Chapter 2

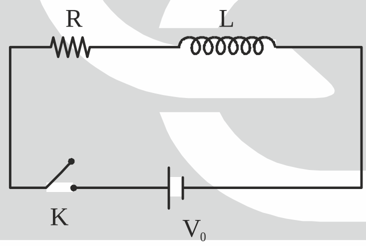
TRANSIENT PHENOMENA & AC CIRCUIT

Introduction:

If an electrical circuit containing inductors or capacitors is suddenly connected source of steady voltage. The current in the inductor or the charge on the capacitor does not become maximum value simultaneously but its takes some time. Similarly, when we remove the connection current in the conductor and charge on the capacitors does not become zero immediately. During the time enterval between switch on and current reach to steady state. The current in the inductor or charge in the capacitor change with time non-periodic manner, this is known as transient phenomena.

Growth and Decay of current in series of LR circuit:

Consider a LR circuit as we have shown in the figure. Initial key K was open and after t connected the key the current in the circuit i,

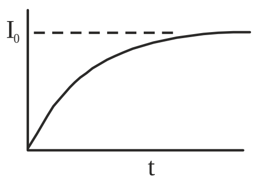


So, from Ohm's law:

$$i = \left(\frac{V_0 - L\frac{di}{dt}}{R} \right) \Rightarrow V_0 - L\frac{di}{dt} = iR \Rightarrow (V_0 - iR) = L\frac{di}{dt} \Rightarrow \frac{di}{i - \frac{V_0}{R}} = -\frac{R}{L}dt$$

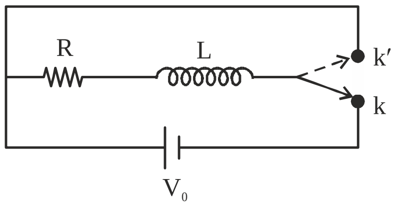
$$\Rightarrow \ln\left(\frac{i - V_0/R}{-V_0/R} \right) = -\frac{R}{L}t \Rightarrow i = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right) \Rightarrow i(t) = I_0\left(1 - e^{-Rt/L}\right)$$

So, current in the LR circuit growth with time as shown in figure and finally it attain maximum value.



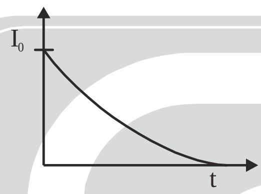
Decay of current:

Suppose steady state current has been set up in the LR circuit. Now, if we open the key from K and connect with K', then we can write,



$$i=\frac{0-L\frac{di}{dt}}{R}\Rightarrow \frac{di}{i}=-\frac{R}{L}dt \Rightarrow \left(\frac{i}{I_0}\right)=e^{-\frac{R}{L}t}\Rightarrow i(t)=I_0\,e^{-Rt/L}$$

So, current in the LR circuit decay exponantially and finally becomes zero.

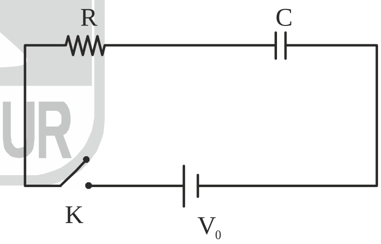


Charging and Discharging of a capacitor in series CR circuit:

(A) Charging of the capacitor:

In the below figure we have shown a circuit which contain capacitor C, resistance R in series with a voltage source V_0. At time $t = 0$ key is closed. At later time t the current in the circuit is i and charge in the capacitor is q. So, we can write,

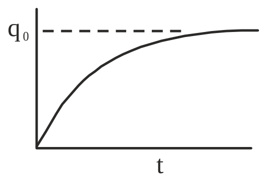
$$V_0=V_R+V_C \Rightarrow V_0=iR+\frac{q}{C} \Rightarrow V_0=R\frac{dq}{dt}+\frac{q}{C}$$



$$\Rightarrow \quad \frac{dq}{(q-V_0C)}=-\frac{dt}{RC} \Rightarrow \ln\left(\frac{q-V_0C}{-V_0C}\right)=-\frac{t}{RC}$$

$$\Rightarrow \quad q(t)=V_0C\left(1-e^{-t/RC}\right)\Rightarrow q(t)=q_0\left(1-e^{-t/RC}\right)$$

So, the charge in the capacitor growth with time as shown in figure and finally it reaches to maximum value q_0.



Time constant:

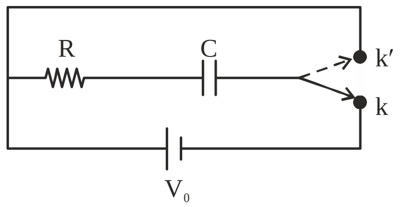
The quantity $RC=\tau$ has the dimension of time is called the time constant of the circuit.

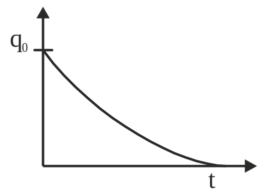
At time $t=\tau$, $q=q_0\left(1-\frac{1}{e}\right)=0.63\,q_0$

So, the time contsant τ may be defined as the time in which the charge on the capacitor grows from zero to 63% of its steady value.

(B) Discharging of the capacitor:

Suppose the capacitor is fully charged, now if we disconnect key from source and connect with k' then we can write,





$$0 = V_R + V_C \Rightarrow \frac{q}{c} + iR = 0 \Rightarrow \frac{dq}{q} = -\frac{1}{RC}dt$$

$$\Rightarrow \quad q(t) = q_0 \; e^{-t/RC}$$

So, charge on the capacitor will decay exponentially.

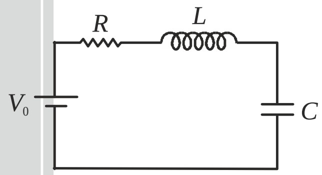
Series LCR circuit:

(A) Charging of the capacitor:

At $t = 0$ key is closed and after time, the charge in the capacitor is q.

So, we can write,

$$V_0 = \frac{q}{C} + L\frac{di}{dt} + iR$$



$$\Rightarrow \quad V_0 = \frac{q}{C} + L\frac{d^2q}{dt^2} + R\frac{dq}{dt} \qquad \left(i = \frac{dq}{dt}\right)$$

$$\Rightarrow \quad L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_0$$

$$\Rightarrow \quad \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = \frac{V_0}{L}$$

Consider $2b = \frac{R}{L}$ and $\omega_0^2 = \frac{1}{LC}$

$$\Rightarrow \quad \frac{d^2q}{dt^2} + 2b\frac{dq}{dt} + \omega_0^2 q = \frac{V_0}{L}$$

$$\Rightarrow \quad \frac{d^2q}{dt^2} + 2b\frac{dq}{dt} + \omega_0^2\left(q - \frac{V_0}{\omega_0^2 L}\right) = 0$$

Consider, $Q = \left(q - \frac{V_0}{\omega_0^2 L}\right) \Rightarrow \frac{dQ}{dt} = \frac{dq}{dt} \Rightarrow \frac{d^2Q}{dt^2} = \frac{d^2q}{dt}$