Chapter 2

MATRICES

2.1 Basic Review of Matrices

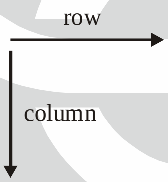
■ Definition:

A square or rectangular array of numbers or functions is known as matrices. Individual numbers or functions are known as elements of the matrix.

Example: $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}, \begin{bmatrix} 1-i & 2+i \\ 4+2i & x+iy \\ 2+4i & 3-2i \end{bmatrix}, \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

■ Order of a matrix:

A matrix of order $m \times n$ has 'm' number of rows and 'n' number of columns.



Every element of matrix is characterised by a row index and a column index:

a_{ij} : 'i' $\rightarrow$ row index , 'j' $\rightarrow$ column index, so, a_{ij} is the element of i^{th} row and j^{th} column

If a matrix A contains m rows and n columns, then it is written as $A = \left[a_{ij} \right]_{m \times n}$

■ Basic Review of matrix operation:

(i) Two matrices A and B are said to be equal if they are of the same order and each element of one is equal to corresponding element of the other matrix i.e. $a_{ij} = b_{ij}$ for all values of 'i' and 'j'.

(ii) Two matrices A and B are said to be conformable for addition/substraction if they have same order.

(iii) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\lambda A = \begin{pmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{pmatrix}$

(iv) Two matrices A and B are said to be conformable for multiplication if number of columns in A is equal to number of rows in B. Say, A matrix is of order '$m \times n$' and B matrix is of order '$p \times q$', then AB is possible only if $n = p$.

■ Various Type of Matrices:

(1) Square matrix: Matrix having number of rows equal to the number of columns i.e. $m = n$

Example: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$

(2) Diagonal matrix: Matrix having elements other than the principal diagonal elements are zero

i.e $a_{ij} = 0$ for $i \neq j$.

Example: $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

(3) Scalar matrix: Diagonal matrix having all principal diagonal elements to be same

i.e. $a_{ij} = 0$ for $i \neq j$ and a_{ii} is same for all i.

Example: $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

(4) Identity/unit matrix: Diagonal matrix having all principal diagonal elements equal to one.

i.e. $a_{ij} = 0$ for $i \neq j$ and a_{ii} is equal to 1 for all i.

Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(5) Row Matrix: Matrix having only one row.

Example: $\begin{bmatrix} a & b & c \end{bmatrix}_{1\times 3}$

(6) Column matrix: Matrix having only one column.

Example: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}_{3\times 1}$

(7) Null matrix: Matrix having all elements equal to zero i.e. $a_{ij} = 0$ for i, j

Example: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(8) Upper triangular matrix: Matrix having elements below the principal diagonal equal to zero

i.e. $a_{ij} = 0$ for all $i > j$

Example: $\begin{bmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$

(9) Lower triangular matrix: Matrix having elements above the principal diagonal equal to zero

i.e. $a_{ij} = 0$ for all $i < j$