rs 680

Answer is (c)

3. Area of sector ACBO = $\dfrac{90^\circ \pi r^2 / 4^2}{360^\circ} = 154$ sqcm



Area of triangle AOB = $\dfrac{14\times14}{2} = 98$ sqcm

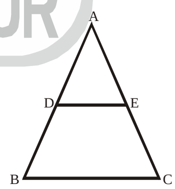
Area of segment ACB = Area of sector ACBO – Area of the triangle AOB = 154 –98 = 56 sqcm.

Answer is (b)

4. In the figure, $PT^2 = PA \times PB$, $6^2 = 5(5 + x)$, $x = 2.2$ cm
Answer is (a)

5. ADBC is a cyclic qudrilateral as all its fair vertices are on the circumference of the circle. Also, the opposite angles of the cyclic qudrilateral is supplementary.
Therefore, $\angle ADB = 180^\circ - 48^\circ = 132^\circ$
Answer is (b)

6. ΔADE is a similar to ΔABC
Thus, DE : BC = 3:5



Area of ΔADE : Area of ΔABC = 9:25
Area of trapezium = area of ABC – area of APE = 25–9 = 16
Area of ΔADE : Area of trapezium EDBC = 9/16
Answer is (b)

7. Speed = 54 km/h

Time = 5 min = $\dfrac{5}{60}$ hrs

Hence, D = Speed × time = $54 \times \dfrac{5}{60} = 4.5$ km

Answer is (c)

8. The radius of incircle of any equilateral triangle $r = \dfrac{a}{2\sqrt{3}}$

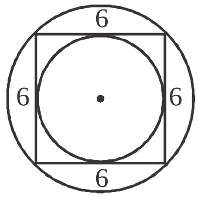
Hence, side of triangle is $2\sqrt{3}.r$
Hence, perimeter = 3×a = $3\times 2\sqrt{3}.r = 6\sqrt{3}\ r$
Answer is (d)

9. Diameter of inscribed circle = side of square = 6

$r = 3$

Hence, area = $\pi\, 3^2 = 9\pi$

Diameter of circumcircle = diagonal of square = $6\sqrt{2}$



$$R = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

Hence, Area = $\pi\left(3\sqrt{2}\right)^2 = 18\pi$

Hence, ratio = $9\pi : 18\pi \quad \Rightarrow 1:2$

Answer is (d)

10. Let width = x, hence length = x + 2

Then area, $x(x+2) = 15$

$\Rightarrow \quad x = 3$. Then length = 3 + 2 = 5

Answer is (b)

11. Area of circle = 308 cm

Then, $\pi r^2 = 308 \quad \Rightarrow \quad r^2 = \frac{308}{22} \times 7 \quad \Rightarrow r = 7\sqrt{2}$

Hence, circumference = $2\pi r = 2 \times \frac{22}{7} \times 7\sqrt{2} = 44\sqrt{2}$ cm

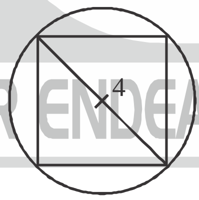
This is equals to perimeter of square = 4 × side = $44\sqrt{2} \quad \Rightarrow$ side = $11\sqrt{2}$

Hence, area $= A^2 = \left(11\sqrt{2}\right)^2 = 242$

Answer is (a)

12. Diagonal of square $= a\sqrt{2}$

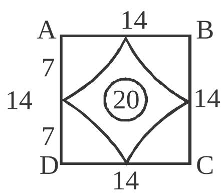
$\Rightarrow \quad a\sqrt{2} = 8 \quad \Rightarrow \quad a = 4\sqrt{2}$



Hence, area = $\left(4\sqrt{2}\right)^2 = 32$

Answer is (c)

13. Area grazed by cow = $4 \times \frac{1}{4} \times \frac{22}{7} \times 7^2 = 154 \text{ m}^2$



Area of pond = $20 m^2$

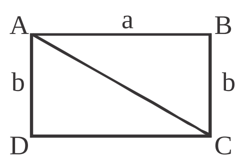
Then area of square is = $14^2 = 196$

Hence, ungrazed area = 196–154 – 20 = $22 m^2$

Answer is (a)

14. Let longer side is 'a' and smaller side is 'b'. If a person walk along a two adjacent side.

Then total distance = a + b and go through diagonal = $\sqrt{a^2 + b^2}$



$\therefore \quad (a+b) - \sqrt{a^2 + b^2} = \frac{a}{2}$

$\therefore \quad a + b - \frac{a}{2} = \sqrt{a^2 + b^2}$

Squaring both sides, we get, $\frac{a^2}{4} + b^2 + 2.\frac{a}{2}.b = a^2 + b^2$

$\Rightarrow \quad a^2 - \frac{a^2}{4} = ab \qquad \Rightarrow \frac{3a^2}{4} = ab \qquad \Rightarrow \frac{b}{a} = \frac{3}{4}$

Answer is (d)

15. Let side of first cube is A and second cube is ‘a’

Hence, $V_I = A^3$ and $V_{II} = a^3$

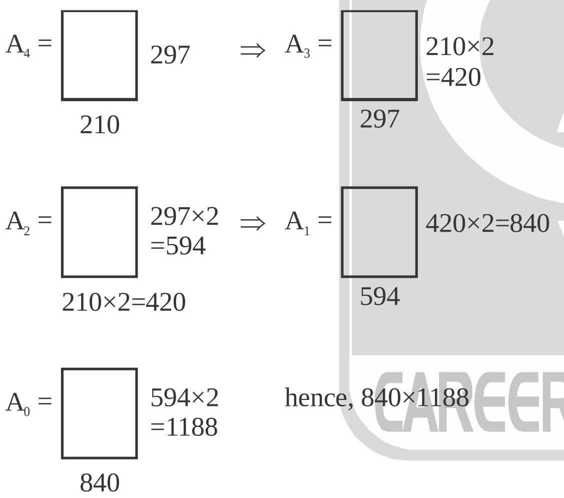
$\frac{V_I}{V_{II}} = \frac{2}{1} = \frac{A^3}{a^3} \Rightarrow a^3 = \frac{A^3}{2} \quad \Rightarrow a = \frac{A}{\sqrt[3]{2}}$

Now, $A^2 = Y \quad \therefore \; a^2 = \left(\frac{A}{\sqrt[3]{2}}\right)^2 = \frac{A^2}{\sqrt[3]{4}} = \frac{Y}{\sqrt[3]{4}}$

Answer is (c)

16. Answer is (d)

17.



hence, 840×1188

Answer is (c)