TEST SERIES NTA-NET DEC. 2019

BOOKLET SERIES C

FULL LENGTH TEST - III

Paper Code 05

Test Type: TEST SERIES

PAPER WITH SOLUTION

PHYSICAL SCIENCES

Duration: 3:00 Hours

Date: 07-12-2019

Maximum Marks: 200

Read the following instructions carefully:

* Single Paper Test is divided into **three** Parts.

Part - A: This part shall carry 20 questions. The candidate shall be required to answer any 15 questions. Each question shall be of **2 marks.**

Part - B: This part shall contain 2**5** questions covering the topics given in the Part 'B' of syllabus. The candidates are required to answer any 20 questions. Each question shall be of **3.5 Marks.**.

Part - C: This part shall contain **30** questions from Part - C of the syllabus. The candidates are required to answer any 20 questions. Each question shall be of **5 Marks**.

* Darken the appropriate bubbles with HB pencil/Ball Pen to write your answer.

* There will be negative marking @25% for each wrong answer.

* The candidates shall be allowed to carry the Question Paper Booklet after completion of the exam.

* For rough work, blank sheet is attached at the end of test booklet.

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USEFUL FUNDAMENTAL CONSTANTS

m	Mass of electron	9.11×10^{-31} Kg
h	Planck's constant	6.63×10^{-34} J sec
e	Charge of electron	1.6×10^{-19} C
k	Boltzmann constant	1.38×10^{-23} J/K
c	Velocity of Light	3.0×10^{8} m/Sec
1_eV	1.6×10^{-19} J	
an[illegible]	1.67×10^{-27} kg	
G	6.67×10^{-11} Nm^{2}kg^{-2}	
R_y	Rydberg constant	1.097×10^{7} m^{-1}
N_A	Avogadro number	6.023×10^{23} mole^{-1}
ε_o	8.854×10^{-12} Fm^{-1}	
μ_o	$4\pi \times 10^{-7}$ Hm^{-1}	
R	Molar Gas constants	8.314 J K^{-1} mole^{-1}

PART – A

1. Five batsmen namely A, B, C, D, E scored an average of 36 run. D scored 5 more runs than what E had scored E scored 8 less than A. B scored as many as D and E combined. B and C scored 107 together. What is average score excluding the score of E ?
(a) 30 (b) 40 (c) 25 (d) 20

2. A box contains 5 green, 4 yellow and 3 white marbles. 3 marbles are drawn at random. What is the probability that they are not of the same colour ?
(a) 13/44 (b) 41/44 (c) 13/55 (d) 152/55

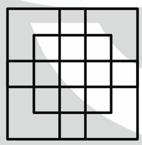
3. Amit purchased 20 dozen notebooks at ₹ 48 per dozen. He sold 8 dozen at 10% profit and the remaining 12 dozen at 20% profit. What is his profit percentage in this transaction ?
(a) 10 % (b) 12 % (c) 8 % (d) 16 %

4. A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 litres of the mixture is taken out and 10 litres of liquid B is poured into the jar, the ratio becomes 2 : 3. How many litres of liquid A was contained in the jar ?
(a) 16 (b) 20 (c) 8 (d) 12

5. A cylinder is filled to $\frac{4}{5}$th of volume. If it is tilted then level of water coincides with one edge of its bottom and top edge of the opposite side. In the process, 30 litre of the water is spilled. What is the capacity of the cylinder?
(a) 75 litre (b) 96 litre (c) 100 litre (d) Data inadequate

6. In a potluck party every two members used a pot of rice between them, every three member used a pot of curry among them and a pot of paneer among four of them, if altogether 65 pots were there, then what is the total number of members in the party ?
(a) 90 (b) 50 (c) 60 (d) 42

7. Count the total number of squares in the given figure ?



(a) 20 (b) 30 (c) 25 (d) 27

8. The next term in the series is

A3E	C10Q	F9L	?

(a) J15T (b) J7P (c) I10P (d) J10R

9. Below is given a table chart which shows the marks obtained by four students A, B, C, D in four different subjects (Physics, Chemistry, Mathematics, Computer Science). Based on the information provided you have to answer the question.

Full Marks in each paper is 100 marks

Students	Physics	Chemistry	Mathematics	Computer
A	50	65	85	70
B	40	60	65	45
C	70	55	75	80
D	35	90	45	40

Marks obtained by C and D together in Physics is how much percentage less than the marks obtained by A and B together in Chemistry ?
(a) 20% (b) 30% (c) 10% (d) 16%

10. Pointing to a photograph of a girl a man says, "She is the daughter of the wife of the only son of my father". How is the girl related to the man ?
(a) Aunty (b) Daughter (c) Mother (d) Niece

11. Arvind is 8^{th} from the left of a line and Monu is 9^{th} from the right of the line. What is the minimum number of students between them ?
(a) 7 (b) 6 (c) 8 (d) 9

12. Six persons namely 1, 2, 3, 4, 5, 6 are there in a family.
(1) '3' is the sister of '6' and both of them are children of '1'.
(2) '2' is the brother of '5's husband.
(3) '4' is the father of '1' and grandfather of '6'.
There are total of two fathers, one mother and three brothers in the family.
Who are sons of '4' ?
(a) 1 and 2 (b) 2 and 3 (c) 6 and 2 (d) 1 and 6

13. Ram and Shyam run a race between points A and B, 5 km apart. Ram starts at 9 a.m. from A at a speed of 5 km/hr, reaches B, and returns to A at the same speed. Shyam starts at 9:45 a.m. from A at a speed of 10 km/hr, reaches B and comes back to A at the same speed.
At what time does Shyam overtake Ram?
(a) 10:20 a.m. (b) 10:30 a.m. (c) 10:40 a.m. (d) 10:50 a.m.

14. A rectangular tank 25 cm long and 20 cm wide contains water to a depth of 5 cm. A metal cube of side 10 cm is placed in the tank so that one face of the cube rests on the bottom of the tank. Find how many litres of water must be poured into the tank so as to just cover the cube ?
(a) 1 litre (b) 1.5 litre (c) 2 litre (d) 2.5 litre

15. A and B undertake to do a work for ` 56. A can do it alone in 7 days and B in 8 days. If with the assistance of a boy they finish the work in 3 days, then the boy gets how much rupees ?
(a) 11 (b) 15 (c) 14 (d) 16

16. The question below consist of a question followed by two statements labelled as (1) and (2). You have to decide if these statements are sufficient to answer the question. Give answer:
What is the value of $(x-y)$?
- Statement-(1) : $x-y=y-x$
- Statement-(2) : $(x-y)=(x^2-y^2)$

(a) If statement (1) alone is sufficient to answer the question, but statement (2) alone is not sufficient to answer the question.
(b) If statement (2) alone is sufficient to answer the question, but statement (1) alone is not sufficient to answer the question.
(c) If you can get the answer from (1) and (2) together although neither statement by itself suffices.
(d) If statement (1) alone is sufficient and statement (2), too, alone is sufficient.

17. What should come next following the same pattern in place of question mark (?) ?

2	10	30	68	?

(a) 95 (b) 130 (c) 110 (d) 120

18. ABCD is a parallelogram, BC is produced to Q such that BC = CQ then which of the following is correct?
(a) $\text{ar}(\Delta ABC)=\text{ar}(\Delta DCQ)$ (b) $\text{ar}(\Delta ABC)>\text{ar}(\Delta DCQ)$
(c) $\text{ar}(\Delta ABC)<\text{ar}(\Delta DCQ)$ (d) $\text{ar}(\Delta ABC)\neq\text{ar}(\Delta DCQ)$

19. Seema starts walking facing the rising Sun and walks for 30 m and takes a right turn and walks 10 m. Again she takes a right turn and walks for 25 m and finally after taking a left turn she walks for 2 m more. How far is she from the starting point ?

(a) 25 m (b) 12 m (c) 10 m (d) 13 m

20. In a certain code language if 'SNAKE' is coded as 'OTBFL', 'DELHI' is coded as 'FEMJI'. What should be the code for 'SPEAK' ?

(a) QREJP (b) QTFLC (c) QTFLB (d) QRFBL

PART – B

21. A symmetrical top molecule with moments of inertia $I_x = I_y$ and I_z in the body axes frame is described by the Hamiltonian

$$\hat{H} = \frac{1}{2I_x}\left(\hat{L}_x^2 + \hat{L}_y^2\right) + \frac{1}{2I_z}\hat{L}_z^2$$

The expectation value of $L_x + L_y + L_z$ for any eigenstate of the Hamiltonian, will be (Symbols have their usual meanings)

(a) 0 (b) $m\hbar$ (c) $-m\hbar$ (d) $m\frac{\hbar}{2}$

22. Consider a particle with mass m in a one-dimensional harmonic oscillator potential

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

At time t = 0, the normalized wavefunction of the particle is given as following:

$$\psi(x) = \left(\frac{1}{\pi\sigma^2}\right)^{1/4} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \text{where } \sigma^2 \neq \frac{\hbar}{m\omega}$$

The probability that the momentum of the particle (at t > 0) will be positive, is

(a) 0 (b) 1/4 (c) 1/2 (d) 1

23. A particle of mass m is moving along x-axis under the influence of the potential $V(x)$. The unnormalized wave function of the particle, in an eigenstate of the Hamiltonian of the system is given as following:

$$\psi(x) = \exp\left(-\alpha^2 x^4 / 4\right)$$

The energy of the particle in the given state, is found to be $\frac{\hbar^2\alpha^2}{m}$. The potential $V(x)$ will be of the form

(a) $Ax^6 + Bx^2 + C$ (b) $Ax^8 + Bx^4 + C$ (c) $Ax^6 + Bx^3 + C$ (d) $Ax^2 + C$

24. A particle of mass m is confined to move in a potential

$$V(x) = \begin{cases} 0 & \text{for } 0 \le x \le L \\ \infty & \text{otherwise} \end{cases}$$

The wave function of the particle at time t = 0 is given by

$$\psi(x,0) = \sum_{p=1}^{n} \sqrt{\frac{2}{nL}} \sin\frac{p\pi x}{L}$$

The average energy of the particle in the given state, is

(a) $\frac{(n+1)(2n+1)\pi^2\hbar^2}{12mL^2}$ (b) $\frac{(n+1)(2n+1)\pi^2\hbar^2}{6mL^2}$

(c) $\frac{(2n+1)\pi^2\hbar^2}{12mL^2}$ (d) $\frac{(n+1)(2n+3)\pi^2\hbar^2}{12mL^2}$

25. Consider the following initial value problem:

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0 \quad \text{with} \quad y(0)=4 \text{ and } y'(0)=-5$$

If we plot the solution $y(x)$ as a function of x for $x \geq 0$, then which of the following statements is CORRECT for $y(x)$?

(a) y increases with increament in x
(b) y decreases with increament in x
(c) y first increases then decreases with increament in x
(d) y first decreases then increases with increament in x

26. Consider the following real-valued function:

$$f(x)=\begin{cases}\pi+x & \text{for } -\pi<x\leq 0\\ \pi-x & \text{for } 0\leq x<\pi\end{cases} \quad ; \quad f(x+2\pi)=f(x)$$

The Fourier Series expansion of the function can be expressed as following:

$$f(x)=a_0+\sum_{n=1}^{\infty}a_n\cos(nx)+\sum_{n=1}^{\infty}b_n\sin(nx)$$

Which of the following statements is INCORRECT?

(a) $a_0=\frac{\pi}{2}$ (b) $b_n=0$ for all n (c) $a_n\neq 0$ for all n (d) $a_n\neq 0$ for odd n

27. Let λ_1, λ_2 and λ_3 be the eigenvalues of the following matrix:

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

Then the value of $\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1$ is equal to

(a) 21 (b) 45 (c) –21 (d) –45

28. Let A and B be two independent random variables, each of which follow normal distribution with means 2, 3 and standard deviation 0.5, 0.75 respectively. Then, the sum $3A+2B$ will follow

(a) normal distribution with mean 12 and standard deviation 2.12
(b) normal distribution with mean 12 and standard deviation 3
(c) normal distribution with mean 2.5 and standard deviation 2.12
(d) normal distribution with mean 2.5 and standard deviation 3

29. If the Fourier Transform of the following function:

$$f(x)=\begin{cases}1 & \text{for } |x|<a\\ 0 & \text{for } |x|>a\end{cases}$$

is given as $F(s)$, then the value of the integral

$$\int_{-\infty}^{\infty} \frac{\sin(as)\cos(sx)}{s} ds$$

will be

(a) $\begin{cases} 2\pi & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$ (b) $\begin{cases} \pi & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$ (c) $\begin{cases} \frac{\pi}{2} & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$ (d) $\begin{cases} \frac{\pi}{2} & \text{for } |x| < a \\ \pi & \text{for } |x| > a \end{cases}$

30. Lagrangian of a particle is $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - mgz$. Which of the following is conserved ?

(a) p_z only (b) L_z only (c) p_z, L_z and energy (d) p_x, p_y, L_z and energy

31. A ball of mass m is thrown up with speed u. If in addition to gravity (g) a constant drag force F also acts on it with what speed will it return to its initial point ?

(a) $u\sqrt{\frac{g+\frac{F}{m}}{g-\frac{F}{m}}}$ (b) $u\sqrt{\frac{g-\frac{F}{m}}{g+\frac{F}{m}}}$ (c) $u\left(\frac{g-\frac{F}{m}}{g+\frac{F}{m}}\right)$ (d) $u\left(\frac{g+\frac{F}{m}}{g-\frac{F}{m}}\right)$

32. A large plane sheet has mass density σ. Gravitational field of the sheet is

(a) $2\pi G\sigma$ (b) $4\pi G\sigma$ (c) $\pi G\sigma$ (d) $\frac{\pi G\sigma}{2}$

33. A transmitter on a spaceship that is going directly away from earth with constant speed emits a pulse, that after being reflected from earth is received back by the transmitter on the spaceship. If the frequency of the received signal is one half that of the emitted one, the speed of the space ship w.r.t. earth is,

(a) $\frac{c}{2}$ (b) $\frac{c}{3}$ (d) $\frac{c}{4}$ (d) $\frac{c}{2\sqrt{2}}$

34. A system at temperature T, consists of three single particle energy levels $0, \in, 2\in$ with degeneracies 2, 1, 3. When the population of second excited state is compared with that of the ground state, then the result is

(a) $\frac{3e^{-2\beta\in}}{2+e^{-\beta\in}+3e^{-2\beta\in}}$ (b) $3e^{-2\beta\in}$ (c) $\frac{1}{3}e^{-2\beta\in}$ (d) $\frac{e^{-2\beta\in}}{2+e^{-\beta\in}+3e^{-2\beta\in}}$

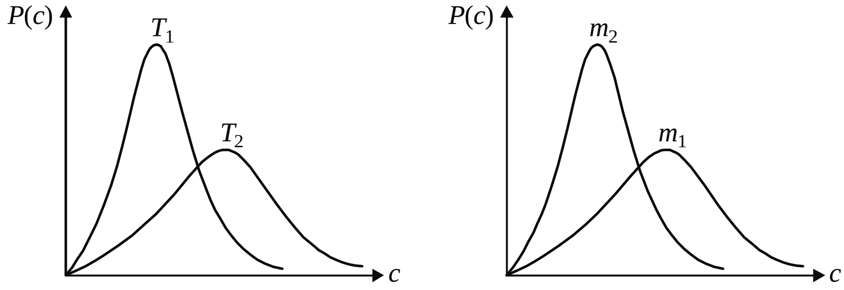
35. For a hypothetical thermodynamical system, the equation of state is $PV^{9/7} = aT$; the number of microstates of the system depends on V as

(a) $\exp\left(-\frac{9}{7}\frac{aV^{-7/9}}{k_B}\right)$ (b) $\exp\left(-\frac{7}{2}\frac{aV^{-2/7}}{k_B}\right)$ (c) $\exp\left(\frac{aV^{-2/7}}{k_B}\right)$ (d) $\exp\left(-\frac{7}{2}\frac{aV^{-7/2}}{k_B}\right)$

36. A system in thermal equilibrium at temperature T has single particle energy levels $\in, 2\in$ with degeneracies 2, 1 respectively. What will be the average value of the square of energy of the system, if the particles are distinguishable and $\beta = \frac{1}{kT}$?

(a) $\frac{\in\left(1+e^{-\beta\in}\right)}{1+0.5\,e^{-\beta\in}}$ (b) $2\in^2\left(\frac{e^{-\beta\in}+e^{-2\beta\in}}{2e^{-\beta\in}+e^{-\beta\in}}\right)$ (c) $\frac{\in^2\left(1+2e^{-\beta\in}\right)}{1+e^{-\beta\in}}$ (d) $\frac{\in^2\left(1+2e^{-\beta\in}\right)}{1+0.5\,e^{-\beta\in}}$

37. The curves (i) and (ii) are the Maxwell speed distributions w.r.t. speed for two particles. In (i) there are two isotherms at temperatures T_1 and T_2 for fixed mass. In (ii) there are two curves each for masses m_1 and m_2 at a fixed temperature. The conclusions that can be drawn from these curves respectively are:



(i) (ii)

(a) $T_1 < T_2$ and $m_1 > m_2$ (b) $T_1 > T_2$ and $m_1 > m_2$

(c) $T_1 > T_2$ and $m_1 < m_2$ (d) $T_1 < T_2$ and $m_1 < m_2$

38. In a non-magnetic material, the magnetic field intensity associated with an electromagnetic wave is given by

$$\vec{H} = \hat{y}\, 30\cos\left(2\pi\times 10^8 t - 6x\right)\ mA/m$$

The time-average power crossing the surface $x = 1, 0 < y < 2$ and $0 < z < 3\ m$ is (in watts)

(a) $0.36\,\pi^2$ (b) $0.036\,\pi^2$ (c) $36\,\pi^2$ (d) $360\,\pi^2$

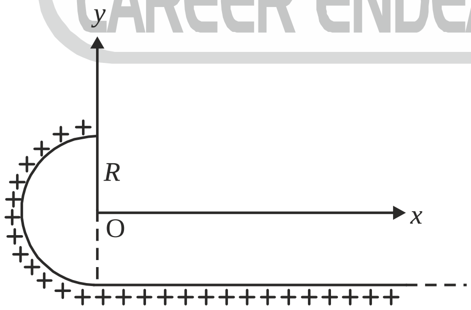
39. The electric field of an electromagnetic wave in rest frame (S) is given by $\vec{E} = \hat{y}\, E_0 e^{i(\omega t - kx)}$. The same wave is observed from an inertial frame S' moving in the x-direction with velocity $\frac{c}{2}$ with respect to the S frame. The intensity of light in the moving frame (S') is

(a) $\frac{1}{2}\varepsilon_0 c E_0^2$ (b) $\frac{1}{4}\varepsilon_0 c E_0^2$ (c) $\frac{1}{6}\varepsilon_0 c E_0^2$ (d) $\frac{1}{8}\varepsilon_0 c E_0^2$

40. Consider the far field diffraction pattern of a double-slits. The fifteen intereference bright fringes fall within the central diffraction maxima. If each slit is 0.25 mm wide, then the width of the opaque part between two slits is

(a) 1.75 mm (b) 1.88 mm (c) 2 mm (d) 0.25 mm

41. In the given figure below we have shown a filament that carries a linear charge density λ. The electric field at the origin is

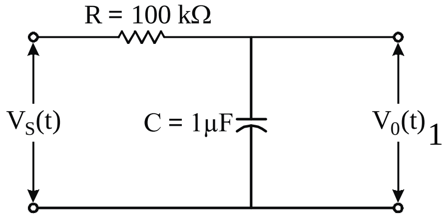


(a) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(\hat{i} - \hat{j}\right)$ (b) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(\hat{i} + \hat{j}\right)$ (c) $\frac{\lambda}{2\pi\varepsilon_0 R}\left(\hat{i} - \hat{j}\right)$ (d) $\frac{\lambda}{2\pi\varepsilon_0 R}\left(\hat{i} + \hat{j}\right)$

42. An analog voltage of 3.41V is converted into eight-bit digital form by an A/D converter with a reference voltage of 5V. The digital output is

(a) 1001 1001 (b) 1111 0001 (c) 1011 0111 (d) 1010 1110

43. For the passive RC low pass filter shown below:

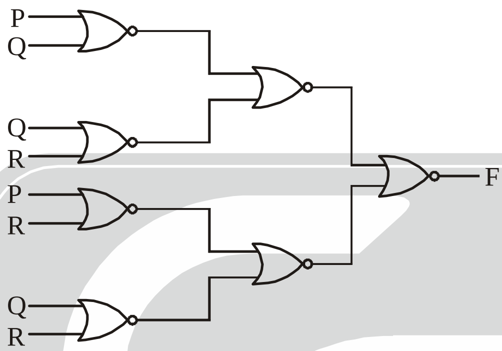


$V_S(t) = \cos t + \cos 100t$ and $V_0(t) = \alpha\cos(t+\theta) + \beta\cos(100t+\phi)$, (where α, β, θ and ϕ are constants)

The value of $|\alpha/\beta|$ is nearly equal to

(a) 1 (b) 10 (c) 100 (d) 1000

44. What is the boolean expression for the output F of the combinational logic circuit of NOR gates given below?



(a) $\overline{Q+R}$ (b) $\overline{P+Q}$ (c) $\overline{P+R}$ (d) $\overline{P+Q+R}$

45. Suppose the moon is rotating around the earth in circular orbit. If the rms error of the linear velocity of the moon, mass of the moon and radius of the orbit are 5%, $\sqrt{2}$ % and 3% respectively, then the rms error of the angular momentum of the the moon is

(a) 6% (b) $\left(8+\sqrt{2}\right)$% (c) 10% (d) 11%

PART – C

46. Consider a particle of spin 1/2. At time t = 0, the particle is in an eigenstate of $\hat{S}_x$ which corresponds to the eigenvalue $-\frac{\hbar}{2}$. The particle is in a magnetic field and its Hamiltonian is given as $\hat{H} = \frac{eB}{mc}\hat{S}_z$. At $t = t_0$, if x-component of the spin angular momentum of the particle is measured, then the probability of getting $-\frac{\hbar}{2}$ and expectation value of $\hat{S}_x$ are respectively

(a) 1 and $\frac{-\hbar}{2}$

(b) $\frac{1}{2}$ and 0

(c) $\cos^2\left(\frac{eBt_0}{2mc}\right), -\frac{\hbar}{2}\cos\left(\frac{eBt_0}{mc}\right)$

(d) $\cos^2\left(\frac{eBt_0}{mc}\right), -\frac{\hbar}{2}\cos\left(\frac{eBt_0}{2mc}\right)$

47. Particles are scattered from the potential $V(r)=\frac{g}{r^2}$, where g is positive constant and the phase shift δ_l corresponding to the l^{th} partial wave is found to be

$$\delta_l=\frac{\pi}{2}\left[l+\frac{1}{2}-\sqrt{\left(l+\frac{1}{2}\right)^2+\frac{2\mu g}{\hbar^2}}\right]$$

For $\frac{2\mu g}{\hbar^2}<<1$, the differential cross section $\frac{d\sigma}{d\theta}$ can be found to be (Symbols have their usual meanings)

(a) $\frac{\pi^2\mu g^2}{8\hbar^2 E}\frac{1}{\sin^2(\theta/2)}$ (b) $\frac{\pi^2\mu g^2}{8\hbar^2 E}\frac{1}{\sin^4(\theta/2)}$ (c) $\frac{\pi^3\mu g^2}{2\hbar^2 E}\cot\frac{\theta}{2}$ (d) $\frac{\pi^3\mu g^2}{2\hbar^2 E}\cot^2\frac{\theta}{2}$

48. The conditions for (i) applicability of the WKB approximation in the case of attractive Coulomb potential $V(r)=-\frac{\alpha}{r^2}$ and (ii) applicability of the condition obtained in (i) in case of Bohr model of the Hydrogen atom, are respectively (Symbols have their usual meanings)

(a) $r>>\frac{\hbar^2}{m\alpha},\quad n>>1$ (b) $r>>\frac{\hbar^2}{m\alpha},\quad n>0$ (c) $r<<\frac{\hbar^2}{m\alpha},\quad n>0$ (d) $r<<\frac{\hbar^2}{m\alpha},\quad n>>1$

49. A particle of mass 'm' having charge 'e', confined to a three dimensional cubical box of side '$2a$', is acted upon by an electric field

$$E=E_0e^{-\alpha t}\,(t>0)\,[\text{where } \alpha \text{ is a positive constant}]$$

along x-direction. The probability that the particle which is ground state at $t=0$, will make a transition to the first excited state (2,1,1) after a long time, is

(a) $\left(\frac{32aeE_0}{9\pi^2\hbar}\right)^2\frac{1}{\alpha^2+\left(\frac{3\pi^2\hbar}{8ma^2}\right)^2}$ (b) $\left(\frac{32aeE_0}{9\pi^2\hbar}\right)^2\frac{1}{\alpha^2+\left(\frac{3\pi^2\hbar}{2ma^2}\right)^2}$

(c) $\left(\frac{24aeE_0}{5\pi^2\hbar}\right)^2\frac{1}{\alpha^2+\left(\frac{3\pi^2\hbar}{2ma^2}\right)^2}$ (d) $\left(\frac{24aeE_0}{5\pi^2\hbar}\right)^2\frac{1}{\alpha^2+\left(\frac{3\pi^2\hbar}{4ma^2}\right)^2}$

50. The value of the real improper integral: $\int_0^\infty \frac{x^2}{x^6+1}dx$ will be

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $-\frac{\pi}{3}$ (d) $-\frac{\pi}{6}$

51. Consider a real valued function $\psi_n(x)$ defined as following:

$$\psi_n(x)=\exp\left(-\frac{x^2}{2}\right)H_n(x)$$

where $H_n(x)$ represents Hermite polynomial of order 'n'. The value of the following integral

$$\int_{-\infty}^{\infty}\psi_m(x)\psi_n'(x)\,dx$$

will be

[Given: $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$, $H_n^{'}(x) = 2nH_{n-1}(x)$]

(a) $2^{n-1}(n-1)!\sqrt{\pi}\delta_{m,(n-1)} - 2^{n+1}(n+1)!\sqrt{\pi}\delta_{m,(n+1)}$

(b) $2^{n-1}n!\sqrt{\pi}\delta_{m,(n-1)} - 2^{n+1}(n+1)!\sqrt{\pi}\delta_{m,(n+1)}$

(c) $2^{n-1}n!\sqrt{\pi}\delta_{m,(n-1)} + 2^{n}(n+1)!\sqrt{\pi}\delta_{m,(n+1)}$

(d) $2^{n-1}n!\sqrt{\pi}\delta_{m,(n-1)} - 2^{n}(n+1)!\sqrt{\pi}\delta_{m,(n+1)}$

52. Let G be the set of all matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$ where x is a non-zero real number. Which of the following statements is CORRECT?

(a) G does not form a group under matrix multiplication as identity element of the group does not exist

(b) G does not form a group under matrix multiplication as inverse element of elements of the group does not exist

(c) G forms a group under matrix multiplication and identity element of the group is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(d) G forms a group under matrix multiplication and inverse element of each element of the group will be of the form $\begin{bmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{bmatrix}$.

53. Consider the following data:

$x =$	1	2	4
$f(x) =$	1	7	61

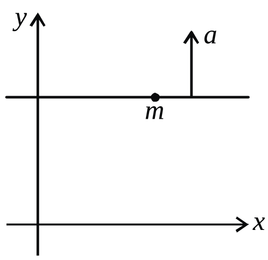
If the function $f(x)$ is obtained using Lagrange's interpolation technique, then $f(10)$ is found to be

(a) 559 (b) 623 (c) 789 (d) 857

54. A particle P of unit mass moves under a force field $\vec{F} = -\frac{\gamma}{r^2}\hat{r}$ (where $\gamma > 0$). Initially P is at a point C, a distance c from O, when it's projected with speed $(\gamma/c)^{1/2}$ in a direction making an angle $\theta = 60°$ with the line OC. The ratio of maximum distance to minimum distance of the particle from the centre 'O' is,

(a) 1 (b) 2 (c) 3 (d) ∞

55. A bead of mass 'm' is constrained to move on a horizontal rod that is accelerated vertically with constant acceleration 'a' as shown in the figure. Which of the following is TRUE about the Hamiltonian H of the system?



(a) H is equal to total energy of the system and is conserved.
(b) H is not equal to the total energy of the system but is conserved.
(c) H is equal to the total energy of the system but is not conserved.
(d) H is not equal to the total energy of the system and is not conserved.

56. Evalute the value of the Poisson bracket:

$$\left\{\vec{a}\cdot\vec{r},\left\{\vec{b}\cdot\vec{p},\vec{L}\right\}\right\}+\left\{\vec{b}\cdot\vec{p},\left\{\vec{L},\vec{a}\cdot\vec{r}\right\}\right\}$$

where $\vec{a}$ and $\vec{b}$ are two constant vectors and symbols have their usual meanings.

(a) $\vec{a}\times\vec{b}$ (b) $\vec{b}\times\vec{a}$ (c) $2(\vec{a}\times\vec{b})$ (d) zero

57. A particle of rest mass m_0 is acted upon by a constant force F. If the particle was initially at rest, then after what time kinetic energy of the particle will be equal to rest mass energy ?

(a) $\dfrac{\sqrt{3}\,m_0 c}{F}$ (b) $\dfrac{m_0 c}{\sqrt{3}\,F}$ (c) $\dfrac{m_0 c}{F}$ (d) $\dfrac{2m_0 c}{F}$

58. Consider a system with equation of state $PV = aT^4$, where a is a constant, the internal energy of the system is given by $U = bT^m \ln\left(\dfrac{V}{V_0}\right) + g(T)$, where b, m and V_0 are constants, $g(T)$ is a function of temperature only, all other symbols have their usual meaning. The values of b and m are:

(a) $3a, 4$ (b) $a, 4$ (c) $4a, 3$ (d) $4a, 1$

59. A system of 2-D ideal gas consists of N indistinguishable particles of mass m at temperature T. The Hamiltonian for energy of a particle is given by $H = \dfrac{p^2}{2m} - \varepsilon'$, where ε' is some kind of surface energy per particle and $p = \left(p_x, p_y\right)$. If the surface area in position space is a. The chemical potential of the system is

(a) $-Nk_BT\left[\ln\left(\dfrac{2\pi mk_BTa}{h^2}\right)+\dfrac{\varepsilon'}{k_BT}\right]$ (b) $k_BT\left[\ln\left(\dfrac{2\pi mk_BTa}{h^2}\right)+\dfrac{\varepsilon'}{kT}\right]$

(c) $-k_BT\left[\ln\left(\dfrac{2\pi mk_BTa}{Nh^2}\right)+\dfrac{\varepsilon'}{kT}\right]$ (d) $k_BT\left[\ln\left(\dfrac{2\pi mk_BTa}{h^2}\right)\right]$

60. A system consisting of particles that obey Maxwell Boltzmann satistics and occupy single particle level is in thermal equilibrium at temperature T. If the non-degenerate energy levels have population as shown below, then the approximate mean value of absolute temperature of the system in terms of $1/k_B$ will be

Energy (eV)	Population %
30	10
20	40
10	50

[Use ln 2 = 0.69, ln 5 = 1.6]

(a) $27/k_B$ (b) $12.5/k_B$ (c) $21.5/k_B$ (d) $7/k_B$

61. Consider a long hollow solenoid of radius R carries an alternating current $I(t) = I_0 \sin(\omega t)$. If the induced electric field inside and outside is given by

$E \propto r^n$ for $r < R$

$E \propto r^m$ for $r > R$

The value of m and n are

(a) $m=-2; n=-1$ (b) $m=-1; n=1$ (c) $m=-1; n=2$ (d) $m=0; n=1$

62. The permitivity tensor of a uniaxial anisotropic medium, in the standard Cartentian basis, is

$$\begin{pmatrix} 9\varepsilon_0 & 0 & 0 \\ 0 & 9\varepsilon_0 & 0 \\ 0 & 0 & 16\varepsilon_0 \end{pmatrix}$$

where, ε_0 is permitivity of free space. We are using this medium as a wave plate. If a circular polarized light propagating along x-axis through this plate, then the minimum thickness of the medium for which the emerging light wil be linear polarized light, is (the wavelength of light in free space is 600 nm)

(a) 300 nm (b) 200 nm (c) 150 nm (d) 100 nm

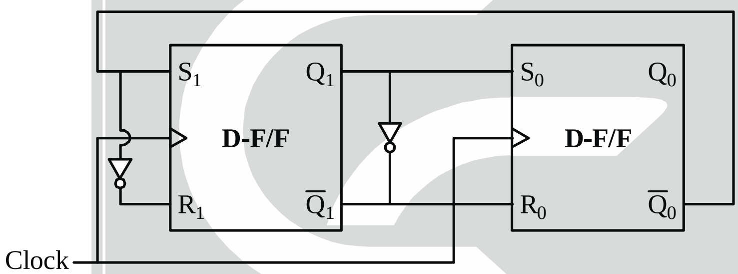
63. The electric field of radiation emitted by an antenna (in free space) in spherical polar coordinate is given by

$$\vec{E} = \frac{5\sin 2\theta}{r}\sin(\omega t - kr)\hat{\theta} \ V/m.$$

The average power radiated by the antenna in far field zone is (in watts). (Given: $\int_0^{\pi} \sin^2 2\beta \sin\beta d\beta = \frac{16}{15}$]

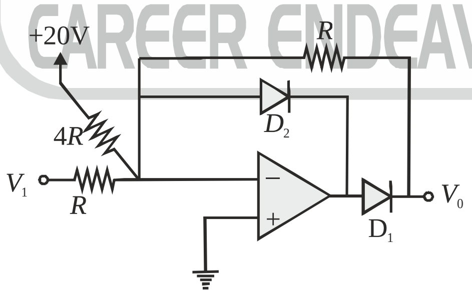
(a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{3}$ (d) $\frac{16}{9}$

64. Determine the value of following counter after 729 pulses; initial value is given as 11

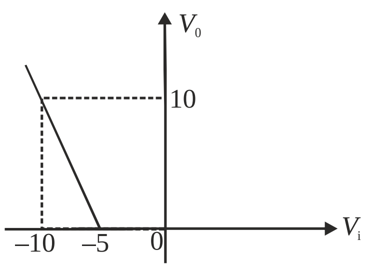


(a) 0 1 (b) 0 0 (c) 1 1 (d) 1 0

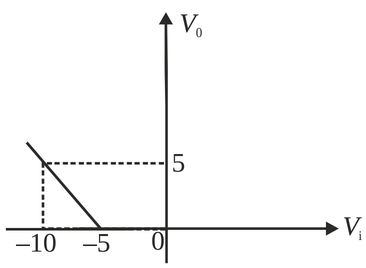
65. The transfer characteristic for the precission rectifier circuit shown below is (assume ideal Op-Amp and practical diodes)

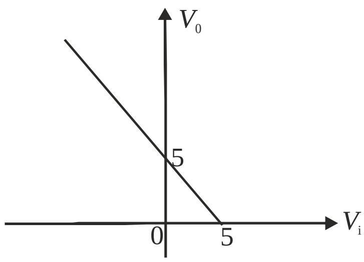


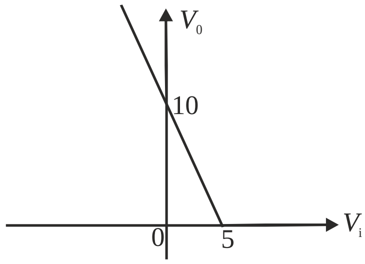
(a)



(b)



(c) 

(d) 

66. Consider an n-channel JFET operating in the saturation region. Its pinch off voltage is –4 volts. For some value of Gate Source Voltage (V_{GS}) the drain current (I_D) becomes $I_{DSS}/2$, the corresponding value of $V_{DS(sat)}$ is

(a) 4 volts (b) –4 volts (c) 2.83 votls (d) –2.83 volts

67. The fundamental and first overtone frequencies of NO molecule are centered at 1876.06 cm^{-1} and 3724.2 cm^{-1}, respectively. The equilibrium vibration frequency and anharmonicity constant of NO molecule are, respectively.

(a) 1876.06 cm^{-1} and 7.33×10^{-2} (b) 7.33×10^{-2} and 1876.06 cm^{-1}

(c) 1903.98 cm^{-1} and 7.33×10^{-3} (d) 1903.98 cm^{-1} and 7.33×10^{-1}

68. A state is denoted by ${}^4D_{5/2}$. The minimum number of electrons which could give rise to this state and a possible electronic configuration are, respectively

(a) 3 and s^1p^2 (b) 2 and p^1d^1 (c) 5 and p^2d^3 (d) 4 and p^2d^2

69. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.99$. The mirrors are separated by a distance $d = 30$ cm and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha_c = 0$. If the wavelength of laser is 632.8 nm, the passive cavity line width $\Delta\nu_p$ and passive cavity life time t_c are, respectively

(a) $0.2\,\mu$s and 8 MHz (b) $2\,\mu$s and 8 MHz (c) $2\,\mu$s and 0.8 MHz (d) $0.2\,\mu$s and 0.8 MHz

70. Consider the following statements:

(i) If a particle X has Isospin $I = \frac{3}{2}$, Baryon number $B = 1$, Strangeness number $S = 0$, then the possible values of electric charges of X are +2, +1, 0 and –1.

(ii) If the reaction $\pi^+ + X \to \Sigma^+ + K^+$ is governed by strong interaction, then the third component of Isospin, Isospin, Baryon number and Strangeness number of X are respectively $\frac{1}{2}, \frac{1}{2}, 1$ and 0.

(iii) The quark content of K^-, Σ^+ and Ξ^0 are $s\bar{u}$, uus and uss respectively.

Which of the above statements are correct ?

(a) Only (i) and (ii) (b) Only (i) and (iii) (c) Only (ii) and (iii) (d) All (i), (ii) and (iii)

71. The electromagnetic radiation emitted when a nucleus makes a transition from spin-parity state $J^p = 1^+$ to $J^p = 3^+$ are

(a) M_2, E_2, M_4 (b) M_2, M_3, E_4 (c) E_2, M_3, E_4 (d) M_1, E_2, M_3

72. Choose the incorrect option from the following:

(a) Deuteron has no excited state and has Isospin = 0.

(b) If the single particle energy difference between the *d* orbitals (i.e., $1d_{5/2}$ and $1d_{3/2}$) of the nucleus $^{114}_{50}\text{Sn}$ is 10 MeV, then the energy difference between the states in its $1f$ orbital is 14 MeV.

(c) If τ decays at rest given by $\tau^- \to \pi^- + \nu_\tau$, then the velocity of π^- is equal to $\frac{\left(m_\tau^2 + m_\pi^2\right)c}{\left(m_\tau^2 - m_\pi^2\right)}$ in terms of rest masses m_τ and m_π.

(d) If the radius of a $^{64}_{29}\text{Cu}$ nucleus is *R*, then the radius of $^{27}_{12}\text{Mg}$ nucleus is equal to $3R/4$.

73. The effective mass of electron in silicon (Si) semiconductor is about $0.2m_0$, where m_0 is free electron mass. The static dielectric constant of Si is 11.7. If Bohr radius of the ground state of Hydrogen atom is 0.53 Å, the Bohr radius of donor in silicon is, approximately

(a) 60 Å (b) 80 Å (c) 120 Å (d) 30 Å

74. The dispersion relation of a magnon in a ferromagnetic cubic lattice with nearest neighbour interaction (for $ka \ll 1$) is $\hbar\omega = (2Jsa^2)k^2$, where J, s, k, a is exchange integral, spin quantum number, anisotropy constant, and lattice constant respectively. The density of modes for magnon in the ferromagnetic is proportional to

(a) ω^2 (b) $\omega^{1/2}$ (c) ω^3 (d) ω

75. The dispersion relation for electron in a semiconductor is given by

$$\varepsilon(k) = \varepsilon_0 - 2A\alpha\left(1 - \frac{k^2a^2}{2}\right)e^{-\alpha a}$$

where ε_0, A, α are constants and *a* is lattice constant of the semiconductor. The effective mass m^* of electron at $\alpha = \frac{1}{a}$ is

(a) $\frac{\hbar^2 e}{2Aa}$ (b) $\frac{\hbar^2 e}{Aa}$ (c) $\frac{\hbar^2 e}{4Aa}$ (d) $\frac{\hbar^2 e}{Aa^2}$

All the very Best for NTA-NET “15th Dec. 2019” Exam

Space for rough work

Date : 07-12-2019

PHYSICAL SCIENCES

TEST SERIES-C

ANSWER KEY

PART-A

1. (b)	2. (b)	3. (d)	4. (a)	5. (c)	6. (c)	7. (d)
8. (a)	9. (d)	10. (b)	11. (b)	12. (a)	13. (b)	14. (b)
15. (a)	16. (a)	17. (b)	18. (a)	19. (d)	20. (c)	

PART-B

21. (b)	22. (c)	23. (a)	24. (a)	25. (d)	26. (c)	27. (c)
28. (a)	29. (b)	30. (d)	31. (b)	32. (a)	33. (b)	34. (b)
35. (b)	36. (d)	37. (d)	38. (b)	39. (c)	40. (a)	41. (b)
42. (d)	43. (b)	44. (a)	45. (a)			

PART-C

46. (c)	47. (c)	48. (a)	49. (a)	50. (b)	51. (d)	52. (d)
53. (a)	54. (c)	55. (d)	56. (d)	57. (a)	58. (a)	59. (c)
60. (c)	61. (b)	62. (c)	63. (a)	64. (a)	65. (b)	66. (c)
67. (c)	68. (a)	69. (d)	70. (d)	71. (c)	72. (c)	73. (d)
74. (b)	75. (a)					

PART – A

1. Total score of five players = 36 × 5 = 180 runs.

D – E = 5 ; D = E + 5

A – E = 8 ; A = E + 8

B = D + E

B + C = 107

Now, A + B + C + D + E = 180

or E + 8 + 107 + E + 5 + E = 180

or 3E = 60

or E = 20

Therefore, average score excluding E = $\frac{180-20}{4}=\frac{160}{4}=40$

Correct option is (b)

2. Total number of balls = 5 + 4 + 3 = 12

$$n(S)={}^{12}C_3=\frac{12\times 11\times 10}{1\times 2\times 3}=220$$

i.e., 3 marbles out of 12 marbles can be drawn in 220 ways.

If all the three marbles are of the same colour, it can be done in

${}^{5}C_3+{}^{4}C_3+{}^{3}C_3=10+4+1=15$ ways

Now, probability(all the 3 marbles of the same colour) + probability(all the 3 marbles are not of the same colour) = 1.

Therefore, P(all the 3 marbles are not of the same colour)

$$=1-\frac{15}{220}=\frac{205}{220}=\frac{41}{44}$$

Correct option is (b)

3. Cost price of 20 dozen notebooks $=20\times 48=₹\,960$

Selling price of 8 dozen notebooks $=₹\,8\times 48\left(110/100\right)$

Selling price of 12 dozen notebooks $=₹\,12\times 48\left(120/100\right)$

Therefore, total selling price $=₹\,\frac{2112}{5}+₹\,\frac{3456}{5}=₹\,\frac{5568}{5}$

Profit $=\frac{5568}{5}-960=₹\,\frac{768}{5}$

Therefore, profit $=\frac{768\times 100}{5\times 960}=16\,\%$

Correct option is (d)

4. In original mixture, % of liquid B $=\frac{1}{4+1}\times 100=20\,\%$

In the resultant mixture, % of liquid B $=\frac{3}{2+3}\times 100=60\,\%$

Replacement is made by the liquid B, so the % of B in second mixture = 100 %.

Then, by the method of alligation:

20 %   100 %

60 %

40 %   40 %

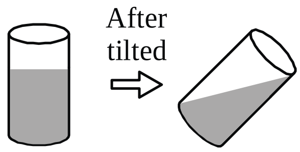
Therefore, ratio in which first and second mixtures should be added is 1 : 1.

Therefore, total mixture = 10 + 10 = 20 litres, and liquid A $=\frac{20}{5}\times 4=16$ litres .

Correct option is (a)

5. Let volume $= x$ litre

Therefore, water present $=\frac{4x}{5}$ litre

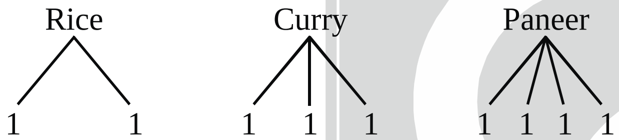


Amount of water spoiled $=\frac{4x}{5}-\frac{x}{2}=\frac{8x-\sqrt{x}}{10}=\frac{3x}{10}$ litre

As per condition $\frac{3x}{10}=30$ or $x=100$

Correct option is (c)

6.



L.C.M. of 2, 3, 4 = 12.

Therefore, for 12 people rice pot $=\frac{12}{2}=6$

Therefore, for 12 people curry pot $=\frac{12}{3}=4$

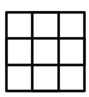
Therefore, for 12 people paneer pot $=\frac{12}{4}=3$

Therefore, for each set of 12 people.
Total of = (6 + 4 + 3) = 13 pot used.
Therefore, 13 pot for 12 people.
Therefore, 1 pot for 12/13 people.

Therefore, 65 pot for $\frac{12}{13}\times 65=60$ people.

Correct option is (c)

7. In the inner squares, 14 squares are there and 4 extras in sides small 4 in corners and 4 by merging and one biggest square.



So, total (14 + 4 + 4 + 4 + 1) = 27.
Correct option is (d)

8.

$$\overset{1}{A}\quad \overset{5}{E}=\frac{5+1}{2}=3$$

$+2$

$$\overset{3}{C}\quad \overset{17}{Q}=\frac{3+17}{2}=10$$

$+3$

$$\overset{6}{F}\quad \overset{12}{L}=\frac{6+12}{2}=9$$

$+4$

$$\overset{10}{J}\quad \overset{20}{T}=\frac{10+20}{2}=15$$

Correct option is (a)

9. (C + D) marks in Physics = 70 + 35 = 105

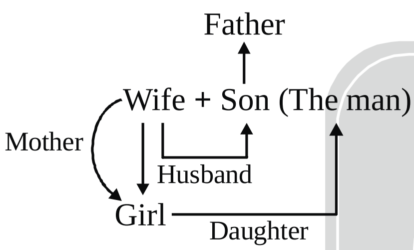
(A + B) marks in Chemistry = 65 + 60 = 125

Therefore, difference = 20

Answer = $\frac{20}{125}\times 100=16\%$

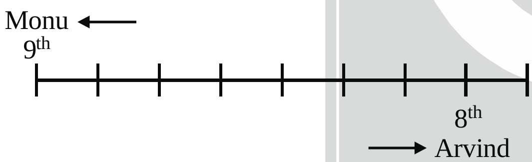
Correct option is (d)

10.



Correct option is (b)

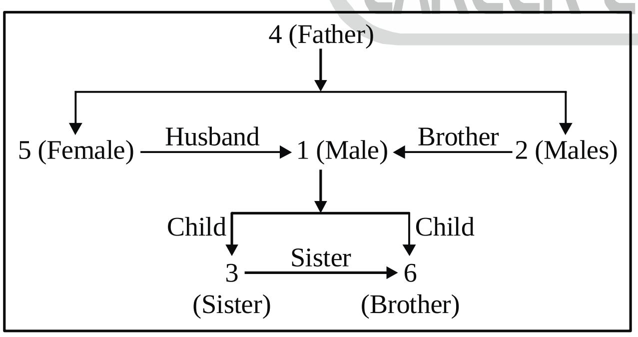
11.



Therefore, minimum students between them is 6.

Correct option is (b)

12.



Hence, Therefore, 1 and 2 are son of 4

Correct option is (a)

13. Let the time at which Shyam overtakes Ram be t minutes past 10. So, distance run by both of them is the same till that moment.

$(60 + t)5 = (15 + t)10 \Rightarrow 300 + 5t = 150 + 10t \Rightarrow 5t = 150 \Rightarrow t = 30$ min.

So, at 10.30 am, Shyam overtakes Ram

Correct option is (b)

14. Volume of water $= l\,b\,h = 25\times 20\times 5 = 2500\text{ cm}^3 = 2.5$ litre

Total required volume $= 25\times 20\times 10 = 5000\text{ cm}^3 = 5$ litre

Volume of cube $= (10\text{ cm})^3 = 1000\text{ cm}^3 = 1$ litre

Now, extra water to be poured $= 5-(2.5+1)$ litre $= 1.5$ litre

Correct option is (b)

15. A's 3 day's work + B's 3 day's work + Boy's 3 day's work = 1

or, $\frac{3}{7}+\frac{3}{8}+$ Boy's 3 day's work = 1

or, Boy's 3 day's work $= 1-\left(\frac{3}{7}+\frac{3}{8}\right) = \frac{11}{56}$.

Ratio of shares $= \frac{3}{7}:\frac{3}{8}:\frac{11}{56} = \frac{3\times 56}{7}:\frac{3\times 56}{8}:\frac{11\times 56}{56} = 24:21:11$

Therefore, Boy's share $= \frac{56}{24+21+11}\times 11 =$ ` 11.

Correct option is (a)

16. The expression $x-y$ involves two unknowns. But the first equation is sufficient. To see this,
$(x-y) = y-x = -(x-y)$
$(x-y) = -(x-y)$ or $2(x-y)=0$
This implies that $x-y=0$. Since a number is equal to its negative in one and only one possible way, that is, if the number is equal to zero. Hence, either (a) or (d) is the answer. The second expression is not sufficient.
Because, $x-y = x^2-y^2 = (x-y)(x+y)(x+y-1)(x-y)=0$
This leads to two possibilities: $x-y=0$ or $x+y=1$.
In this case, the value of $(x-y)$ is not accurately determined. Hence, (a) is the answer.
Correct option is (a)

17. $1^3 + 1 = 2$

$2^3 + 2 = 10$

$3^3 + 3 = 30$

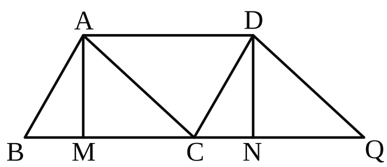
$4^3 + 4 = 68$

$5^5 + 5 = 130$

Correct option is (b)

18. Area of $\Delta ABC = \frac{1}{2}\times BC\times h$ $[AM = DN = h]$

Area of $\Delta DCQ = \frac{1}{2}\times CQ\times h = \frac{1}{2}\times BC\times h$ $[CQ = BC]$



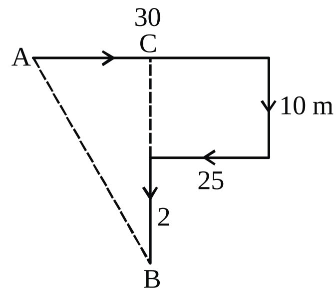
Therefore, area of both triangles are equal.
Correct option is (a)

19. AB distance $= \sqrt{\mathrm{AC}^2 + \mathrm{BC}^2}$

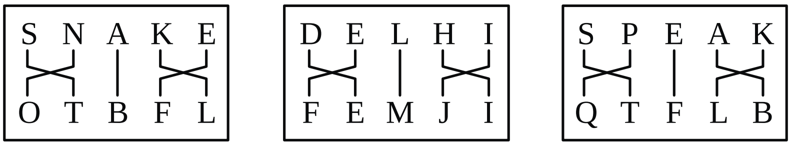
$$= \sqrt{5^2 + 12^2}$$

$$= 13 \text{ m}$$

Correct option is (d)



20.



Correct option is (c)

PART – B

21. The given Hamiltonian is,

$$\hat{H} = \frac{1}{2I_x}\left(\hat{L}_x^2 + \hat{L}_y^2\right) - \frac{1}{2I_z}\hat{L}_z^2$$

$$= \frac{\hat{L}^2 - \hat{L}_z^2}{2I_x} - \frac{1}{2I_z}\hat{L}_z^2 \qquad \left[\because\ \hat{L}_x^2 + \hat{L}_y^2 = \hat{L}^2 - \hat{L}_z^2\right]$$

$$\hat{H} = \frac{1}{2I_x}\hat{L}^2 - \frac{1}{2}\left(\frac{1}{I_x} + \frac{1}{I_z}\right)\hat{L}_z^2$$

Since, the Hamiltonian commutes with $\hat{L}^2$ and $\hat{L}_z^2$, the spherical Harmonics $\{Y_{l,m}\}$ will be the eigenstates of the system.

We know that, $\langle L_x \rangle\big|_{Y_{l,m}} = \langle L_y \rangle\big|_{Y_{l,m}} = 0$ and $\langle L_z \rangle\big|_{Y_{l,m}} = m\hbar$

Therefore, $\langle L_x + L_y + L_z \rangle\big|_{Y_{l,m}} = m\hbar$

Correct option is (b)

22. The position space wavefunction of the particle is,

$$\psi(x) = \left(\frac{1}{\pi\sigma^2}\right)^{1/4} \exp\left(-\frac{x^2}{2\sigma^2}\right), \text{ where } \sigma^2 \neq \frac{\hbar}{m\omega}$$

Since, it's Gaussian, the momemtum space wavefunction $\phi(p)$ will also be a Gaussian.
Therefore, the momentum space wavefunction will be symmetric in 'p' and the probability of finding a positive momentum will be 1/2.

Correct option is (c)

23. Given eigenstate is, $\psi(x) = \exp\left(-\frac{\alpha^2 x^4}{4}\right)$.

If $V(x)$ be the potential, then

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = \frac{\hbar^2\alpha^2}{m}\psi \qquad \ldots(1) \qquad \left(\because\ E = \frac{\hbar^2\alpha^2}{m}\right)$$

Now, $\frac{d\psi}{dx}=\frac{d}{dx}\exp\left(-\frac{\alpha^2x^4}{4}\right)=-\alpha^2x^3\exp\left(-\frac{\alpha^2x^4}{4}\right)=-\alpha^2x^3\psi$

$$\Rightarrow \frac{d^2\psi}{dx^2}=-\alpha^2\left\{x^3\frac{d\psi}{dx}+\psi\left(3x^2\right)\right\}$$

$$=-\alpha^2\left\{x^3\times\left(-\alpha^2x^3\right)+3x^2\right\}\psi$$

$$=-\alpha^2\left\{-\alpha^2x^6+3x^2\right\}\psi$$

Substituting this into Eq. (1), we have

$$\frac{\hbar^2\alpha^2}{2m}\left\{3x^2-\alpha^2x^6\right\}\psi+V\psi=\frac{\hbar^2\alpha^2}{m}\psi$$

$\Rightarrow V(x)=Ax^6+Bx^2+C$

where A, B and C are constants.

Correct option is (a)

24. The eigenstates of the given 1-D infinite potential well are,

$$\psi_p(x)=\sqrt{\frac{2}{L}}\sin\frac{p\pi x}{L};\ (p=1,2,3,...)$$

The given wavefunction of the particle is,

$$\psi(x,0)=\sum_{p=1}^{n}\frac{1}{\sqrt{n}}\sqrt{\frac{2}{L}}\sin\frac{p\pi x}{L}=\sum_{p=1}^{n}\frac{1}{\sqrt{n}}\psi_p(x)=\sum_{p=1}^{n}C_p\psi_p(x)$$

The energy spectrum is, $E_p=\frac{p^2\pi^2\hbar^2}{2mL^2}$

Therefore, the average energy for the state is,

$$\langle E\rangle\big|_\psi=\sum_{p=1}^{n}\left|C_p\right|^2E_p=\sum_{p=1}^{n}\frac{1}{n}\frac{\pi^2\hbar^2}{2mL^2}p^2$$

$$=\frac{\pi^2\hbar^2}{2n\,mL^2}\sum_{p=1}^{n}p^2=\frac{n(n+1)(2n+1)\pi^2\hbar^2}{12n\,mL^2}\qquad\left[\because\sum_{1}^{n}p^2=\frac{n(n+1)(2n+1)}{6}\right]$$

$$=\frac{(n+1)(2n+1)\pi^2\hbar^2}{12mL^2}$$

Correct option is (a)

25. Given differential equation is $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$

Assuming a solution of the form $y(x)=e^{imx}$, we have,

$\Rightarrow m^2+m-2=0\Rightarrow(m+2)(m-1)=0\Rightarrow m=-2,1$

Therefore, the general solution is,

$\Rightarrow y(x)=Ae^{-2x}+Be^{x}\Rightarrow y'(x)=-2Ae^{-2x}+Be^{x}$

Now, $y(0)=4 \Rightarrow A+B=4$ (1)

Also, $y'(0)=-5 \Rightarrow -2A+B=-5$ (2)

Substracting equation (2) from equation (1), we have $3A=9 \Rightarrow A=3$

Therefore, $B=1$

Therefore, the solution is $y(x)=3e^{-2x}+e^{x}$

Since $\left.\frac{dy}{dx}\right|_{x=0}=-4$, i.e. it is negative, the value of y first decreases with increment in 'x'.

Also, for large x,

$$\frac{dy}{dx}=-6e^{-2x}+e^{x}>0 \qquad \left[\because\ e^{x}\gg e^{-2x} \text{ for large } x\right]$$

Therefore, dy/dx is positive after a certain value of 'x' and hence, y increases.

Therefore, y first decreases then increases with increment in x.

Correct option is (d)

26. Since, $f(x)=\pi-|x|$ and therefore $f(x)$ is an even function.

Therefore, $f(x)=a_0+\sum_{n=1}^{\infty} a_n \cos nx \Rightarrow b_n=0$

$$\text{Also, } a_0=\frac{1}{\pi}\int_0^{\pi} f(x)\,dx=\frac{1}{\pi}\int_0^{\pi}(\pi-x)\,dx=\frac{1}{\pi}\left[\pi x-\frac{x^2}{2}\right]_0^{\pi}=\frac{\pi}{2}$$

$$\text{And } a_n=\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx=\frac{2}{\pi}\int_0^{\pi}(\pi-x)\cos nx\,dx$$

$$=\frac{2}{\pi}\left[(\pi-x)\left(\frac{\sin nx}{n}\right)-(-1)\left(\frac{\cos nx}{n^2}\right)\right]_0^{\pi}=\frac{2}{\pi}\left[-\frac{\cos n\pi}{n^2}+\frac{1}{n^2}\right]$$

$$=\frac{2}{n^2\pi}\left\{1-(-1)^n\right\}=\begin{cases}0, & \text{if } n \text{ is even}\\ \dfrac{4}{n^2\pi}, & \text{if } n \text{ is odd}\end{cases}$$

Therefore, $a_n=0$ for even n and hence, the statement in (c) is incorrect.

Correct option is (c)

27. Let $\{\lambda\}$ be the eigenvalues. Then the secular equation is

$$\begin{vmatrix}-(2+\lambda) & 2 & -3\\ 2 & 1-\lambda & -6\\ -1 & -2 & -\lambda\end{vmatrix}=0$$

$$\Rightarrow -(2+\lambda)\left\{\lambda(\lambda-1)-12\right\}+2(6+2\lambda)-3(-4+1-\lambda)=0$$

$$\Rightarrow -(2+\lambda)\left\{\lambda^2-\lambda-12\right\}+12+4\lambda+9+3\lambda=0$$

$$\Rightarrow -\left(2\lambda^2-2\lambda-24+\lambda^3-\lambda^2-12\lambda\right)+12+4\lambda+9+3\lambda=0$$

$$\Rightarrow -\lambda^3-\lambda^2+21\lambda+45=0 \qquad (1)$$

Now, we know that if x_1, x_2 and x_3 are the roots of a cubic equation of the form:

$$ax^3 + bx^2 + cx + d = 0 \text{ then } x_1 x_2 + x_2 x_3 + x_3 x_1 = \frac{c}{a}$$

Comparing with Eq. (1), we have,

$$\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1 = \frac{21}{-1} = -21$$

Correct option is (c)

28. We know that if $X_1, X_2, ..., X_n$ are mutually independent normal random variables with means $\mu_1, \mu_2, ..., \mu_n$ and variances $\sigma_1^2, \sigma_2^2, ..., \sigma_n^2$ respectively, then the linear combination $Y = \sum_{i=1}^{n} C_i X_i$ follows a normal distribution N with mean $\mu = \sum_{i=1}^{n} C_i \mu_i$ and variance $\sigma^2 = \sum_{i=1}^{n} C_i^2 \sigma_i^2$.

In this case, $Y = 3A + 2B$; $\mu_A = 2, \mu_B = 3$; $\sigma_A = 0.5, \sigma_B = 0.75$

Therefore, the new mean is $\mu = 3\mu_A + 2\mu_B = 3\times 2 + 2\times 3 = 12$

Also, the new variance is, $\sigma^2 = 3^2\sigma_A^2 + 2^2\sigma_B^2$

Therefore, the standard deviation will be,

$$\sigma = \sqrt{9\sigma_A^2 + 4\sigma_B^2} = \sqrt{9\times\frac{1}{4} + 4\times\frac{9}{16}} = \frac{3}{2}\sqrt{2} = 2.12$$

Correct option is (a)

29. $F(s)$ is the Fourier transform of the given $f(x)$.

Therefore, $f(x) = \int_{-\infty}^{\infty} F(s) e^{isx}\, ds$ ... (1)

where $F(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-isx}\, dx$

Given: $f(x) = \begin{cases} 1 & ; \quad |x| < a \\ 0 & ; \quad |x| > a \end{cases}$

Therefore, $F(s) = \frac{1}{2\pi}\int_{-a}^{a} e^{-isx}\, dx = \frac{1}{2\pi}\times \left.\frac{e^{-isx}}{(-is)}\right|_{-a}^{a}$

$$= \frac{1}{\pi s}\times\frac{e^{isa} - e^{-isa}}{2i} = \frac{\sin(sa)}{\pi s}$$

Therefore, putting this into Eq. (1), we have

$$f(x) = \int_{-\infty}^{\infty} \frac{\sin(sa)}{\pi s} e^{isx}\, dx$$

Since, $f(x)$ is real, it can be equated with the real part of the above integral.

i.e., $f(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin(sa)}{s}\cos(sx)\,ds$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{\sin(sa)}{s}\cos(sx)\,ds=\pi f(x)=\begin{cases}\pi & ; \quad |x|<a \\ 0 & ; \quad |x|>a\end{cases}$$

Correct option is (b)

30. The given Lagrangian is, $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-mg\,z$

Since, x, y are cyclic co-ordinates, p_x, p_y are constants.

Also, $L\neq L(t)$, i.e., the Lagrangian is independent of time and the Hamiltonian is,

$$H=\frac{p_x^2+p_y^2+p_z^2}{2m}-mg\,z$$

Therefore, energy is conserved for the system.

The value of L_z is, $L_z=x\,p_y-y\,p_x$

$$\Rightarrow \frac{d}{dt}(L_z)=\dot{x}\,p_y-\dot{y}\,p_x \qquad (\because\ p_x, p_y \text{ are constants})$$

$$=\frac{p_x\,p_y}{m}-\frac{p_y\,p_x}{m} \qquad (\because\ p_x=m\dot{x},\ p_y=m\dot{y})$$

$$=0$$

$\Rightarrow L_z$ is constant

Correct option is (d)

31. A constant drag force acts on it. Therefore, when the particle is going up the equation of motion will be,

$$m\frac{d^2y}{dt^2}=-mg-F$$

$$\Rightarrow mv\frac{dv}{ds}=-mg-F \Rightarrow v\,dv=-\left(g+\frac{F}{m}\right)ds$$

Let the particle reach a height H_0

$$\Rightarrow \left.\frac{v^2}{2}\right|_u^0=-\left(g+\frac{F}{m}\right)H_0 \Rightarrow -\frac{u^2}{2}=-H_0\left(g+\frac{F}{m}\right) \Rightarrow H_0=\frac{u^2}{2\left(g+\frac{F}{m}\right)}$$

The equation of motion when the particle falls, will be,

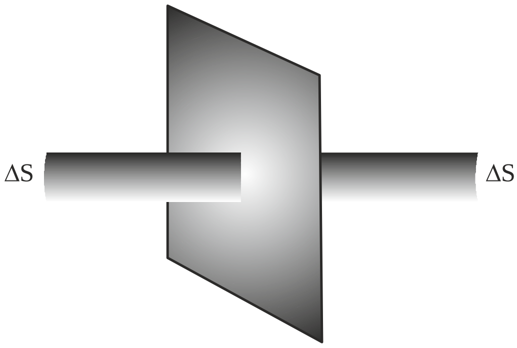
$$\int_0^{v_0}v\,dv=-\left(g-\frac{F}{m}\right)\int_{H_0}^{0}ds \Rightarrow \frac{v_0^2}{2}=\left(g-\frac{F}{m}\right)H_0=\frac{u^2}{2}\frac{\left(g-\frac{F}{m}\right)}{\left(g+\frac{F}{m}\right)}$$

Implying the speed of the particle when it hits ground will be,

$$v_0 = u\sqrt{\frac{g-\frac{F}{m}}{g+\frac{F}{m}}}$$

Correct option is (b)

32. According to Gauss's law,



$$\oint \vec{E}\cdot\overrightarrow{dS} = -4\pi G\, m_{enclosed}$$

$$\Rightarrow \quad \vec{E}\cdot\Delta\vec{S} + \vec{E}\cdot\Delta\vec{S} - = -4\pi G\sigma\Delta S$$

$$\Rightarrow \quad \vec{E} = -2\pi G\sigma\hat{n}$$

$$\therefore \quad \left|\vec{E}\right| = 2\pi G\sigma$$

Correct option is (a)

33. Let ν be the frequency at emitter. Since the spaceship is moving away from Earth, the frequency of the signal at Earth will be,

$$\nu' = \nu\sqrt{\frac{1-\beta}{1+\beta}} \quad \left(\text{where } \beta = \frac{v}{c}, v \text{ being the speed of spaceship}\right)$$

This signal of frequency ν' will be reflected from Earth and will reach the spaceship with frequency ν'', such that,

$$\nu'' = \nu'\sqrt{\frac{1-\beta}{1+\beta}} \Rightarrow \nu'' = \nu\left(\frac{1-\beta}{1+\beta}\right) \qquad \ldots(1)$$

$$\text{Given}: \nu'' = \frac{\nu}{2} \Rightarrow \frac{1-\beta}{1+\beta} = \frac{1}{2} \Rightarrow 2-2\beta = 1+\beta \Rightarrow \beta = \frac{1}{3} \Rightarrow v = \frac{c}{3}$$

Correct option is (b)

34. $$N_i = N_0 P_i = \frac{g_i\, e^{-\beta\epsilon_i}}{\sum g_i\, e^{-\beta\epsilon_i}} \qquad P_2 = \frac{3\,e^{-2\beta\epsilon}}{2+e^{-\beta\epsilon}+3\,e^{-2\beta\epsilon}}$$

$$\frac{N_2}{N_1} = \frac{N_0 P_2}{N_0 P_0} = \frac{3}{2}e^{-2\beta\epsilon} \qquad P_0 = \frac{2}{2+e^{-\beta\epsilon}+3e^{-2\beta\epsilon}}$$

Approximately correct option is (b)

Correct option is (b)

35. Equation of state is $PV^{9/7} = aT$

$$S = k\ln\Omega \Rightarrow \Omega = \exp\left(\frac{S}{k_B}\right)$$

$$\text{Also, } \frac{\partial S}{\partial V} = \frac{P}{T} \Rightarrow \frac{\partial S}{\partial V} = aV^{-9/7} \Rightarrow S(V) = a\int V^{-9/7}\partial V$$

$$\Rightarrow S(V) = \frac{aV^{-2/7}}{(-2/7)} \Rightarrow \Omega(V) = \exp\left(-\frac{7}{2}\frac{aV^{-2/7}}{k_B}\right)$$

Correct option is (b)

36. $$\langle E^2\rangle = \frac{\sum g_i \in_i^2 e^{-\beta\in_i}}{\sum g_i e^{-\beta\in_i}} = \frac{2\in^2 e^{-\beta\in} + 4\in^2 e^{-2\beta\in}}{2e^{-\beta\in} + e^{-2\beta\in}}$$

$$= \frac{2\in^2 e^{-\beta\in}\left(1+2e^{-\beta\in}\right)}{2e^{-\beta\in}\left(1+\frac{1}{2}e^{-\beta\in}\right)} = \frac{\in^2\left(1+2e^{-\beta\in}\right)}{1+0.5e^{-\beta\in}}$$

Correct option is (d)

37. The most probable speed $C_m \propto \sqrt{\frac{T}{m}}$

From figure-(i), we can write,

$$(C_m)_2 > (C_m)_1$$

$$\Rightarrow \quad \sqrt{\frac{T_2}{m}} > \sqrt{\frac{T_1}{m}}$$

$$\Rightarrow \quad T_2 > T_1 \ \Rightarrow T_1 < T_2$$

From figure-(ii), we can write,

$$(C_m)_1 > (C_m)_2$$

$$\Rightarrow \quad \sqrt{\frac{T}{m_1}} > \sqrt{\frac{T}{m_2}} \ \Rightarrow m_1 < m_2$$

Correct option is (d)

38. Given: $\vec{H} = \hat{y}\, 30\cos\left(2\pi\times10^8 t - 6x\right)\ mA/m$

The speed of wave in the given medium is

$$v = \frac{\omega}{k} = \frac{2\pi\times10^8}{6}\ m/\sec = \frac{\pi}{3}\times10^8\ m/\sec$$

Therefore, the intensity of the wave is $I = \frac{1}{2}\mu_0 v H_0^2$

Therefore, the power crossing the surface, $0<y<2; 0<z<3$ is

$$P = \frac{1}{2}\mu_0 v H_0^2 A = \frac{1}{2}\times 4\pi\times10^{-7}\times\frac{\pi}{3}\times10^8\times900\times10^{-6}\times6 = 0.036\,\pi^2 W$$

Correct option is (b)

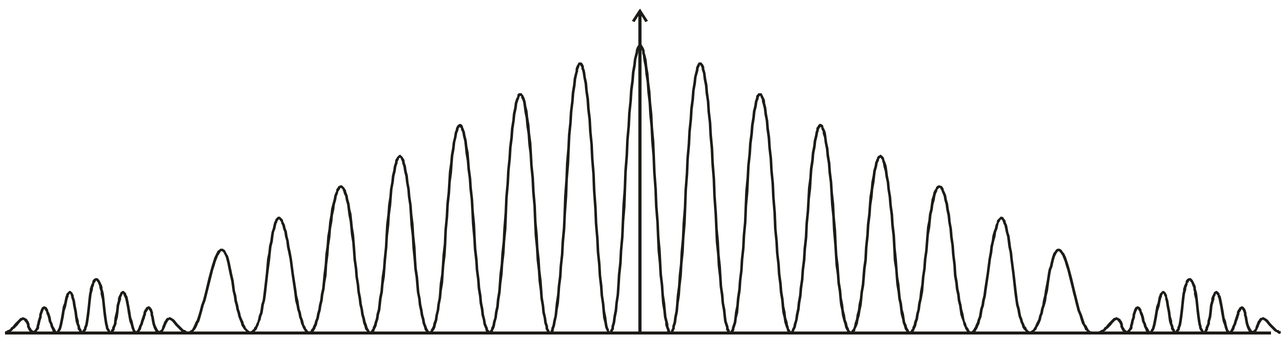
39. The intensity of wave in rest frame, $I = \frac{1}{2}\varepsilon_0 c E_0^2$

The intensity of wave with respect to moving frame,

$$I' = I\frac{1-v/c}{1+v/c} = \frac{1}{2}\varepsilon_0 c E_0^2 \frac{1-1/2}{1+1/2} = \frac{1}{6}\varepsilon_0 c E_0^2$$

Correct option is (c)

40. Let a is the slit width, b is the opaque space between two slits and slit separation is d = a + b



The missing order is 8.

$\therefore \quad \frac{d}{a} = 8 \Rightarrow d = 8a \Rightarrow d = 8 \times 0.25 = 2\ mm$

$\Rightarrow \quad a + b = 2mm$

$\Rightarrow \quad b = 1.75\ mm$

Correct option is (b)

41. The electric field at the origin due to semicircular region,

$$\vec{E}_1 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{i}$$

The electric field due to linear part at O is

$$\vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 R}\left(-\hat{i} + \hat{j}\right)$$

Therefore, the net electric field at the origin is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 R}\left(\hat{i} + \hat{j}\right)$$

Correct option is (b)

42. The resolution of the analog to digital converter is

$$S = \frac{5}{2^8 - 1} = \frac{5}{255} = 0.0196078$$

Therefore, the number of steps corresponding 3.41 volts

$$m = \frac{3.41}{0.0196078} = (174)$$

Therefore, the digital output is (10101110)

2	174	
2	87	0
2	43	1
2	21	1
2	10	1
2	5	0
2	2	1
	1	0

Correct option is (d)

43. Given: Input voltage $V_S(t) = \cos t + \cos 100t$

The input signal is the superposition of two signal with frequencies, $\omega_1 = 1$ and $\omega_2 = 100$

The output corresponding, $V_{in_1} = \cos t$ is

$$V_{out_1} = \frac{\cos t}{1 + j\omega cR} = \frac{1}{1 + j \times 0.1}\cos t = \frac{1}{\sqrt{1+(0.1)^2}}\cos(t+\theta)$$

$$\therefore \quad |\alpha| = \frac{1}{\sqrt{1+(0.1)^2}} = \frac{1}{\sqrt{1.01}}$$

The output corresponding, $V_{in_2} = \cos 100\,t$ is

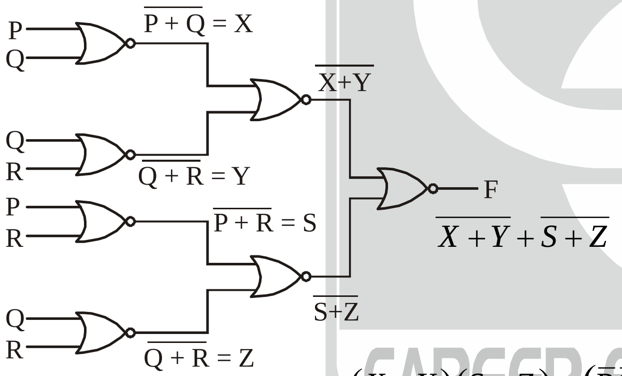
$$V_{out_2} = \frac{\cos 100t}{1 + j\omega cR} = \frac{1}{1 + j10}\cos(100t) = \frac{1}{\sqrt{101}}\cos(100t+\phi)$$

$$\therefore \quad |\beta| = \frac{1}{\sqrt{101}}$$

$$\therefore \quad \frac{|\alpha|}{|\beta|} = \sqrt{\frac{101}{1.01}} = \sqrt{100} = 10$$

Correct option is (b)

44.



$$= (X+Y)(S+Z) = \left(\bar{P}\bar{Q} + \bar{Q}\bar{R}\right)\left(\bar{P}\bar{R} + \bar{Q}\bar{R}\right)$$

$$= \bar{P}\bar{Q}\bar{R} + \bar{P}\bar{Q}\bar{R} + \bar{P}\bar{Q}\bar{R} + \bar{Q}\bar{R}$$

$$= \bar{P}\bar{Q}\bar{R} + \bar{Q}\bar{R} = \bar{Q}\bar{R}\left(1+\bar{P}\right) = \bar{Q}\bar{R} = \overline{Q+R}$$

Correct option is (a)

45. The angular momentum of the moon is $L = rmv$
Therefore, the rms error in angular momentum is

$$\frac{\Delta L}{L} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta v}{v}\right)^2}$$

$$= \sqrt{2+9+25}\,\% = \sqrt{36}\,\% = 6\%$$

Correct option is (a)

PART – C

46. Initial state is $\left|\chi(0)\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$. The state at time t will be $\left|\chi(t)\right\rangle = e^{-\frac{i\hat{H}t}{\hbar}}\left|\chi(0)\right\rangle$, where $\hat{H}$ is the Hamiltonian of the system.

$$\exp\left(-\frac{i\hat{H}t}{\hbar}\right) = \exp\left(-\frac{ieB\hbar t}{2mc\hbar}\right)\sigma_z = \cos\theta I - i\sin\theta\,\sigma_z = \begin{pmatrix}e^{-i\theta} & 0\\0 & e^{i\theta}\end{pmatrix} \quad \left(\text{Here, } \theta = \frac{eBt}{2mc}\right)$$

Therefore, $\left|\chi(t)\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\theta} & 0\\0 & e^{i\theta}\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\theta}\\-e^{i\theta}\end{pmatrix}$

Therefore, probability of finding $\left(-\frac{\hbar}{2}\right)$ at t_0 while measuring $\hat{S}_x$ is,

$$P = \left|\frac{1}{2}\left(e^{i\theta} - e^{-i\theta}\right)\begin{pmatrix}1\\-1\end{pmatrix}\right|^2 = \left|\frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)\right|^2 = \cos^2\theta = \cos^2\left(\frac{eBt_0}{2mc}\right)$$

Also, the expectation value of $\hat{S}_x$ at $t = t_0$ is,

$$\left\langle\hat{S}_x\right\rangle\Big|_{t_0} = \frac{\hbar}{2}\times\frac{1}{2}\left(e^{i\theta} - e^{-i\theta}\right)\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}e^{-i\theta}\\-e^{i\theta}\end{pmatrix} = \frac{\hbar}{4}\left(e^{i\theta} - e^{-i\theta}\right)\begin{pmatrix}-e^{i\theta}\\e^{-i\theta}\end{pmatrix}$$

$$= \frac{\hbar}{4}\left(-e^{2i\theta} - e^{-2i\theta}\right) = -\frac{\hbar}{2}\times\cos 2\theta = -\frac{\hbar}{2}\cos\left(\frac{eBt_0}{mc}\right)$$

Correct option is (c)

47. For $2\mu g/\hbar^2 \ll 1$, we get

$$\delta_\ell = \frac{\pi}{2}\left\{\ell + \frac{1}{2} - \left(\ell + \frac{1}{2}\right)\left[1 + \frac{2\mu g}{\hbar^2\left(\ell + \frac{1}{2}\right)^2}\right]^{1/2}\right\}$$

$$= \frac{\pi}{2}\left\{\ell + \frac{1}{2} - \left(\ell + \frac{1}{2}\right)\left[1 + \frac{\mu g}{\hbar^2\left(\ell + \frac{1}{2}\right)^2}\right]\right\} = -\frac{\pi}{2}\frac{\mu g}{\hbar^2\left(\ell + \frac{1}{2}\right)} \ll 1$$

Thus, we have

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}\left(2\ell + 1\right)e^{i\delta_\ell}\sin\delta_\ell\, P_\ell\left(\cos\theta\right)\right|^2$$

$$= \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)\frac{\pi}{2}\frac{\mu g}{\hbar^2\left(\ell+\frac{1}{2}\right)}P_\ell(\cos\theta)\right|^2$$

$$= \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}\frac{\pi\mu g}{\hbar^2}P_\ell(\cos\theta)\right|^2 = \frac{\pi^2\mu^2 g^2}{\hbar^4 k^2}\left|\sum_{\ell=0}^{\infty}P_\ell(\cos\theta)\right|^2$$

In order to sum the series we use the generating function of $P_\ell(x)$:

$$\sum_{\ell=0}^{\infty}P_\ell(x)\,y^\ell = \frac{1}{\sqrt{1-2yx+y^2}} \Rightarrow \sum_{\ell=0}^{\infty}P_\ell(x) = \frac{1}{\sqrt{2(1-x)}}$$

$$\Rightarrow \sum_{\ell=0}^{\infty}P_\ell(\cos\theta) = \frac{1}{\sqrt{2(1-\cos\theta)}} = \frac{1}{2\sin(\theta/2)}$$

Therefore, $\dfrac{d\sigma}{d\Omega} = \dfrac{\pi^2\mu^2 g^2}{4\hbar^4 k^2}\dfrac{1}{\sin^2(\theta/2)} = \dfrac{\pi^2\mu g^2}{8\hbar^2 E}\dfrac{1}{\sin^2(\theta/2)}$

Finally, using $d\Omega = \sin\theta\, d\theta\, d\phi$, we get

$$\frac{d\sigma}{d\theta} = \frac{\pi^2\mu g^2}{8\hbar^2 E}\frac{\sin\theta}{\sin^2(\theta/2)}\int_0^{2\pi}d\phi = \frac{\pi^3\mu g^2}{4\hbar^2 E}\frac{\sin\theta}{\sin^2(\theta/2)}$$

$$= \frac{\pi^3\mu g^2}{4\hbar^2 E}\frac{2\sin(\theta/2)\cos(\theta/2)}{\sin^2(\theta/2)} = \frac{\pi^3\mu g^2}{2\hbar^2 E}\cot(\theta/2)$$

Correct option is (c)

49. The Hamiltonian for the given perturbation $\vec{E} = E_0\, e^{-\alpha t}\,\hat{i}$ will be, $H(t) = -E_0\, x e^{-\alpha t}\quad(\alpha, t > 0)$.

The energy spectrum of a 3-D cubical box of side '2*a*' is,

$$E_{n_x,n_y,n_z} = \frac{\pi^2\hbar^2}{2m(2a)^2}\left(n_x^2+n_y^2+n_z^2\right)$$

$$= \left(n_x^2+n_y^2+n_z^2\right)\frac{\pi^2\hbar^2}{8ma^2}$$

Since, the probability of transition between $(1,-1,1)\to(2,1,1)$ is desired, we have,

$$\omega_{fi} = \frac{E_f - E_i}{\hbar} = \frac{(6-3)}{\hbar}\times\frac{\pi^2\hbar^2}{8ma^2} = \frac{3\pi^2\hbar}{8ma^2}$$

Now, the probability of transition from $|\psi_i\rangle \to |\psi_f\rangle$ after a long time under a perturbation $H'(t)$ is given by

$$P_{if}(\infty) = \left|-\frac{i}{\hbar}\int_0^{\infty}\langle\psi_f|H(t)|\psi_i\rangle\, e^{i\omega_{fi}t}\,dt\right|$$

Now, the eigenstate corresponding to (n_x, n_y, n_z) is $\psi_{n_x,n_y,n_z} = \psi_{n_x}(x)\,\psi_{n_y}(y)\,\psi_{n_z}(z)$, where $\psi_n(x)=\sqrt{\frac{2}{2a}}\sin\frac{n\pi x}{2a}\left(0\le x\le 2a\right)$

The value of $\left\langle \psi_f \left| H(t) \right| \psi_i \right\rangle$ for $\psi_f=\psi_{2,1,1}$ and $\psi_i=\psi_{1,1,1}$ is

$$=-eE_0e^{-\alpha t}\int_0^{2a}x\psi_2(x)\psi_1(x)\,dx\int_0^{2a}\psi_1(y)\,\psi_1(y)\,dy\int_0^{2a}\psi_1(z)\,\psi_1(z)\,dz$$

$$=-\frac{eE_0e^{-\alpha t}}{a}\int_0^{2a}x\sin\frac{\pi x}{2a}\sin\frac{\pi x}{a}dx \qquad \left[\because \int_0^{2a}\psi_n^2(x)\,dx=1\right]$$

$$=-\frac{eE_0e^{-\alpha t}}{2a}\int_0^{2a}x\left[\cos\left(\frac{\pi x}{2a}-\frac{\pi x}{a}\right)-\cos\left(\frac{\pi x}{2a}+\frac{\pi x}{a}\right)\right]dx$$

$$=-\frac{eE_0e^{-\alpha t}}{2a}\int_0^{2a}\left(x\cos\frac{\pi x}{2a}-x\cos\frac{3\pi x}{2a}\right)dx$$

$$=-\frac{eE_0e^{-\alpha t}}{2a}\left[-\int_0^{2a}\frac{\sin\frac{\pi x}{2a}}{\frac{\pi}{2a}}dx+\int_0^{2a}\frac{\sin\frac{3\pi x}{2a}}{\frac{3\pi}{2a}}dx\right]=-\frac{eE_0e^{-\alpha t}}{2a}\times\frac{2a}{\pi}\left[\frac{1}{3}\times\left.\frac{\cos\frac{3\pi x}{2a}}{\frac{3\pi}{2a}}\right|_{2a}^{0}-\left.\frac{\cos\frac{\pi x}{2a}}{\frac{\pi}{2a}}\right|_{2a}^{0}\right]$$

$$=-\frac{eE_0e^{-\alpha t}}{\pi}\times\frac{2a}{\pi}\left[\frac{1}{9}\left(1-(-1)\right)-\left(1-(-1)\right)\right]=-\frac{4a\,eE_0e^{-\alpha t}}{\pi^2}\left(-\frac{8}{9}\right)=\frac{32\,ae\,E_0e^{-\alpha t}}{9\pi^2}$$

$$P_{if}(\infty)=\left|\frac{i}{\hbar}\int_0^{\infty}\left\langle\psi_f\left|H(t)\right|\psi_i\right\rangle e^{i\omega_{fi}t}\,dt\right|^2=\left(\frac{32\,ae\,E_0}{9\pi^2\hbar}\right)^2\left|\int_0^{\infty}e^{i\omega_{fi}t}\cdot e^{-\alpha t}dt\right|^2$$

$$=\left(\frac{32\,ae\,E_0}{9\pi^2\hbar}\right)^2\left|\frac{1}{\alpha-i\omega_{fi}}\right|^2 \qquad \left[\because \text{Laplace transform}, L\left(e^{at}\right)=\frac{1}{s-a}\right]$$

$$=\left(\frac{32\,e\,E_0\,a}{9\pi^2\hbar}\right)^2\times\frac{1}{\alpha-i\omega_{fi}}\times\frac{1}{\alpha+i\omega_{fi}}$$

$$=\left(\frac{32\,e\,E_0\,a}{9\pi^2\hbar}\right)^2\times\frac{1}{\alpha^2+\left(\frac{3\pi^2\hbar}{8ma^2}\right)^2} \qquad \left[\because\ \omega_{fi}=\frac{3\pi^2\hbar}{8ma^2}\right]$$

Correct option is (a)

50. Given integral is $I=\int_0^{\infty}\frac{x^2\,dx}{x^6+1}=\frac{1}{3}\int_0^{\infty}\frac{3x^2\,dx}{(x^3)^2+1}=\frac{1}{3}\int_0^{\infty}\frac{dy}{y^2+1}=\frac{1}{3}\int\frac{dy}{(y+i)\,(y-i)}$, where $(y=x^3)$.

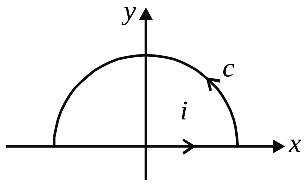
Now, the function $f(y)=\frac{1}{1+y^2}$ has a simple pole at $y=i$ in the upper half plane.

Taking a Contour c, we have

$$\int_{-\infty}^{\infty} \frac{dy}{y^2+1} = 2\pi i \ \text{Res}\left[f(y)\right]\Big|_{y=i}$$ [Since, the integral around the curved Contour will vanish]

$$\Rightarrow 2\int_0^{\infty} \frac{dy}{y^2+1} = 2\pi i \times \lim_{y\to i} \frac{(y-i)}{(y-i)(y+i)} = 2\pi i \times \frac{1}{2i} = \pi$$

$$\Rightarrow \int_0^{\infty} \frac{dy}{y^2+1} = \frac{\pi}{2}$$



Given integral $I = \int_0^{\infty} \frac{dx}{x^6+1} = \frac{1}{3}\int_0^{\infty} \frac{dy}{y^2+1} = \frac{1}{3}\times\frac{\pi}{2} = \frac{\pi}{6}$

Correct option is (b)

51. Given: $\psi_n(x) = \exp\left(-\frac{x^2}{2}\right) H_n(x)$

The derivative of $\psi_n(x)$ w.r.t. x is,

$$\frac{d\psi_n}{dx} = \psi_n'(x) = \exp\left(-\frac{x^2}{2}\right) H_n'(x) + H_n(x)\exp\left(-\frac{x^2}{2}\right)(-x)$$

$$= 2n H_{n-1}(x)\exp\left(-\frac{x^2}{2}\right) - \exp\left(-\frac{x^2}{2}\right)\left(nH_{n-1}(x) + \frac{H_{n+1}(x)}{2}\right)$$

$$= \exp\left(-\frac{x^2}{2}\right)\left[2n H_{n-1}(x) - nH_{n-1}(x) - \frac{H_{n+1}(x)}{2}\right]$$

$$\therefore \quad \psi_n'(x) = \frac{1}{2}\exp\left(-\frac{x^2}{2}\right)\left[2nH_{n-1} - H_{n+1}\right]$$

Substituting these in the given integral, we have

$$\int_{-\infty}^{\infty} \psi_m(x)\psi_n'(x)\,dx = \int_{-\infty}^{\infty} \frac{\exp(-x^2)}{2}\left\{H_m(x)\left(2nH_{n-1} - H_{n+1}\right)\right\}dx$$

$$= \frac{1}{2}\left[2n\sqrt{\pi}\, 2^m\, m!\,\delta_{m,n-1} - \sqrt{\pi}\, 2^m\, m!\,\delta_{m,n+1}\right]$$

$$= n\sqrt{\pi}\, 2^m\, m!\,\delta_{m,n-1} - \sqrt{\pi}\, 2^{m-1}\, m!\,\delta_{m,n+1}$$

$$= n\sqrt{\pi}\, 2^{n-1}\,(n-1)!\,\delta_{m,n-1} - 2^{n+1-1}\sqrt{\pi}\,(n+1)!\,\delta_{m,n+1}$$

$$= 2^{n-1}\, n!\sqrt{\pi}\ \delta_{m,n-1} - 2^n\sqrt{\pi}\,(n+1)!\,\delta_{m,n+1}$$

Correct option is (d)

52. Consider the set G and let's take two matrices of the form

$$M_1 = x_1\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \in G, \text{ and } M_2 = x_2\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \in G$$

The product is, $M_1 M_2 = x_1 x_2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 2x_1 x_2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \in G$

Therefore, closure holds for the given set G.

Now, $x\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \times \frac{1}{4x}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Let's see if $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ is the identity element for the group G or not. We have

$x\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \times \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{x}{2}\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = x\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Therefore, $\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ is the identity element and $\frac{1}{4x}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ is the inverse of each element of the group.

Correct option is (d)

53. Given:

$x_0 = 1$	$f_0 = 1$
$x_1 = 2$	$f_1 = 7$
$x_2 = 4$	$f_2 = 61$

Now, from Lagrange's interpolation technique, we have,

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f_2$$

Here, $x = 10$

$$f(10) = \frac{(10-2)(10-4)}{(1-2)(1-4)} \times 1 + \frac{(10-1)(10-4)}{(2-1)(2-4)} \times 7 + \frac{(10-1)(10-2)}{(4-1)(4-2)} \times 61$$

$$= \frac{8\times 6\times 1}{(-1)\times(-3)} + \frac{9\times 6\times 7}{1\times(-2)} + \frac{9\times 8\times 61}{3\times 2}$$

$$= 16 - \frac{54\times 7}{2} + 12\times 61 = 559$$

Correct option is (a)

54. The potential energy for the particle is,

$$V(r) = -\int_{\infty}^{r} F(r)\, dr = -\frac{\gamma}{r} \qquad [\because V(\infty) = 0]$$

Therefore, the initial total energy of the particle is,

$$E = \frac{1}{2}\left(\sqrt{\frac{\gamma}{c}}\right)^2 + V(c) = \frac{\gamma}{2c} - \frac{\gamma}{c} = -\frac{\gamma}{2c} \qquad [\because m = 1]$$

Since the total energy is negative for the given attractive inverse square force field, the trajectory of the particle will be an ellipse. Also, the angular momentum of the particle is ,

$$L = \left(\frac{\gamma}{c}\right)^{1/2} c \sin(60^\circ) = \sqrt{\frac{3\gamma c}{4}} \qquad [\because L = vr\sin\theta, \text{ where } \theta \text{ is the angle between } v \text{ and } r]$$

Therefore, the eccentricity of the orbit is,

$$e=\sqrt{1+\frac{2EL^2}{\gamma^2}}=\sqrt{1+\frac{2\times\left(-\frac{\gamma}{2c}\right)\times\frac{3\gamma c}{4}}{\gamma^2}}=\frac{1}{2}$$

The maximum and minimum distances are

$r_{\max}=a(1+e)$, $r_{\min}=a(1-e)$, where 'a' is the semi-major axis length.

Therefore, $\frac{r_{\max}}{r_{\min}}=\frac{1+e}{1+e}=\frac{3/2}{1/2}=3$

Correct option is (c)

55. Given: $\frac{d^2y}{dt^2}=a$

$\Rightarrow\ \dot{y}=at$ and $y=\frac{at^2}{2}$ (Ignoring constants)

Therefore, the Lagrangian of the system is,

$$L=\frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right)-mg\,y=\frac{m}{2}\left(\dot{x}^2+a^2t^2\right)-\frac{mg\,at^2}{2}$$

Therefore, the generalized momentum along 'x' is, $p_x=\frac{\partial L}{\partial\dot{x}}=m\dot{x}$

Therefore, the Hamiltonian is,

$$H=p_x\,\dot{x}-L=\frac{p_x^2}{m}-\left[\frac{m}{2}\left(\dot{x}^2+a^2t^2\right)-\frac{mg\,at^2}{2}\right]$$

$$=\left(\frac{p_x^2}{2m}-\frac{ma^2t^2}{2}\right)+\frac{mg\,at^2}{2}$$

If T, V are kinetic and potential energy, we have

$$T=\frac{p_x^2}{2m}+\frac{ma^2t^2}{2},V=\frac{mg\,at^2}{2}\Rightarrow H\neq T+V$$

Therefore, the Hamiltonian is not equal to the total energy of the system and since $H=H(t)$, the Hamiltonian is not conserved as well.

Correct option is (d)

56. We know from Jacobi identity that

$$\{A,\{B,C\}\}+\{B,\{C,A\}\}=-\{C,\{A,B\}\}$$

$\therefore$ Here, $A=\vec{a}\cdot\vec{r}$, $B=\vec{b}\cdot\vec{p}, C=\vec{L}$

$\therefore\ \{A,\{B,C\}\}+\{B\{C,A\}\}$

$=-\left\{\vec{L},\left\{\vec{a}\cdot\vec{r},\vec{b}\cdot\vec{p}\right\}\right\}$

$=-\left\{L_i,\left\{a_j\,x_j,\,b_k\,p_k\right\}\right\}=-\left\{L_i,\,a_j\,b_k\,\delta_{j,k}\right\}$ $\left(\because\ \left\{x_j,p_k\right\}=\delta_{j,k}\right)$

$=-\left\{\vec{L},\vec{a}\cdot\vec{b}\right\}=0$ $\left(\therefore\ \vec{a},\vec{b}\text{ and constant vectors.}\right)$

Correct option is (d)

57. The momentum (p) of the particle of rest mass m_0 moving with speed v is,

$$p = \gamma m_0 v, \text{ where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

If kinetic energy equals rest mass energy, we have

$$(\gamma - 1)m_0c^2 = m_0c^2 \Rightarrow \gamma = 2$$

$$\Rightarrow \quad \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = \frac{\sqrt{3}}{2}c$$

The equation of motion of the particle is

$$\frac{dP}{dt} = F \Rightarrow \frac{d}{dt}\left(\frac{v}{\sqrt{1-v^2/c^2}}\right) = \left(\frac{F}{m_0}\right)$$

$$\frac{v}{\sqrt{1-v^2/c^2}} = \frac{F_0t_0}{m_0} \Rightarrow \frac{\sqrt{3}/2 \times c}{1/2} = \frac{F_0t_0}{m_0}$$

$$\Rightarrow \quad t_0 = \frac{\sqrt{3}\, m_0c}{F_0}$$

Correct option is (a)

58. $TdS = dU + PdV \Rightarrow dS = \dfrac{1}{T}(dU + PdV)$

$$dS = \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT + PdV\right]$$

$$= \left[\frac{1}{T}\left(\frac{\partial U}{\partial V}\right)_T + \frac{P}{T}\right]dV + \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V dT$$

$$= \left[\frac{1}{T}\frac{bT^m}{V} + \frac{aT^3}{V}\right]dV + \frac{1}{T}\left[mbT^{m-1}\ln\left(\frac{V}{V_0}\right) + g'(T)\right]dT$$

Since, S is a state function: $\dfrac{\partial^2 S}{\partial T\,\partial V} = \dfrac{\partial^2 S}{\partial V\,\partial T}$

$$\Rightarrow \frac{\partial}{\partial T}\left(\frac{bT^{m-1}}{V} + \frac{aT^3}{V}\right) = \frac{\partial}{\partial V}\left[mbT^{m-2}\ln\frac{V}{V_0} + \frac{1}{T}g'(T)\right]$$

$$\frac{b(m-1)T^{m-2}}{V} + \frac{3aT^2}{V} = mbT^{m-2}\frac{1}{V}$$

$$\frac{3aT^2}{V} = \frac{bT^{m-2}}{V}$$

$\Rightarrow b = 3a$ and $m - 2 = 2 \Rightarrow m = 4$

Correct option is (a)

59. Single particle partition function, $Q_1(T,V)$

$$Q_1(T,V)=\frac{1}{h^2}\iiiint e^{-\beta\left(\frac{p^2}{2m}-\varepsilon'\right)}dx\,dy\,dp_x\,dp_y$$

$$=\frac{a}{h^2}\left[\int_0^\infty e^{-\frac{\beta p^2}{2m}}2\pi p\,dp\right]e^{\beta\varepsilon'}=\frac{2\pi pa}{h^2}\frac{e^{\beta\varepsilon'}}{2}\left(\frac{2m}{\beta}\right)=\left(\frac{2\pi mk_BTa}{h^2}\right)e^{\varepsilon'/k_BT}$$

Partition function for system $Q_N(T,V)=\frac{1}{N!}\left[Q_1(T,V)\right]^N$

$$Q_N(T,V)=\frac{1}{N!}\left[\frac{2\pi mk_BTa}{h^2}\right]^N e^{N\varepsilon'/k_BT}$$

Chemical potential, $\mu=\frac{\partial A}{\partial N}$ ... (1)

$$A=-kT\ln Q_N(T,V)$$

$$=-kTN\left[\ln a-\frac{1}{N}(N\ln N-N)+\ln\left(\frac{2\pi mkT}{h^2}\right)+\frac{\varepsilon'}{kT}\right]$$

$$\mu=\frac{\partial A}{\partial N}$$

$$\mu=-k_BT\left[\ln a-\ln N+1+\ln\left(\frac{2\pi mk_BT}{h^2}\right)+\frac{\varepsilon'}{kT}\right]-NkT\left(-\frac{1}{N}\right)$$

$$\mu=-k_BT\left[\ln\frac{a}{N}+1+\ln\left(\frac{2\pi mk_BT}{h^2}\right)+\frac{\varepsilon'}{kT}-1\right]$$

$$\mu=-k_BT\left[\ln\frac{a}{N}+\ln\frac{2\pi mk_BT}{h^2}+\frac{\varepsilon'}{kT}\right]$$

$$\mu=-k_BT\left[\ln\frac{2\pi mk_BTa}{Nh^2}+\frac{\varepsilon'}{kT}\right]$$

Correct option is (c)

60. The population of a level, $n_1\propto\exp\left(-\frac{\varepsilon_1}{k_BT}\right)$

Therefore, $\frac{n_2}{n_1}=\exp\left(\frac{\varepsilon_1-\varepsilon_2}{k_BT}\right)\Rightarrow T=\frac{\varepsilon_1-\varepsilon_2}{k_B}\frac{1}{\ln\left(n_2/n_1\right)}$

For $\begin{cases}n_2=40,\ \varepsilon_2=20\\ n_1=10,\ \varepsilon_1=30\end{cases}$

$$T=\frac{10}{k_B}\frac{1}{\ln(4)}=\frac{10}{k_B(1.38)}\cong\frac{7}{k_B}$$

For $\begin{cases} n_2 = 50, \ \varepsilon_2 = 10 \\ n_1 = 40, \ \varepsilon_1 = 20 \end{cases}$

$$T = \frac{10}{k_B} \times \frac{1}{\ln(5/4)} = \frac{10}{k_B(0.22)} = \frac{1000}{22k_B} \simeq \frac{45}{k_B}$$

For $\begin{cases} n_2 = 50, \ \varepsilon_2 = 10 \\ n_1 = 10, \ \varepsilon_1 = 30 \end{cases}$

$$T = \frac{20}{k_B} \times \frac{1}{\ln(5)} = \frac{20}{k_B(1.6)} = \frac{12.5}{k_B}$$

Mean value of temperature is

$$T = \frac{1}{k_B}\left(\frac{7+45+12.5}{3}\right) = \frac{1}{k_B}\left(\frac{64.5}{3}\right) = \frac{21.5}{k_B}$$

Correct option is (b)

61. The magnetic field due to solenoid is given by

$$B = \begin{cases} \mu_0 n I(t) & \text{for } r < R \\ 0 & \text{for } r > R \end{cases}$$

Now, according to Maxwell's equation,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \quad \iint \left(\vec{\nabla} \times \vec{E}\right) \cdot \overrightarrow{dS} = -\mu_0 n \iint \frac{\partial I}{\partial t} dS$$

$$\Rightarrow \quad \oint \vec{E} \cdot \overrightarrow{d\ell} = -\mu_0 n I_0 \omega \cos \omega t \iint dS$$

For $r < R$

$$E \cdot 2\pi r = -\mu_0 n I_0 \omega \cos \omega t \pi r^2$$

$$\Rightarrow \quad E \propto r$$

For $r > R$

$$E \cdot 2\pi r = -\mu_0 n I_0 \omega \cos \omega t \pi R^2$$

$$\Rightarrow \quad E \propto \frac{1}{r}$$

$\therefore \quad m = -1$ and $n = 1$

Correct option is (b)

62. The circular polarized will be linear polarized if plate interduced a phase difference $\frac{\pi}{2}$ between O-ray and E-ray , when it will travelling through the plate.

$$\therefore \quad \frac{2\pi}{\lambda_0} d_{\min}\left(n_o \sim n_e\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad d_{\min} = \frac{\lambda_0}{4\left(n_o \sim n_e\right)} = \frac{\lambda_0}{4(4-3)} = \frac{\lambda_0}{4} = \frac{600}{4} = 150 \ nm$$

Correct option is (c)

63. Given: $\vec{E} = \frac{5\sin 2\theta}{r}\sin\left(\omega t - kr\right)\hat{\theta}\ \ V/m$

Therefore, the intensity of the wave is

$$I = \frac{1}{2}\varepsilon_0 c\frac{25}{r^2}\sin^2 2\theta$$

Therefore, the power radiated by the antenna is

$$P = \frac{1}{2}\varepsilon_0 c\ 25\iint\frac{\sin^2 2\theta}{r^2}r^2\sin\theta d\theta d\phi$$

$$= \frac{25}{2}\varepsilon_0 c\times 2\pi\times\frac{16}{15} = \frac{80}{3}\pi\varepsilon_0 c = \frac{20}{3}\times 4\pi\varepsilon_0\times c = \frac{20}{3}\times\frac{3\times 10^8}{9\times 10^9} = \frac{2}{9}\ \ \text{W}$$

Correct option is (a)

64.

CLK	D_1	D_0	Q_1	Q_0
×	×	×	1	1
1	0	1	0	1
2	0	0	0	0
3	1	0	1	0
4	1	1	1	1

$D_1 = \bar{Q}_0\ \ ;\ \ D_0 = Q_1$

So, output will repeat after 4 pulses.
729 pulse = 182×4+1 = 1 pulse

Correct option is (a)

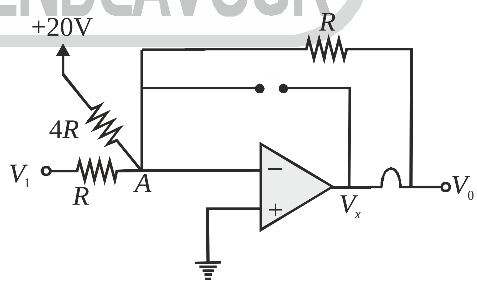
65. Assume that D_2 is OFF and D_1 is ON
Applying nodal analysis at A, we can write,

$$\frac{0 - V_{in}}{R} + \frac{0 - 20}{4R} + \frac{0 - V_{out}}{R} = 0$$

$$V_{out} = \left(-V_{in} - 5\right)$$

Now, V_{out} will be positive if $V_{in} < -5$ volts

If $V_{in} > -5$ volts, V_x will be negative
our assumption will be wrong, D_2 will be ON.
D_1 will be OFF Vout will be zero.



Now, for $V_{in} = -10$

$V_{out} = +10 - 5 = 5$ volts

For $V_{in} = -5$

$V_{out} = +5 - 5 = 0$

For $V_{in} > -5$ volts

$V_{out} = 0$

Correct option is (b)

66. Given: Pinch OFF voltage $V_P = -4$ volts

We know, $$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$$

$$\Rightarrow \quad \frac{I_{DSS}}{2} = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2 \Rightarrow \frac{V_{GS}}{V_P} = \left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad V_{GS} = -4\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) = -1.171 \text{ volts}$$

Therefore, $V_{DS\,(sat)} = V_{GS} - V_P = -1.171 + 4\,Volts = 2.83$ volts

Correct option is (c)

67. For fundamental band : $\omega_e(1-2x_e) = 1876.06\ \text{cm}^{-1}$ ... (1)

and for first overtone : $2\omega_e(1-3x_e) = 3724.2\ \text{cm}^{-1}$ ... (2)

Multiplying Eq. (1) by 3 and subtracting Eq. (2), we get

$$\left(3\omega_e - 6\omega_e x_e\right) - \left(2\omega_e - 6\omega_e x_e\right) = 3\times 1876.06 - 3724.2$$

$$\Rightarrow \ \omega_e = 1903.98\ \text{cm}^{-1}$$

Substituting value of ω_e in Eq. (1), we get

$$1903.98 - 2\times(1903.98)\times x_e = 1876.06$$

$$\Rightarrow \ x_e = \frac{1903.98 - 1876.06}{2\times 1903.98} = 0.00733$$

$$\Rightarrow \ x_e = 7.33\times 10^{-3}$$

Correct option is (c)

68. For the state ${}^4\text{D}_{5/2} : 2S+1 = 4$

$$\Rightarrow \ S = \frac{3}{2},\ J = \frac{5}{2} \text{ and } L = 2$$

Minimum number of electrons which could give $S = \frac{3}{2}$ is 3, since each electron has $s = \frac{1}{2}$. The possible combinations to get $L = 2$ with 3 electrons are: $l_1 = 0,\ l_2 = 0,\ l_3 = 2$ and $l_1 = 0,\ l_2 = 1$ and $l_3 = 1$.

These two correspond to the possible electronic configuration s^2d^1 and s^1p^2. Out of these two s^2d^1 is not possible since the net spin is 1/2. Hence, a possible configuration is s^1p^2.

Correct option is (a)

69. The passive cavity life time,

$$t_c = \frac{2n_0 d}{c\ln\left(\frac{1}{1-x}\right)} \qquad \text{... (1)}$$

where $x = 1 - R_1 R_2 \exp\left(-2\alpha_c d\right)$

$$= 1 - 1 \times 0.99 \times \exp(-2 \times 0 \times d)$$
$$= 1 - 0.99 = 0.01 \qquad \dots (2)$$

From Eq. (2), substituting value of $x = 0.01$ into Eq. (1), we get

Cavity life time, $t_c = \dfrac{2 \times 1 \times 30 \text{ cm}}{3 \times 10^{10} \times \ln\left(\dfrac{1}{1-0.01}\right)} = 1989.98 \times 10^{-10} \simeq 0.2 \times 10^{-6} \text{ sec} = 0.2\ \mu\text{sec}$

The passive cavity line width $\Delta \nu_p = \dfrac{1}{2\pi t_c} = \dfrac{1}{2 \times 3.14 \times 0.2\ \mu\text{sec}}$

$$= \frac{1}{2 \times 3.14 \times 0.2 \times 10^{-6}}$$

$$= 0.7961 \times 10^6 \text{ Hz} \cong 0.8 \text{ MHz}$$

Correct option is (d)

70. **For (i):** For $I = \frac{3}{2}$, $I_3 = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}$ and $-\frac{3}{2}$ and $Q = I_3 + \frac{B+S}{2} \Rightarrow Q = +2, +1, 0$ and -1.

For (ii): After applying conservation laws, particle X is proton having $I_3 = +\frac{1}{2}, I = +\frac{1}{2}, B = 1, S = 0$.

For (iii): Quark content of $K^- \to s\bar{u}$, $\Sigma^+ \to uus$ and $\Xi \to uss$.

Thus all statements are correct.

Correct option is (d)

71. $\Delta J = |3-1|$ to $|3+1| = 2, 3, 4$ and parity change = No

$\Rightarrow$ E_2, M_3 and E_4

Correct option is (c)

72. Statement (a) is correct

For statement (b), $\Delta E \propto (2l+1) \Rightarrow \dfrac{\Delta E(d)}{\Delta E(f)} = \dfrac{(2 \times 2 + 1)}{(2 \times 3 + 1)}$

$\Rightarrow \Delta E(f) = \dfrac{7}{5} \times \Delta E(d) = \dfrac{7}{5} \times 10 = 14$ MeV

For statement (c), $E_\pi = \dfrac{\left(m_\tau^2 + m_\pi^2\right)c^2}{2m_\tau}$ and $E_\pi = \dfrac{m_\pi c^2}{\sqrt{1 - \dfrac{v_\pi^2}{c^2}}}$

Solving we get, $v_\pi = \dfrac{\left(m_\tau^2 - m_\pi^2\right)c^2}{\left(m_\tau^2 + m_\pi^2\right)}$. Thus it is incorrect.

For statement (d),

$R_{\text{Cu}} = R = R_0 (64)^{1/3} = 4R_0 \Rightarrow R_0 = \dfrac{R}{4}$

$$R_{\text{Mg}} = R_0(27)^{1/3} = 3R_0 = \frac{3R}{4}$$

Correct option is (c)

73. The Bohr radius of donor in silicon $r = a_H \times \varepsilon \times \frac{m}{m^*}$

a_H = Bohr radius

ε = Dielectric constant

m^* = effective mass of silicon

$$r = \frac{0.53 \times 11.7 \times m}{0.2\,m} = \frac{62.01}{2} = 31.005 \text{ Å}$$

Correct option is (d)

74. The number of modes wavevector is $\left(\frac{1}{2\pi}\right)^3 \frac{4}{3}\pi k^3$ per unit volume.

Number of magnons $D(\omega)\,d\omega = \left(\frac{1}{2\pi}\right)^3 \cdot 4\pi k^2 dk$

$$\omega = \frac{2Jsa^2}{\hbar}k^2 \Rightarrow k^2 = \frac{\hbar\omega}{2Jsa^2}$$

$$\frac{d\omega}{dk} = \frac{4Jsa^2k}{\hbar}$$

$$D(\omega) = \left(\frac{1}{2\pi}\right)^3 \cdot 4\pi k^2 \cdot \frac{dk}{d\omega}$$

$$= \left(\frac{1}{2\pi}\right)^3 \cdot 4\pi k^2 \frac{\hbar}{4Jsa^2k}$$

$$= \left(\frac{1}{2\pi}\right)^3 \cdot \frac{4\pi\hbar}{4Jsa^2} \cdot \left(\frac{\hbar\omega}{2Jsa^2}\right)^{1/2} = \omega^{1/2}$$

Correct option is (b)

75. $$\varepsilon(k) = \varepsilon_0 - 2A\alpha\left(1 - \frac{k^2a^2}{2}\right)e^{-\alpha a}$$

$$\frac{d\varepsilon}{dk} = 2A\alpha k a^2 e^{-\alpha a}\ ;\ \frac{d^2\varepsilon}{dk^2} = 2A\alpha a^2 e^{-\alpha a}$$

Effective mass $m^* = \dfrac{\hbar^2}{d^2E/dk^2} = \dfrac{\hbar^2}{2A\alpha a^2 e^{-\alpha a}}$

For $\alpha = \dfrac{1}{a};\ \ m^* = \dfrac{\hbar^2}{2A\frac{1}{a}\cdot a^2 e^{-\frac{1}{a}\cdot a}} = \dfrac{\hbar^2 e}{2Aa}$

Correct option is (a)