Chapter 4

Collision & Scattering

Types of Collision:

1. **Elastic:** Kinetic energy of colliding objects doesn't change. It implies that kinetic energy is not converted into internal energy. Therefore, in cae of elastic collision, we have following conservation laws

(a) Conservation of kinetic energy: Sum of kinetic energy of all colliding objects remains constant.

i.e. $K.E._{initial} = K.E_{final}$

(b) Convervation of momentum: Sum of momentum, of all colliding objects remains constant.

i.e. $\vec{p}_{initial} = \vec{p}_{final}$

or, $\left(p_{initial}\right)_x = \left(p_{final}\right)_x, \left(p_{initial}\right)_y = \left(p_{final}\right)_y$

2. **Inelastic:** Internal energy of colliding objects changes due to which kinetic energy doesn't remain conserved. Therefore, in case of inelastic collision we can only apply conservation of momentum. **Pefectly inelastic collision** is a special case of inelastic collision in which colliding objects stick to each other.

How to distinguish elastic and inelastic collision:

If collision is elastic then the in the problem it is explicitly mentioned in most cases that collision is elastic. However, if collision is inelastic then it may be explicitly mentioned or phrases such as 'there is loss of kinetic energy' or 'breaking of objects' or 'objects stick to each other' etc are used.

Coefficient of restitution:

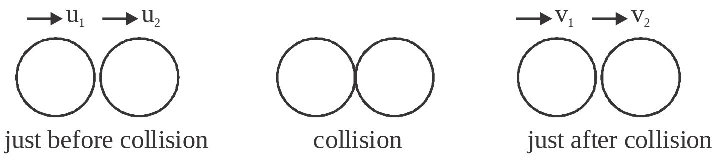
It is defined as

$$e = \frac{|\text{velocity of separation after collision}|}{|\text{velocity of approach before collision}|}$$

For elastic collision (involving no rotation) $e = 1$. For inelastic collision $0 \le e \le 1$. If objects stick to each other after collision. Then $e = 0$

4.1 Elastic collision in one dimension in Lab Frame:

Let m_1 and m_2 be masses of two objects having velocities u_1 and u_2 just before collision and velocities v_1 and v_2 just after collision.



From conservation of momentum we get,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{... (i)}$$

From conservation of kinetic energy we get,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + m_2v_2^2 \qquad \text{... (ii)}$$

Solving (i) and (ii), we get

$$\boxed{\begin{aligned} v_1 &= \frac{u_1(m_1 - m_2) + 2m_2u_2}{m_1 + m_2} \\ v_2 &= \frac{u_2(m_2 - m_1) + 2m_1u_1}{m_1 + m_2} \end{aligned}}$$

Using above results we can calculate velocities of objects after collision in one dimensional elastic collision if velocities before collision is either given or can be determined by some method. The above results have been derived by assuming that objects are moving in same direction. If in a problem objects are moving in opposite direction then one velocity must be taken to be negative.

Example: Two objects of masses m and 4m are moving with speed u and 2u in opposite direction. What is their velocities after collision

Soln. Let us take, $m_1 = m, u_1 = u$ and $m_2 = 4m$, $u_2 = -2u$

Therefore, velocity of m after collision

$$v_1 = \frac{u_1(m_1 - m_2) + 2m_2u_2}{m_1 + m_2} = \frac{u(m - 4m) + 2(4m)(-2u)}{5m} = \frac{-19}{5}u$$

Velocity of 4m after collision,

$$v_2 = \frac{u_2(m_2 - m_1) + 2m_1u_1}{m_1 + m_2} = \frac{-2u(4m - m) + 2mu}{5m} = -\frac{4}{5}u$$

Therefore, after collision both the masses move in the direction of velocity of 4m before collision.

Special case

- I-d Elastic collision of two equal masses

If $m_1 = m_2$ then, $v_1 = u_2$ and $v_2 = u_1$ i.e. the two objects exchange their velocities.

- One object is much heavier than the other object and heavier object is initially at rest.

$m_1 >> m_2$ and $u_1 = 0$

then, $v_1 = 0$ and $v_2 = -u_2$