

**Q.** 1 gm molecule of a monatomic ( $\gamma = \frac{5}{3}$ ) perfect gas at  $27^\circ\text{C}$  is adiabatically compressed in a reversible process from an initial pressure of 1 atmosphere to a final pressure of 50 atmosphere. Calculate the resulting difference in temperature.

**Soln.** Given  $P_1 = 1 \text{ atm}$   $P_2 = 50 \text{ atm}$   $T_1 = 27^\circ\text{C} = 300\text{K}$

$$T_2 = ? \quad \gamma = \frac{5}{3}$$

$$\frac{P_1^{\gamma-1}}{T_1^\gamma} = \frac{P_2^{\gamma-1}}{T_2^\gamma} \quad \text{or} \quad \left(\frac{P_2}{P_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^\gamma$$

$$(50)^{\frac{5}{3}-1} = \left(\frac{T_2}{300}\right)^{\frac{5}{3}} \Rightarrow 50^{\frac{2}{3}} = \left(\frac{T_2}{300}\right)^{\frac{5}{3}}$$

$$\frac{2}{3} \log 50 = \frac{5}{3} \log T_2 - \frac{5}{3} \log 300$$

$$T_2 = 1434 \text{ K} = 1161^\circ\text{C}$$

### Work done during an isothermal process

**Q.** A certain gas obeys the eqn of state  $P(V-b) = RT$

(i) Derive an expression for heat and work in a reversible isothermal expansion of the gas

(ii) Derive an expression for heat and work in a reversible adiabatic expansion of the gas

(iii) Calculate  $\Delta U$ ,  $Q$  and  $W$  when 1 mol of the gas expands reversibly at  $300\text{K}$  from  $0.220$  to  $20.02$  litre

(i) isothermally (ii) adiabatically. Assume that  $b = 0.020 \text{ lit mol}^{-1}$ ,  $C_v = \frac{3}{2}R$  and  $\gamma = \frac{5}{3}$

**Soln.** (i)  $dW = PdV$  or  $W = \int_{V_1}^{V_2} PdV$

Given equation of state is  $P = \frac{RT}{(V-b)}$

$$\text{Thus } W = \int_{V_1}^{V_2} \frac{RT}{V-b} dV = RT \ln \left( \frac{V_2 - b}{V_1 - b} \right)$$

$$\begin{aligned} \text{Now, } dU &= \left( \frac{\partial U}{\partial V} \right) dV + \left( \frac{\partial U}{\partial T} \right) dT \quad \left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial P}{\partial T} \right)_V - P = T \left( \frac{\partial \left( \frac{RT}{V-b} \right)}{\partial T} \right)_V - P \\ &= \frac{RT}{V-b} - P = P - P = 0 \quad \therefore \Delta U = 0 \end{aligned}$$

$$\text{Thus, } \Delta Q = \Delta U - W = -W = RT \ln \left( \frac{V_2 - b}{V_1 - b} \right)$$

(ii) for adiabatic reversible expansion  $\rightarrow Q = 0$

$$\text{Hence } dU = -dW = -\frac{RT}{V-b}dV$$

$$\Rightarrow \int_{T_1}^{T_2} \frac{C_V dT}{T} = -\int_{V_1}^{V_2} \frac{R}{V-b} dV \Rightarrow C_V \ln\left(\frac{T_2}{T_1}\right) = -R \ln\left(\frac{V_2-b}{V_1-b}\right)$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_2-b}{V_1-b}\right)^{-R/C_V} = \left(\frac{V_2-b}{V_1-b}\right)^{-(\gamma-1)}$$

$$\therefore W = -\Delta U = -C_V(T_2 - T_1) = -C_V T_1 \left[ \left(\frac{V_1-b}{V_2-b}\right)^{\gamma-1} - 1 \right]$$

(iii)  $T_1 = T_2 = 300\text{K}$ ,  $V_1 = 0.220$  litre,  $V_2 = 20.020$  litre,  $b = 0.020$  litre

For isothermal reversible expansion

$$W = -\Delta Q = -RT_1 \ln\left(\frac{V_2-b}{V_1-b}\right) = -(8.314)(300) \ln\left(\frac{20.0}{0.20}\right) = -11.548 \text{ KJ mol}^{-1}$$

$$\Delta U = 0$$

Now, for an adiabatic reversible expansion

$$\Delta Q = 0$$

$$\Delta U = -W = -C_V T_1 \left[ \left(\frac{V_1-b}{V_2-b}\right)^{\gamma-1} - 1 \right]$$

Given  $C_V = \frac{3}{2}R$ ,  $\gamma = \frac{5}{3}$  and  $T_1 = 300\text{K}$

$$\Delta U = -W = -\left(\frac{3}{2}R\right)(300) \left[ \left(\frac{200}{20000}\right)^{\frac{2}{3}} - 1 \right] = 3.554 \text{ kJ.}$$

### Slope of Adiabatic and Isothermal Process

(a) **Isothermal Process** : For an ideal gas, we have

$$PV = \text{constant} \Rightarrow PdV + VdP = 0 \Rightarrow \left(\frac{dP}{dV}\right)_{\text{iso}} = -\frac{P}{V}$$

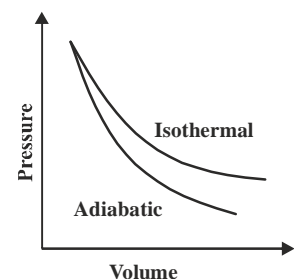
(b) **Adiabatic Process** : For an ideal gas, we have

$$PV^\gamma = \text{Constant} \Rightarrow P\gamma V^{\gamma-1}dV + dP \cdot V^\gamma = 0 \Rightarrow \frac{dP}{dV} = -\frac{PV^{\gamma-1}\gamma}{V^\gamma}$$

$$\Rightarrow \left(\frac{dP}{dV}\right)_{\text{adia}} = -\gamma \frac{P}{V}$$

$$\therefore \boxed{\frac{(dP/dV)_{\text{adia.}}}{(dP/dV)_{\text{iso.}}} = \gamma > 1}$$

... (35)



This shows that slope of adiabatic is  $\gamma$ -times than that of isothermal. Moreover, the adiabatic constant ( $\gamma$ ) can also be defined as the ratio of slope of  $P$ - $V$  curve for an adiabatic process to slope of  $P$ - $V$  curve for an isothermal process.