Example: If the density function of continuous random variable X is given by

$$f(x)=\begin{cases} 0 & ; \quad x<0 \\ ax & ; \quad 0\le x\le 2 \\ (4-x)a & ; \quad 2\le x\le 4 \\ 0 & ; \quad x>4 \end{cases}$$

Then find $P(X>2.5)$?

(a) 11/32 (b) 9/32 (c) 12/32 (d) None of these

Soln: Since, $f(x)$ is a pdf function, we have

$\int_0^4 f(x)dx=1$ **or** $\int_0^2 ax\,dx+\int_2^4 a(4-x)dx=1$

$2a+2a=1\Rightarrow a=\frac{1}{4}$

Hence, required probability $P(X>2.5)=\int_{2.5}^4 \frac{1}{4}(4-x)dx=\frac{9}{32}$.

Correct option is (b)

Example: The amount of time in hours that a computer functions before breaking down is a continuous random variable with probability density functions given by

$$f(x)=\begin{cases} \lambda\, e^{-x/100} & x\ge 0 \\ 0 & x<0 \end{cases}$$

What is the probability that a computer will function between 50 and 150 hours before breaking down?

(a) 0.633 (b) 0.384 (c) 0.434 (d) 0.334

Soln: Since, $\int_{-\infty}^{\infty} f(x)dx=\lambda\int_0^{\infty} e^{-x/100}dx=1 \Rightarrow \boxed{\lambda=\frac{1}{100}}$

Hence, the probability that computer will function between 50 and 150 hr before breaking down is given by

$$P(50<x<150)=\int_{50}^{150}\frac{1}{100}e^{-x/100}dx\simeq 0.384$$

Correct option is (b)

Example: The joint probability distribution function of the random variables x and y, is given as following:

$$f(x,y)=K\left(xy+y^2\right) \quad [0\le x\le 1,\ 0\le y\le 2]$$

The probability that $x+y\le 1$, will be

(a) 3/22 (b) 3/44 (c) 3/88 (d) 1/11

Soln: $f(x,y)=K(xy+y^2) \qquad [0\le x<<1, O\le y\le 1]$

Let us first find the value of K

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(xy+y^2)\,dx\,dy=1$$

$$\Rightarrow \int_{x=0}^{x=1}\int_{y=0}^{2} K(xy+y^2)\,dx\,dy = 1 \qquad \Rightarrow \int_{x=0}^{1} K\left(\frac{xy^2}{2}+\frac{y^3}{3}\right)dx = 1$$

$$\Rightarrow K\int_0^1\left(2x+\frac{8}{3}\right)dx = 1$$

$$\Rightarrow K\left(\frac{2x^2}{2}+\frac{8}{3}x\right)_0^1 = 1 \qquad \Rightarrow K\left(1+\frac{8}{3}\right) = 1$$

$$\Rightarrow K = \frac{3}{11}$$

Therefore, $f(x,y) = \frac{3}{11}(xy+y^2)$

Now $P(x+y\le 1) = P(x+y\le 1) = \int\int_{x+y\le 1}(xy+y^2)dx\,dy = \int_{x=1}^{1}\int_{y=0}^{1-x} K(xy+y^2)dydx$

$$= \int_0^1 K\left(\frac{xy^2}{2}+\frac{y^3}{3}\right)_0^{1-x} dx = \int_0^1 K\left(\frac{x(1-x)^2}{2}+\frac{(1-x)^3}{3}\right)dx = \int_0^1 K\left(x\left(\frac{1-x}{2}\right)^2+\left(\frac{1-x}{3}\right)^3\right)dx$$

$$= K\int_0^1 \frac{x+x^3-2x^2}{2}+\frac{(1-x^3+3x^2-3x)}{3}dx$$

$$= K\left(\frac{x^2}{4}+\frac{x^4}{4\times 2}-\frac{2x^3}{2x^3}+\frac{x}{3}-\frac{x^4}{4\times 3}+\frac{2x^3}{2\times 3}+\frac{x}{3}-\frac{x^4}{4\times 3}+\frac{3x^3}{3\times 3}-\frac{3x^2}{3\times 2}\right)_0^1$$

$$= K\left(\frac{1}{4}+\frac{1}{8}-\frac{1}{3}+\frac{1}{3}-\frac{1}{12}+\frac{1}{3}-\frac{1}{2}\right) = \frac{3}{11}\left(\frac{6+3-2+8-12}{24}\right) = \frac{3}{11}\times\frac{3}{24} = \frac{3}{88}$$

Correct option is (c)

Binomial Distribution

Binomial distribution can be used under the following conditions

(a) A random experiment is repeated many times but the number of trials will be finite

(b) There are only two possible outcomes i.e. success and failure, for each trial.

(c) All trials are independent of each other

If a random experiment is repeated 'n' times (i.e. there are 'n' trials of a random experiment), the probability that event A will occur 'r' times is

$$P(r) = {}^nC_r\ p^r q^{n-r}$$

where, $n =$ total number of independent trials
$r =$ number of success
$p =$ probability of success in a single trial
$q =$ probability of failure in a single trial.