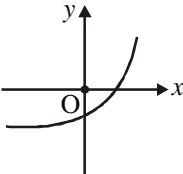
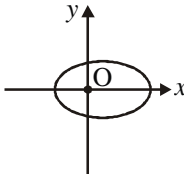


CSIR-UGC-NET/JRF- JUNE - 2018
PHYSICAL SCIENCES BOOKLET - [A]

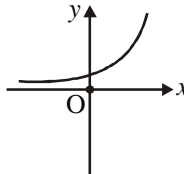
PART - B

21. Consider the three vectors $\vec{v}_1 = 2\hat{i} + 3\hat{k}$, $\vec{v}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{v}_3 = 5\hat{i} + \hat{j} + \alpha\hat{k}$, where \hat{i} , \hat{j} and \hat{k} are the standard unit vectors in a three-dimensional Euclidean space. These vectors will be linearly dependent if the value of α is
 (a) $31/4$ (b) $23/4$ (c) $27/4$ (d) 0
22. The fourier transform $\int_{-\infty}^{\infty} dx f(x) e^{ikx}$ of the function $f(x) = e^{-|x|}$ is
 (a) $-\frac{2}{1+k^2}$ (b) $-\frac{1}{2(1+k^2)}$ (c) $\frac{2}{1+k^2}$ (d) $\frac{2}{(2+k^2)}$
23. The value of the integral $\int_{-\pi/2}^{+\pi/2} dx \int_{-1}^{+1} dy \delta(\sin 2x) \delta(x-y)$ is
 (a) 0 (b) $1/2$ (c) $1/\sqrt{2}$ (d) 1
24. Consider the following ordinary differential equation : $\frac{d^2x}{dt^2} + \frac{1}{x} \left(\frac{dx}{dt}\right)^2 - \frac{dx}{dt} = 0$
 with the boundary conditions $x(t=0) = 0$ and $x(t=1) = 1$. The value of $x(t)$ at $t=2$ is
 (a) $\sqrt{e-1}$ (b) $\sqrt{e^2+1}$ (c) $\sqrt{e+1}$ (d) $\sqrt{e^2-1}$
25. What is the value of α for which $f(x, y) = 2x + 3(x^2 - y^2) + 2i(3xy + \alpha y)$ is an analytic function of complex variable $z = x + iy$?
 (a) 1 (b) 0 (c) 3 (d) 2
26. A particle moves in the one-dimensional potential $V(x) = \alpha x^6$, where $\alpha > 0$ is a constant. If the total energy of the particle is E , its time period in a periodic motion is proportional to
 (a) $E^{-1/3}$ (b) $E^{-1/2}$ (c) $E^{1/3}$ (d) $E^{1/2}$
27. Which of the following figures best describes the trajectory of a particle moving in a repulsive central potential $V(r) = \frac{\alpha}{r}$ ($\alpha > 0$ is a constant)?
- 

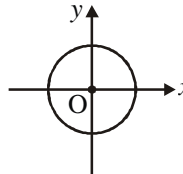
(a)



(b)

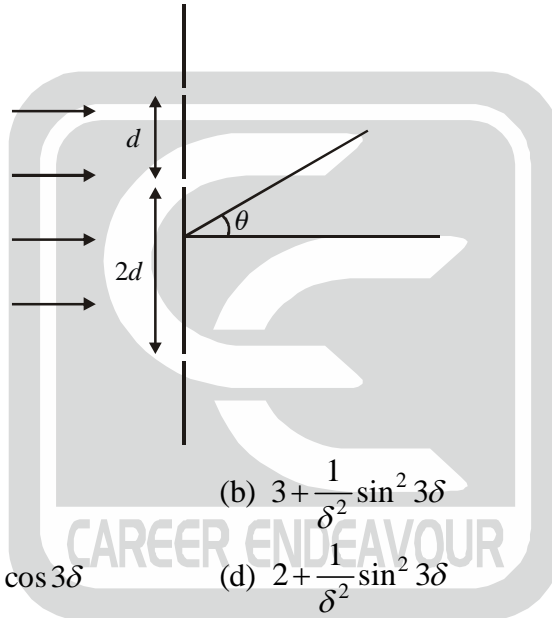


(c)



(d)
28. Two particles A and B move with relativistic velocities of equal magnitude v , but in opposite directions, along the x -axis of an inertial frame of reference. The magnitude of the velocity of A, as seen from the rest frame of B, is
 (a) $\frac{2v}{\left(1 - \frac{v^2}{c^2}\right)}$ (b) $\frac{2v}{\left(1 + \frac{v^2}{c^2}\right)}$ (c) $2v\sqrt{\frac{c-v}{c+v}}$ (d) $\frac{2v}{\sqrt{1 - \frac{v^2}{c^2}}}$

29. A particle of mass m , kept in a potential $V(x) = -\frac{1}{2}kx^2 + \frac{1}{4}\lambda x^4$, [where k and λ are positive constants], undergoes small oscillations about an equilibrium point. The frequency of oscillations is
- (a) $\frac{1}{2\pi} \sqrt{\frac{2\lambda}{m}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{k}{m}}$ (c) $\frac{1}{2\pi} \sqrt{\frac{2k}{m}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{\lambda}{m}}$
30. Two point charges $+2Q$ and $-Q$ are kept at points with Cartesian coordinates $(1, 0, 0)$ and $(2, 0, 0)$, respectively, in front of an infinite grounded conducting plate at $x = 0$. The potential at $(x, 0, 0)$ for $x \gg 1$ depends on x as
- (a) x^{-3} (b) x^{-5} (c) x^{-2} (d) x^{-4}
31. The following configuration of three identical narrow slits are illuminated by monochromatic light of wavelength λ (as shown in the figure below). The intensity is measured at an angle θ (where θ is the angle with the incident beam) at a large distance from the slits. If $\delta = \frac{2\pi d}{\lambda} \sin \theta$, the intensity is proportional to



- (a) $2 \cos \delta + 2 \cos 2\delta$ (b) $3 + \frac{1}{\delta^2} \sin^2 3\delta$
- (c) $3 + 2 \cos \delta + 2 \cos 2\delta + 2 \cos 3\delta$ (d) $2 + \frac{1}{\delta^2} \sin^2 3\delta$
32. The electric field of a plane wave in a conducting medium is given by
- $$\vec{E}(z, t) = \hat{i} E_0 e^{-z/3a} \cos\left(\frac{z}{\sqrt{3}a} - \omega t\right),$$
- where ω is the angular frequency and $a > 0$ is a constant. The phase difference between the magnetic field \vec{B} and the electric field \vec{E} is
- (a) 30° and \vec{E} lags behind \vec{B} (b) 30° and \vec{B} lags behind \vec{E}
- (c) 60° and \vec{E} lags behind \vec{B} (d) 60° and \vec{B} lags behind \vec{E}
33. The electric field \vec{E} and the magnetic field \vec{B} corresponding to the scalar and vector potentials, $V(x, y, z, t) = 0$ and $\vec{A}(x, y, z, t) = \frac{1}{2} \hat{k} \mu_0 A_0 (ct - x)$, where A_0 is a constant, are
- (a) $\vec{E} = 0$ and $\vec{B} = \frac{1}{2} \hat{j} \mu_0 A_0$ (b) $\vec{E} = -\frac{1}{2} \hat{k} \mu_0 A_0 c$ and $\vec{B} = \frac{1}{2} \hat{j} \mu_0 A_0$
- (c) $\vec{E} = 0$ and $\vec{B} = -\frac{1}{2} \hat{i} \mu_0 A_0$ (d) $\vec{E} = \frac{1}{2} \hat{k} \mu_0 A_0 c$ and $\vec{B} = -\frac{1}{2} \hat{i} \mu_0 A_0$



34. A particle of mass m is confined in a three-dimensional box by the potential

$$V(x, y, z) = \begin{cases} 0, & 0 \leq x, y, z \leq a \\ \infty, & \text{otherwise} \end{cases}$$

The number of eigenstates of Hamiltonian with energy $\frac{9\hbar^2\pi^2}{2ma^2}$ is

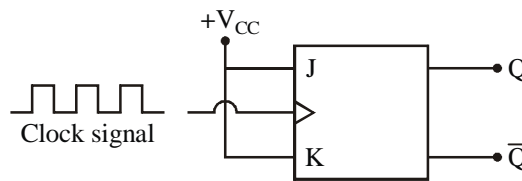
- (a) 1 (b) 6 (c) 3 (d) 4
35. The Hamiltonian of a spin- $\frac{1}{2}$ particle in a magnetic field \vec{B} is given by $H = -\mu\vec{B}\cdot\vec{\sigma}$, where μ is a real constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. If $\vec{B} = (B_0, B_0, 0)$ and the spin state at time $t = 0$ is an eigenstate of σ_x , then of the expectation values $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$
- (a) Only $\langle\sigma_x\rangle$ changes with time (b) Only $\langle\sigma_y\rangle$ changes with time
(c) Only $\langle\sigma_z\rangle$ changes with time (d) All three change with time
36. Two Stern-Gerlach apparatus S_1 and S_2 are kept in a line (x -axis). The directions of their magnetic fields are along the positive z - and y -axes, respectively. Each apparatus only transmits particles with spins aligned in the direction of its magnetic field. If an initially unpolarized beam of spin- $\frac{1}{2}$ particles passes through this configuration, the ratio of intensities $I_0 : I_f$ of the initial and final beams, is



- (a) 16 : 1 (b) 2 : 1 (c) 4 : 1 (d) 1 : 0
37. A particle of mass m is constrained to move in a circular ring to radius R . When a perturbation
- $$V' = \frac{a}{R^2} \cos^2 \phi$$
- (where a is a real constant) is added, the shift in energy of the ground state, to first order in a , is
- (a) a/R^2 (b) $2a/R^2$ (c) $a/(2R^2)$ (d) $a/(\pi R^2)$
38. Which of the following statements concerning the coefficient of volume expansion α and the isothermal compressibility κ of a solid is true ?
- (a) α and κ are both intensive variables.
(b) α is an intensive and κ is an extensive variable.
(c) α is an extensive and κ is an intensive variable.
(d) α and κ are both extensive variables.
39. The van der Waals equation for one mole of a gas is $\left(p + \frac{a}{V^2}\right)(V - b) = RT$. The corresponding equation of state for n moles of this gas at pressure p , volume V and temperature T , is
- (a) $\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$ (b) $\left(p + \frac{a}{V^2}\right)(V - nb) = nRT$
(c) $\left(p + \frac{an^2}{V^2}\right)(V - nb) = RT$ (d) $\left(p + \frac{a}{V^2}\right)(V - nb) = RT$

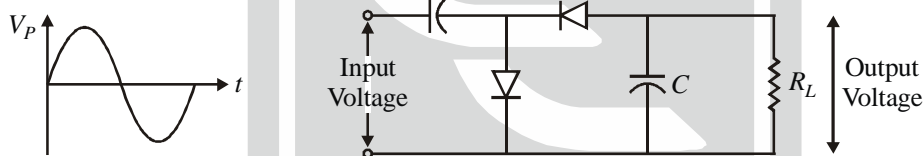
40. The number of ways of distributing 11 indistinguishable bosons in 3 different energy levels is
 (a) 3^{11} (b) 11^3 (c) $\frac{(13)!}{2!(11)!}$ (d) $\frac{(11)!}{3!8!}$
41. In a system of N distinguishable particles, each particle can be in one of two states with energies 0 and $-E$, respectively. The mean energy of the system at temperature T , is
 (a) $-\frac{1}{2}N(1+e^{E/k_B T})$ (b) $-\frac{NE}{(1+e^{E/k_B T})}$ (c) $-\frac{1}{2}NE$ (d) $-\frac{NE}{(1+e^{-E/k_B T})}$

42. In the following JK flip-flop circuit, J and K inputs are tied together to $+V_{CC}$. If the input is a clock signal of frequency f , the frequency of the output Q is



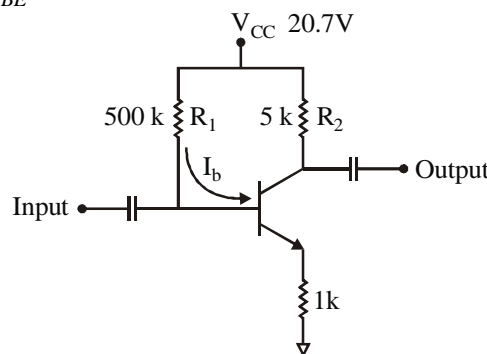
- (a) f (b) $2f$ (c) $4f$ (d) $f/2$
43. Which of the following gates can be used as a parity checker ?
 (a) an OR gate (b) a NOR gate
 (c) an exclusive OR (XOR) gate (d) an AND gate

44. A sinusoidal signal with a peak voltage V_p and average value zero, is an input to the following circuit.



Assuming ideal diodes, the peak value of the output voltage across the load resistor R_L , is

- (a) V_p (b) $V_p/2$ (c) $2V_p$ (d) $\sqrt{2}V_p$
45. In the following circuit, the value of the common-emitter forward current amplification factor β for the transistor is 100 and V_{BE} is 0.7 V.



The base current I_B is

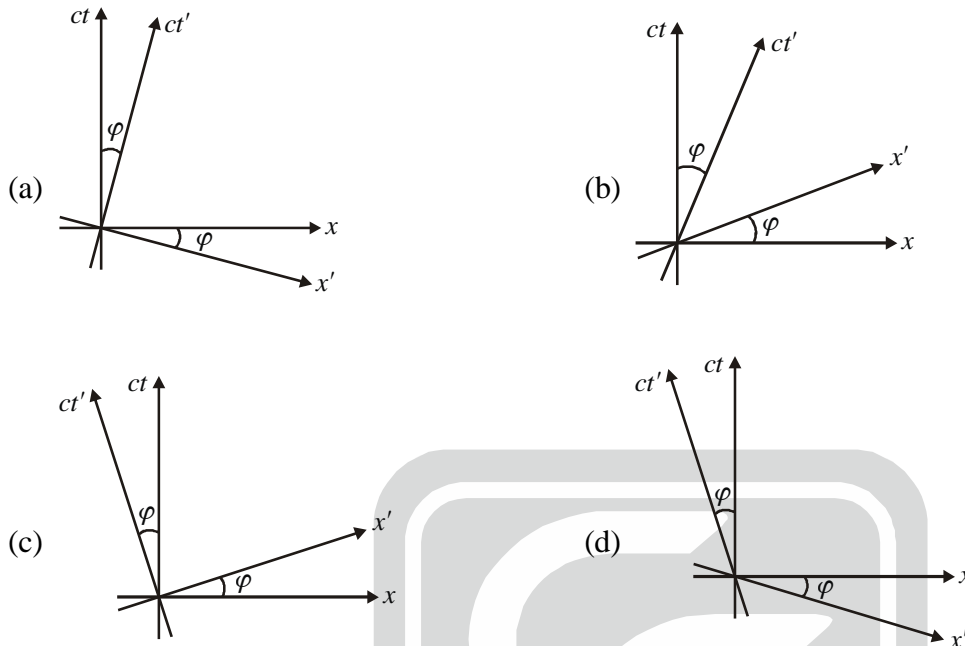
- (a) $40 \mu A$ (b) $30 \mu A$ (c) $44 \mu A$ (d) $33 \mu A$



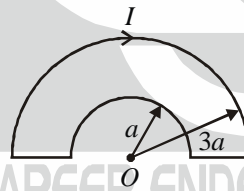
PART - C

46. In the function $P_n(x)e^{-x^2}$ of a real variable x , $P_n(x)$ is a polynomial of degree n . The maximum number of extrema that this function can have is
 (a) $n + 2$ (b) $n - 1$ (c) $n + 1$ (d) n
47. The Green's function $G(x, x')$ for the equation $\frac{d^2 y(x)}{dx^2} + y(x) = f(x)$, with the boundary values $y(0) = y\left(\frac{\pi}{2}\right) = 0$, is
- (a) $G(x, x') = \begin{cases} x\left(x' - \frac{\pi}{2}\right), & 0 < x < x' < \frac{\pi}{2} \\ \left(x - \frac{\pi}{2}\right), & 0 < x' < x < \frac{\pi}{2} \end{cases}$ (b) $G(x, x') = \begin{cases} -\cos x' \sin x, & 0 < x < x' < \frac{\pi}{2} \\ -\sin x' \cos x, & 0 < x' < x < \frac{\pi}{2} \end{cases}$
- (c) $G(x, x') = \begin{cases} \cos x' \sin x, & 0 < x < x' < \frac{\pi}{2} \\ \sin x' \cos x, & 0 < x' < x < \frac{\pi}{2} \end{cases}$ (d) $G(x, x') = \begin{cases} x\left(\frac{\pi}{2} - x'\right), & 0 < x < x' < \frac{\pi}{2} \\ x'\left(\frac{\pi}{2} - x\right), & 0 < x' < x < \frac{\pi}{2} \end{cases}$
48. The fractional error in estimating the integral $\int_0^1 x dx$ using Simpson's $\frac{1}{3}$ -rule, using a step size 0.1, is nearest to
 (a) 10^{-4} (b) 0 (c) 10^{-2} (d) 3×10^{-4}
49. Which of the following statements is true for a 3×3 real orthogonal matrix with determinant +1 ?
 (a) the modulus of each of its eigenvalues need not be 1, but their product must be 1.
 (b) at least one of its eigenvalues is +1.
 (c) all of its eigenvalues must be real.
 (d) none of its eigenvalues need be real.
50. A particle of mass m moves in a central potential $V(r) = -\frac{k}{r}$ in an elliptic orbit $r(\theta) = \frac{a(1-e^2)}{1+e \cos \theta}$, where $0 \leq \theta \leq 2\pi$ and a and e denote the semi-major axis and eccentricity, respectively. If its total energy is $E = -\frac{k}{2a}$, the maximum kinetic energy is
 (a) $E(1-e^2)$ (b) $E \frac{(e+1)}{(e-1)}$ (c) $\frac{E}{(1-e^2)}$ (d) $E \frac{(1-e)}{(1+e)}$
51. The Hamiltonian of a one-dimensional system is $H = \frac{xp^2}{2m} + \frac{1}{2}kx$, where m and k are positive constants. The corresponding Euler-Lagrange equation for the system is
 (a) $m\ddot{x} + k = 0$ (b) $m\ddot{x} + 2\dot{x} + kx^2 = 0$
 (c) $2m\ddot{x} - m\dot{x}^2 + kx^2 = 0$ (d) $m\ddot{x} - 2m\dot{x}^2 + kx^2 = 0$

52. An inertial frame K' moves with a constant speed v with respect to another inertial frame K along their common x -axis in the positive x -direction. Let (x, ct) and (x', ct') denote the space-time coordinates in the frame K and K' , respectively. Which of the following space-time diagrams correctly describes the t' -axis ($x' = 0$ line) and the x' -axis ($t' = 0$ line) in the $x - ct$ plane? (In the following figures $\tan \varphi = v/c$).



53. The loop shown in the figure below carries a steady current I .



The magnitude of the magnetic field at the point O is

- (a) $\frac{\mu_0 I}{2a}$ (b) $\frac{\mu_0 I}{6a}$ (c) $\frac{\mu_0 I}{4a}$ (d) $\frac{\mu_0 I}{3a}$
54. In the region far from a source, the time dependent electric field at a point (r, θ, ϕ) is

$$\vec{E}(r, \theta, \phi) = \hat{\phi} E_0 \omega^2 \left(\frac{\sin \theta}{r} \right) \cos \left[\omega \left(t - \frac{r}{c} \right) \right]$$

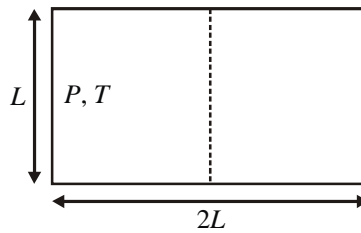
where ω is angular frequency of the source. The total power radiated (average over a cycle) is

- (a) $\frac{2\pi}{3} \frac{E_0^2 \omega^4}{\mu_0 c}$ (b) $\frac{4\pi}{3} \frac{E_0^2 \omega^4}{\mu_0 c}$ (c) $\frac{4}{3\pi} \frac{E_0^2 \omega^4}{\mu_0 c}$ (d) $\frac{2}{3} \frac{E_0^2 \omega^4}{\mu_0 c}$
55. A hollow waveguide supports transverse electric (TE) modes with the dispersion relation $k = \frac{1}{c} \sqrt{\omega^2 - \omega_{mn}^2}$, where ω_{mn} is the mode frequency. The speed of flow of electromagnetic energy at the mode frequency is
- (a) c (b) ω_{mn}/k (c) 0 (d) ∞

61. Consider a particle diffusing in a liquid contained in a large box. The diffusion constant of the particle in the liquid is $1.0 \times 10^{-2} \text{ cm}^2/\text{s}$. The minimum time after which the root-mean-squared displacement becomes more than 6 cm is

(a) 10 min (b) 6 min (c) 30 min (d) $\sqrt{6}$ min

62. A thermally insulated chamber of dimensions $(L, L, 2L)$ is partitioned in the middle. One side of the chamber is filled with n moles of an ideal gas at a pressure P and temperature T , while the other side is empty. At $t = 0$, the partition is removed and the gas is allowed to expand freely. The time to reach equilibrium varies as



(a) $n^{1/3} L^{-1} T^{1/2}$ (b) $n^{2/3} L T^{-1/2}$ (c) $n^0 L T^{-1/2}$ (d) $n L^{-1} T^{1/2}$

63. The maximum intensity of solar radiation is at the wavelength of $\lambda_{\text{sun}} \sim 5000 \text{ \AA}$ and corresponds to its surface temperature $T_{\text{sun}} \sim 10^4 \text{ K}$. If the wavelength of the maximum intensity of an X-ray star is 5 \AA , its surface temperature is of the order of

(a) 10^{16} K (b) 10^{14} K (c) 10^{10} K (d) 10^7 K

64. The full scale of a 3-bit digital-to-analog (DAC) converter is 7 V. Which of the following tables represents the output voltage of this 3-bit DAC for the given set of input bits ?

(a)

Input bits	Output voltage
000	0
001	1
010	2
011	3

(b)

Input bits	Output voltage
000	0
001	1.25
010	2.5
011	3.75

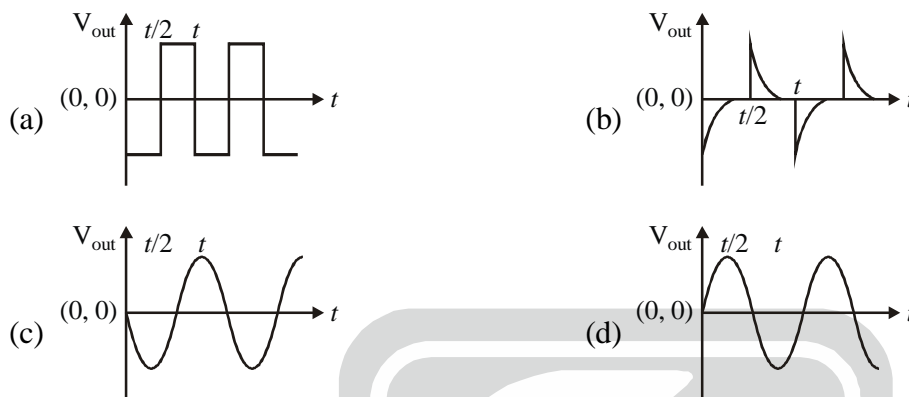
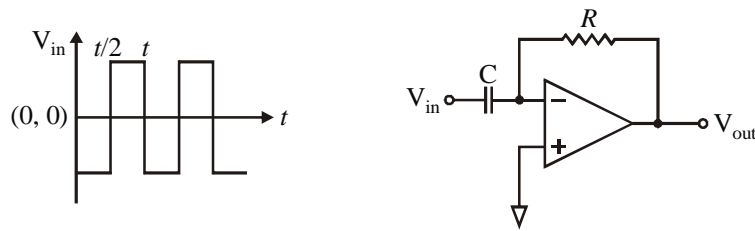
(c)

Input bits	Output voltage
000	1.25
001	2.5
010	3.75
011	5

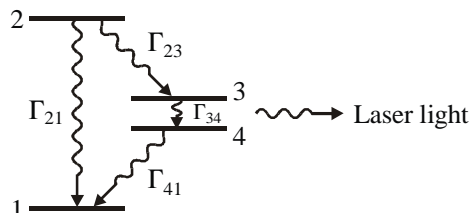
(d)

Input bits	Output voltage
000	1
001	2
010	3
011	4

65. The input V_i to the following circuit is a square wave as shown in the following figure :
Which of the waveforms V_o best describes the output ?



66. Two signals $A_1 \sin(\omega t)$ and $A_2 \cos(\omega t)$ are fed into the input and the reference channels, respectively, of a lock-in amplifier. The amplitude of each signal is 1V. The time constant of the lock-in amplifier is such that any signal of frequency larger than ω is filtered out. The output of the lock-in amplifier is
(a) 2V (b) 1V (c) 0.5V (d) 0V
67. A photon energy 115.62 keV ionizes a K-shell electron of a Be atom. One L-shell electron jumps to the K-shell to fill this vacancy and emits a photon of energy 109.2 keV in the process. If the ionization potential for the L-shell is 6.4 keV, the kinetic energy of the ionized electron is
(a) 6.42 keV (b) 12.82 keV (c) 20 eV (d) 32 eV
68. The value of the Lande g -factor for a fine structure level defined by the quantum numbers $L = 1$ and $J = 2$ and $S = 1$, is
(a) 11/6 (b) 4/3 (c) 8/3 (d) 3/2
69. The electronic energy level diagram of a molecule is shown in the following figure.



Let Γ_{ij} denote the decay rate for a transition from the level i and j . The molecules are optically pumped from level 1 to 2. For the transition from level 3 to level 4 to be a lasing transition, the decay rates have to satisfy

- (a) $\Gamma_{21} > \Gamma_{23} > \Gamma_{41} > \Gamma_{34}$ (b) $\Gamma_{21} > \Gamma_{41} > \Gamma_{23} > \Gamma_{34}$
(c) $\Gamma_{41} > \Gamma_{23} > \Gamma_{21} > \Gamma_{34}$ (d) $\Gamma_{41} > \Gamma_{21} > \Gamma_{34} > \Gamma_{23}$

70. Sodium Chloride (NaCl) crystal is a face centred cubic lattice, with a basis consisting of Na^+ and Cl^- ions separated by half the body diagonal of a unit cube. Which of the planes corresponding to the Miller indices given below will not give rise to Bragg reflection of X-rays ?
 (a) (2 2 0) (b) (2 4 2) (c) (2 2 1) (d) (3 1 1)
71. The dispersion relation for the electrons in the conduction band of a semiconductor is given by $E = E_0 + \alpha k^2$, where α and E_0 are constants. If ω_c is the cyclotron resonance frequency of the conduction band electrons in a magnetic field B , the value of α is
 (a) $\frac{\hbar^2 \omega_c}{4eB}$ (b) $\frac{2\hbar^2 \omega_c}{eB}$ (c) $\frac{\hbar^2 \omega_c}{eB}$ (d) $\frac{\hbar^2 \omega_c}{2eB}$
72. Hard discs of radius R are arranged in a two-dimensional triangular lattice. What is the fractional area occupied by the discs in the closest possible packing ?
 (a) $\frac{\pi\sqrt{3}}{6}$ (b) $\frac{\pi}{3\sqrt{2}}$ (c) $\frac{\pi\sqrt{2}}{5}$ (d) $\frac{2\pi}{7}$
73. Which of the following elementary particle processes does not conserve strangeness ?
 (a) $\pi^0 + p \rightarrow K^+ + \Lambda^0$ (b) $\pi^- + p \rightarrow K^0 + \Lambda^0$
 (c) $\Delta^0 \rightarrow \pi^0 + n$ (d) $K^0 \rightarrow \pi^+ + \pi^-$
74. A deuteron d captures a charged pion π^- in the $l=1$ state, and subsequently decays into a pair of neutrons (n) via strong interaction. Given that the intrinsic parities of π^- , d and n are -1 , $+1$ and $+1$ respectively, the spin-wavefunction of the final state neutrons is a
 (a) linear combination of a singlet and a triplet (b) singlet
 (c) triplet (d) doublet
75. The reaction ${}^{63}\text{Cu}_{29} + p \rightarrow {}^{63}\text{Zn}_{30} + n$ is followed by a prompt β -decay of zinc
 ${}^{63}\text{Zn}_{30} \rightarrow {}^{63}\text{Cu}_{29} + e^+ + \nu_e$. If the maximum energy of the positron is 2.4 MeV, the Q -value of the original reaction in MeV is nearest to
 [Take the masses of electrons, proton and neutron to be $0.5 \text{ MeV}/c^2$, $938 \text{ MeV}/c^2$ and $939.5 \text{ MeV}/c^2$, respectively].
 (a) -4.4 (b) -2.4 (c) -4.8 (d) -3.4