Chapter 5

Electric Dipole & Dielectrics

Electric dipolemoment:

Dipole moment for a collection of charges, q_i having position vectors $\vec{r}_i$ is

$$\vec{p} = \sum_i q_i \vec{r}_i$$

For continuous system the dipole moment is written as

$$\vec{p} = \int_v \vec{r}\rho(\vec{r})dV$$

Show that, if the total charge is zero then dipole moment $(\vec{p})$ is independent of the choice of origin of the coordinate system:

Suppose we consider a new coordinate system whose origin O' is at $\vec{\ell}$ with respect to the origin O of the old coordinate system. The dipole moment with respect to old coordinate system is,

$$\vec{p}_{\text{old}} = \int \vec{r}\rho dV$$

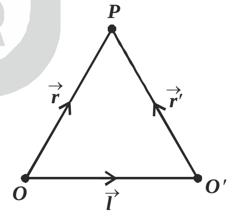
Let a point which is denoted by $\vec{r}\,'$ with respect to the old coordinate system and with respect to the new coordinate system.

$$\vec{r}\,' = \left(\vec{r} - \vec{\ell}\right)$$

The dipole moment with respect to the new system is

$$\vec{p}_{\text{new}} = \int \vec{r}\,'\rho dV = \int\left(\vec{r} - \vec{\ell}\right)\rho dV$$

$$= \int \vec{r}\rho dV + \vec{\ell}\int \rho dV = \int \vec{r}\rho dV + \vec{\ell}Q$$



where, $Q = \int \rho dV$ = total charge

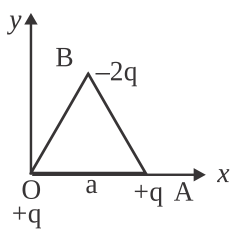
If $Q = 0$, then $\vec{p}_{\text{new}} = \int \vec{r}\rho dV = \vec{p}_{\text{old}}$ dipole moment in the new system is equal to dipole moment in the old system.

Example: Suppose three point charges $+q$, $+q$, $-2q$ place at the vertices of an equilateral triangle.

Soln. Total charge, $Q = +q + q - 2q = 0$

Therefore, dipole moment is independent of choice of origin.

The dipole moment with respect to origin.



$$= q\times 0 + q\left(a\hat{i}\right) - 2q\left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}}{2}a\hat{j}\right)$$

$$= 0 + qa\hat{i} - qa\hat{i} - q\sqrt{3}a\hat{j} = -\hat{j}\sqrt{3}aq = -\hat{j}\sqrt{3}aq$$

Example: Consider a sphere of radius r with a charge distribution $\sigma(\theta)=\sigma_0\cos\theta$. Find the dipole moment of the sphere.

Soln. Let elementary area,

$$ds = r^2\sin\theta d\theta d\phi \qquad \text{and} \qquad \sigma(\theta_0)=\sigma_0\cos\theta$$

$$\therefore \qquad dq=\sigma ds=\sigma_0\cos\theta\, r^2\sin\theta d\theta d\phi \Rightarrow q=\sigma_0 r\int_0^{2\pi}d\phi\int_0^{\pi}\cos\theta\sin\theta\, d\theta=0$$

The dipole moment is independent of choice of origin becomes q to total charge is zero.

Let any point on the surface element given by

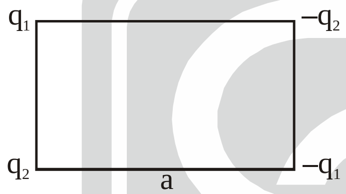
$$\vec{r}=r\sin\theta\cos\phi\hat{i}+r\sin\theta\sin\phi\hat{j}+r\cos\theta\hat{k}$$

Therefore, the dipole moment,

$$\vec{P}=\int\vec{r}dq=r^3\sigma_\theta\int_0^\pi\int_0^{2\pi}\left[\left(\sin^2\theta\cos\theta\cos\phi\right)\hat{i}+\left(\sin^2\theta\cos\theta\sin\theta\right)\hat{j}+\left(\cos\theta\sin\theta\right)\hat{k}\right]d\theta d\phi$$

$$=0+0+r^3\sigma_0\int_0^\pi\cos^2\theta\sin\theta d\theta\int_0^\pi d\phi\hat{k}=\frac{4}{3}\pi r^3\sigma_0\hat{k}$$

Example : Four point charges $\pm q_1$ and $\pm q_2$ are placed at the corners of a rectangle of sides a and b as shown in the figure :



What is the magnitude of he dipole moment of the system ?

(a) $(q_1+q_2)\sqrt{a^2+b^2}$

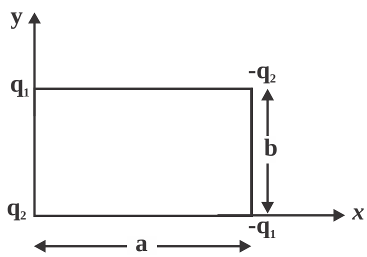
(b) $(q_1-q_2)(a-b)$

(c) $\sqrt{(q_1+q_2)^2a^2+(q_1-q_2)^2b^2}$

(d) The dipole moment will on the choice of origin

Soln. **(c)** Total charge is zero, therefore dipole moment will not depend on the choice of origin.

Let us take the axes as shown in the figure.



$$\vec{p}\sum_i q_i\vec{r}_i=-q_1a\hat{i}-q_2\left(a\hat{i}+b\hat{j}\right)+q_1b\hat{j} = -(q_1+q_2)a\hat{i}+(q_1-q_2)b\hat{j}$$

$$\therefore \qquad |\vec{p}|=\sqrt{(q_1+q_2)^2a^2+(q_1-q_2)^2b^2}$$