

Example 3: Solve $e^{dy/dx} = (x+1)$; given $y = 3$ at $x = 0$

Soln: Taking log of both sides we get, $\frac{dy}{dx} = \ln(x+1) \Rightarrow dy = \ln(x+1) dx$

on integration, $\int dy = \int \ln(x+1) dx$

$$\Rightarrow y = x \ln(x+1) - \int \frac{x}{(x+1)} dx + C \quad \Rightarrow y = x \ln(x+1) - \int \frac{(x+1)-1}{(x+1)} dx + C$$

$$\Rightarrow y = x \ln(x+1) - \int \frac{(x+1)}{(x+1)} dx + \int \frac{1}{(x+1)} dx + C = x \ln(x+1) - x + \ln(x+1) + C$$

$$\Rightarrow y = (x+1) \ln(x+1) - x + C$$

Given: at $x = 0$ $y = 3 \Rightarrow C = 3$

Therefore, $y = (x+1) \ln(x+1) - x + 3$

Example 4: Solve $\frac{dy}{dx} = e^y (e^x + x^2 e^{x^3})$

Soln: $\frac{dy}{dx} = e^y (e^x + x^2 e^{x^3}) \Rightarrow \int e^{-y} dy = \int (e^x + x^2 e^{x^3}) dx \Rightarrow -e^{-y} = e^x + (1/3)e^{x^3} + C$

Example 5: Solve $(x+y)(dx-dy) = dx+dy$

Soln: Re-writing the given equation, we get $(x+y-1)dx = (x+y+1)dy \Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x+y+1}$

Let $x+y = v \Rightarrow \frac{dy}{dx} = (dv/dx) - 1$

Therefore the given equation becomes,

$$\frac{dv}{dx} - 1 = \frac{v-1}{v+1} \Rightarrow \frac{dv}{dx} = \frac{2v}{v+1} \Rightarrow \int 2dx = \int \left(1 + \frac{1}{v}\right) dv$$

$$\Rightarrow 2x + c = v + \ln v \quad \Rightarrow -y + c = \ln(x+y)$$

Example 6: Solve $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$

Soln: The given equation may be re-written as $\frac{dy}{dx} = \frac{2(2x+3y)+5}{(2x+3y)+4}$

Taking $2x+3y = v \Rightarrow 2+3\frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{3} \left(\frac{dv}{dx} - 2 \right)$

Therefore, the given equation becomes,

$$\frac{1}{3} \left(\frac{dv}{dx} - 2 \right) = \frac{2v+5}{v+4} \Rightarrow \frac{dv}{dx} = \frac{3(2v+5)}{v+4} + 2 = \frac{8v+23}{v+4}$$