

Average

Average : The average of a number is a measure of the central tendency of a set of numbers. In other words, it is an estimate of where the centre point of a set of numbers lies.

The basic formula for the average of n numbers $x_1, x_2, x_3, \dots, x_n$ is

$$A_n = (x_1 + x_2 + x_3 + \dots + x_n) / n = (\text{Total sum of } n \text{ numbers}) / n$$

This also means $A_n \times n = \text{total sum of 'n' numbers}$.

The average is always calculated for a set of numbers.

$$\text{Average} = \left(\frac{\text{Sum of observations}}{\text{Number of observation}} \right)$$

Weighted average: When we have 2 or more groups whose individual average are known then to find the combined average of all the elements of all the groups we use weighted average. Thus if we have k groups with average A_1, A_2, \dots, A_k and having n_1, n_2, \dots, n_k elements then the weighted average is given by the formula.

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \dots + n_k A_k}{n_1 + n_2 + n_3 + \dots + n_k}$$

Example-1: There are two sections A and B of a class consisting of 36 and 44 students respectively. If the average weight of section A is 40 kg and that of section B is 35 kg. Calculate average weight of whole class.

$$A_w = \frac{36 \times 40 + 44 \times 35}{36 + 44} = 37.25$$

Important points about average:

- (1) The average always lies above the lowest number of the set and the below highest number of the set.
- (2) The net deficit due to the numbers below the average always equals the net surplus due to the numbers above the average.
- (3) Ages and averages: If the average age of a group of persons is x years today then after ' n ' years their average age will be $(x+n)$.

Also, n years ago their average age would have been $(x-n)$. This happens due to the fact that for a group of people, 1 years is added to each persons' age every year.

Median: Median is the middle value that separates the higher half from the lower half of the set of observations.
 Example: To find the median of the set of observations {2, 10, 15, 12, 3, 55, 4, }, arrange the set of observation in the increasing order as {2, 3, 4, 10, 12, 15, 55}. Since, there are 7 observations odd, the median is the 4th term, that is 10.

Modes: The mode is the most frequently appearing value in the set of observation.

Example: The mode of the set of observation {1, 3, 5, 4, 5, 7, 2, 6, 5}. Here, 5 is the most frequently appearing value. So, 5 is the mode.

Standard Deviation: Standard deviation is a measure that is used to quantify the amount of variation of a set of values. A standard deviation close to 0 indicates that the data points tend to be very close to the mean of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values. Variance for N observations is defined as:

$$\text{Variance} = \sum \frac{1}{N} (x_i - \text{Mean})^2$$

$$\text{Standard Deviation} = \sqrt{\text{variance}}$$

Example: Consider the set of observations {2, 5, 7, 10}

$$\text{Here the mean} = \frac{2+5+7+10}{4} = 6$$

$$\text{Variance} = \frac{(2-6)^2 + (5-6)^2 + (7-6)^2 + (10-6)^2}{4} = \frac{34}{4} = \frac{17}{2}$$

$$\text{Standard Deviation} = \sqrt{\text{Variance}} = \sqrt{\frac{17}{2}}$$

SOLVED EXAMPLES

1. There were 35 students in a hostel. Due to the admission of 7 new students the expenses of the mess were increased by Rs. 42 per day while the average expenditure per head diminished by Rs. 1. The original expenditure of the mess was
 (a) 450 rs (b) 320 rs (c) 550 rs (d) 420 rs

Soln. Let original average expenditure = x rs
 then original expenditure for 35 students = $35x$ rs.
 After admission of 7 new students
 average expenditure = $(x - 1)$ rs
 hence total expenditure = $42(x - 1)$ rs

$$\begin{aligned} \therefore 42(x - 1) - 35x &= 42 \\ 42x - 42 - 35x &= 42 \\ 7x &= 84 \Rightarrow 12 = x \end{aligned}$$

then original expenditure = $35 \times 12 = 420$ rs

2. 9 persons went to a hotel for taking their meals. Eight of them spent 12 rs each on their meals and the ninth spent 8 rs. more than the average expenditure. What was the total money spent by them?
 (a) 115rs (b) 116rs (c) 117rs (d) 118rs

Soln. Eight of them spent 12 rs each on their meals.
 Therefore, total expenditure for eight of them = $12 \times 8 = 96$
 and the 9th person spend 8 rupees more than average expenditure i.e. $12 + 8 = 20$
 \therefore Total expenditure equals to $96 + 20 = 116$

Correct option is (b)