CASE 3:

Suppose a function $f(t)$ is defined in the interval $\left(0, \frac{2\pi}{\omega}\right)$, then the Fourier series expansion of $f(x)$ is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$a_0 = \frac{\omega}{\pi}\int_0^{2\pi/\omega} f(t)dt\ ;\ a_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega} f(t)\cos n\omega t\, dt\ ;\ b_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega} f(t)\sin n\omega t\, dt$$

CASE 4:

Suppose a function $f(t)$ is defined in the interval $\left(-\frac{\pi}{\omega}, \frac{\pi}{\omega}\right)$, then the Fourier series expansion of $f(x)$ is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$a_0 = \frac{\omega}{\pi}\int_{-\pi/\omega}^{\pi/\omega} f(t)dt\ ;\ a_n = \frac{\omega}{\pi}\int_{-\pi/\omega}^{\pi/\omega} f(t)\cos n\omega t\, dt\ ;\ b_n = \frac{\omega}{\pi}\int_{-\pi/\omega}^{\pi/\omega} f(t)\sin n\omega t\, dt$$

■ Nature of the fourier series for even function and odd function:

CASE 1: EVEN FUNCTION

A function is said to be even if $f(-x) = f(x)$ i.e. nature of the function is symmetric about $x = 0$.
We know that

$$\int_{-L}^{L} F(x)\,dx = \begin{cases} 2\int_0^L F(x)\,dx & \text{if } F(x) \text{ is an even function} \\ 0 & \text{if } F(x) \text{ is an odd function} \end{cases}$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{2}{L}\int_0^L f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx = 0$$

This series will contain only cosine terms and the fourier series expansion can be written as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}$$