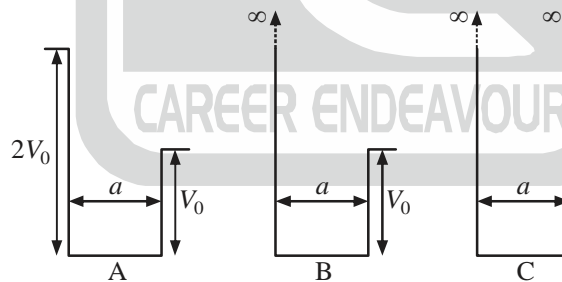


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PHYSICAL SCIENCES

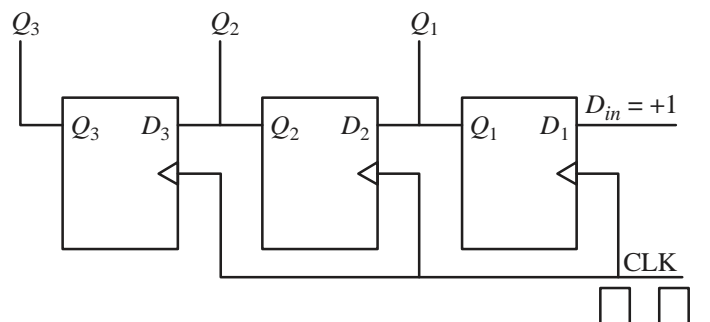
PART - B

1. The Boolean equation $Y = \bar{A}BC + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C}$ is to be implemented using only two-input NAND gates. The minimum number of gates required is
 (a) 3 (b) 4 (c) 5 (d) 6
2. The eigenvalues of the 3×3 matrix $M = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$ are
 (a) $a^2 + b^2 + c^2, 0, 0$ (b) $b^2 + c^2, a^2, 0$
 (c) $a^2 + b^2, c^2, 0$ (d) $a^2 + c^2, b^2, 0$
3. The wavelength of the first Balmer line of hydrogen is 656 nm. The wavelength of the corresponding line for a hydrogenic atom with $Z = 6$ and nuclear mass of 19.92×10^{-27} kg is
 (a) 18.2 nm (b) 109.3 nm (c) 143.5 nm (d) 393.6 nm
4. An idealised atom has a non-degenerate ground state at zero energy and a g -fold degenerate excited state of energy E . In a non-interacting system of N such atoms, the population of the excited state may exceed that of the ground state above a temperature $T > \frac{E}{2k_B \ln 2}$. The minimum value of g for which this is possible is
 (a) 8 (b) 4 (c) 2 (d) 1
5. For the one dimensional potential wells A, B and C, as shown in the figure, let E_A, E_B and E_C denote the ground states energies of a particle, respectively.



The correct ordering of the energies is

- (d) $E_C > E_B > E_A$ (b) $E_A > E_B > E_C$ (c) $E_B > E_C > E_A$ (d) $E_B > E_A > E_C$
6. In the 3-bit register shown below, Q_1 and Q_3 are the least and the most significant bits of the output, respectively.





If Q_1, Q_2 and Q_3 are set to zero initially, then the output after the arrival of the second falling clock (CLK) edge is

- (a) 001 (b) 100 (c) 011 (d) 110

7. The temperatures of two perfect black bodies A and B are 400 K and 200 K respectively. If the surface area of A is twice that of B, the ratio of total power emitted by A to that by B is

- (a) 4 (b) 2 (c) 32 (d) 16

8. Two coupled oscillators in a potential $V(x, y) = \frac{1}{2}kx^2 + 2kxy + \frac{1}{2}ky^2$ ($k > 2$) can be decoupled into two independent harmonic oscillators (coordinates: x', y') by means of an appropriate transformation

$\begin{pmatrix} x' \\ y' \end{pmatrix} = S \begin{pmatrix} x \\ y \end{pmatrix}$. The transformation matrix S is

- (a) $\begin{pmatrix} 1/\sqrt{2} & 1 \\ 1 & -1/\sqrt{2} \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

- (c) $\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

9. Two time dependent non-zero vectors $\vec{u}(t)$ and $\vec{v}(t)$, which are not initially parallel to each other, satisfy $\vec{u} \times \frac{d\vec{v}}{dt} - \vec{v} \times \frac{d\vec{u}}{dt} = 0$ at all time t . If the area of the parallelogram formed by $\vec{u}(t)$ and $\vec{v}(t)$ be

$A(t)$ and the unit normal vector to it be $\hat{n}(t)$, then

- (a) $A(t)$ increases linearly with t , but $\hat{n}(t)$ is a constant.
 (b) $A(t)$ increases linearly with t , and $\hat{n}(t)$ rotates about $\vec{u}(t) \times \vec{v}(t)$.
 (c) $A(t)$ is a constant, but $\hat{n}(t)$ rotates about $\vec{u}(t) \times \vec{v}(t)$.
 (d) $A(t)$ and $\hat{n}(t)$ are constants.

10. A function of a complex variable z is defined by the integral $f(z) = \oint_{\Gamma} \frac{w^2 - 2}{w - z} dw$, where Γ is a circular contour of radius 3, centred at origin, running counter-clockwise in the w -plane. The value of the function at $z = (2 - i)$ is

(a) 0 (b) $1 - 4i$ (c) $8\pi + 2\pi i$ (d) $-\frac{2}{\pi} - \frac{i}{2\pi}$

11. The Hamiltonian of a system of N non-interacting particles, each of mass m , in one dimension is

$$H = \sum_{i=1}^N \left(\frac{p_i^2}{2m} + \frac{\lambda}{4} x_i^4 \right)$$

where $\lambda > 0$ is a constant and p_i and x_i are the momentum and position respectively of the i -th particle. The average internal energy of the system is

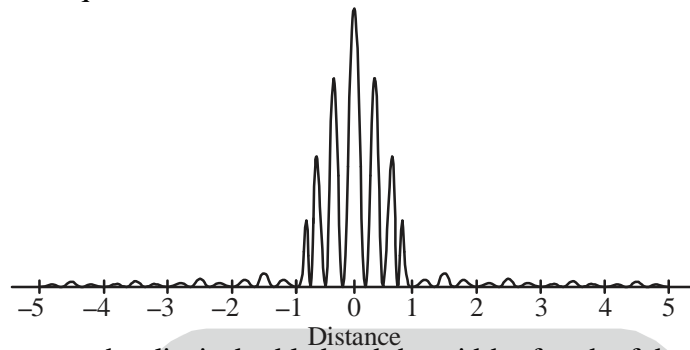
- (a) $\frac{4}{3}k_B T$ (b) $\frac{3}{4}k_B T$ (c) $\frac{3}{2}k_B T$ (d) $\frac{1}{3}k_B T$

12. An angular momentum eigenstate $|j, 0\rangle$ is rotated by an infinitesimally small angle ε about the positive y-axis in the counter-clockwise direction. The rotated state, to order ε (Upto a normalisation constant), is

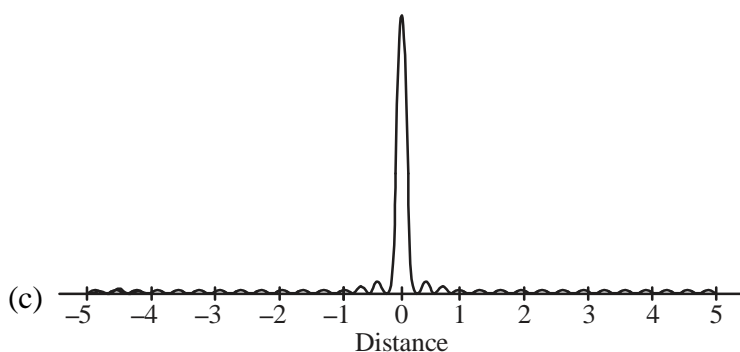
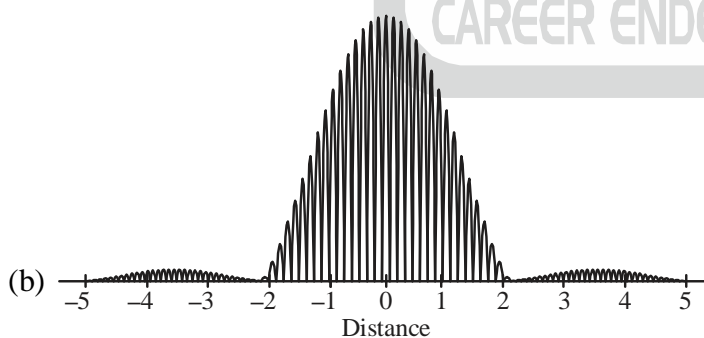
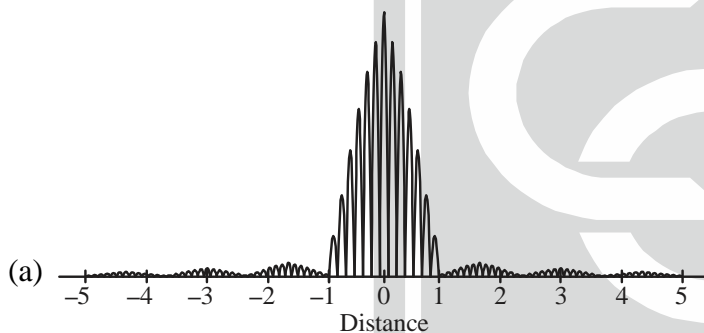
(a) $|j, 0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}(|j, 1\rangle + |j, -1\rangle)$ (b) $|j, 0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}(|j, 1\rangle - |j, -1\rangle)$

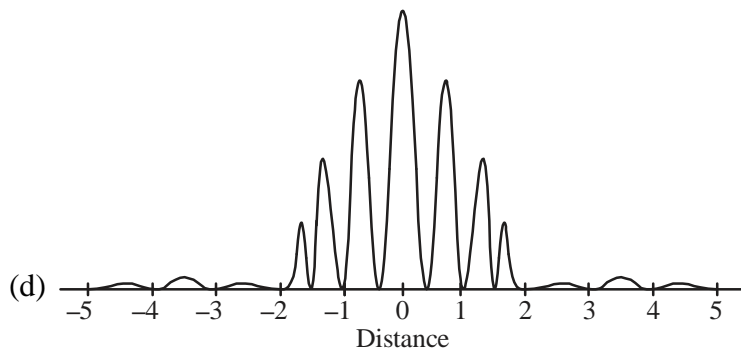
(c) $|j, 0\rangle - \frac{\varepsilon}{2}\sqrt{j(j-1)}(|j, 1\rangle - |j, -1\rangle)$ (d) $|j, 0\rangle - \frac{\varepsilon}{2}\sqrt{j(j+1)}|j, 1\rangle - \frac{\varepsilon}{2}\sqrt{j(j-1)}|j, -1\rangle$

13. The following figure show the intensity of the interference pattern in the Young's double-slit experiment with two slits of equal width is observed on a distant screen.



If the separation between the slits is doubled and the width of each of the slits is halved, then the new interference pattern is best represented by





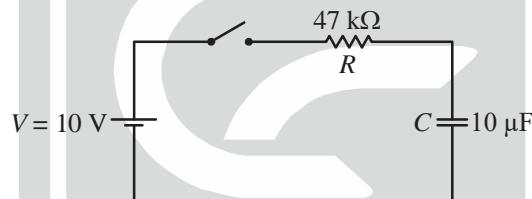
14. Let $\vec{E}(x, y, z, t) = \vec{E}_0 \cos(2x + 3y - \omega t)$, where ω is a constant, be the electric field of an electromagnetic wave travelling in vacuum. Which of the following vectors is a valid choice for \vec{E}_0 ?

(a) $\hat{i} - \frac{3}{2}\hat{j}$ (b) $\hat{i} + \frac{3}{2}\hat{j}$ (c) $\hat{i} + \frac{2}{3}\hat{j}$ (d) $\hat{i} - \frac{2}{3}\hat{j}$

15. An inductor L , a capacitor C and a resistor R are connected in series to an AC source, $V = V_0 \sin \omega t$. If the net current is found to depend only on R , then

(a) $C = 0$ (b) $L = 0$ (c) $\omega = \frac{1}{\sqrt{LC}}$ (d) $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

16. A 10 V battery is connected in series to a resistor R and a capacitor C , as shown in the figure.



The initial charge on the capacitor is zero. The switch is turned on and the capacitor is allowed to charge to its full capacity. The total work done by the battery in this process is

(a) 10^{-3} J (b) 2×10^{-3} J (c) 5×10^{-4} J (d) 47×10^{-2} J

17. A basket consists of an infinite number of red and black balls in the proportion $p : (1 - p)$. Three balls are drawn at random without replacement. The probability of their being two red and one black is a maximum for

(a) $p = \frac{3}{4}$ (b) $p = \frac{3}{5}$ (c) $p = \frac{1}{2}$ (d) $p = \frac{2}{3}$

18. Three infinitely long wires, each carrying equal current are placed in the xy -plane along $x = 0, +d$ and $-d$. On the xy -plane, the magnetic field vanishes at

(a) $x = \pm \frac{d}{2}$ (b) $x = \pm d \left(1 + \frac{1}{\sqrt{3}}\right)$ (c) $x = \pm d \left(1 - \frac{1}{\sqrt{3}}\right)$ (d) $x = \pm \frac{d}{\sqrt{3}}$

19. A frictionless horizontal circular table is spinning with a uniform angular velocity ω about the vertical axis through its centre. If a ball of radius 'a' is placed on it at a distance 'r' from the centre of the table, its linear velocity will be

(a) $-r\omega\hat{r} + a\omega\hat{\theta}$ (b) $r\omega\hat{r} + a\omega\hat{\theta}$ (c) $a\omega\hat{r} + r\omega\hat{\theta}$ (d) 0 (zero)



25. Two ideal gases in a box are initially separated by a partition. Let N_1, V_1 and N_2, V_2 be the numbers of particles and volumes occupied by the two systems. When the partition is removed, the pressure of the mixture at an equilibrium temperature T , is

(a) $k_B T \left(\frac{N_1 + N_2}{2(V_1 + V_2)} \right)$ (b) $k_B T \left(\frac{N_1 + N_2}{V_1 + V_2} \right)$ (c) $k_B T \left(\frac{N_1}{V_1} + \frac{N_2}{V_2} \right)$ (d) $\frac{1}{2} k_B T \left(\frac{N_1}{V_1} + \frac{N_2}{V_2} \right)$

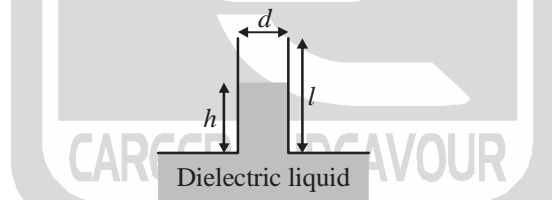
PART - C

1. Using the following values of x and $f(x)$

x	0	0.5	1.0	1.5
$f(x)$	1	a	0	$-5/4$

the integral $I = \int_0^{1.5} f(x) dx$, evaluated by the Trapezoidal rule is $5/16$. The value of ' a ' is

- (a) $3/4$ (b) $3/2$ (c) $7/4$ (d) $19/24$
2. The energies of the 3 lowest states of an atom are $E_0 = -14$ eV, $E_1 = -9$ eV and $E_2 = -7$ eV. The Einstein co-efficients are $A_{10} = 3 \times 10^8 \text{ sec}^{-1}$, $A_{20} = 1.2 \times 10^8 \text{ sec}^{-1}$ and $A_{21} = 8 \times 10^7 \text{ sec}^{-1}$. If a large number of atoms are in the energy level E_2 , the mean radiative lifetime of this excited state is
- (a) $8.3 \times 10^{-9} \text{ sec}$ (b) $1 \times 10^{-8} \text{ sec}$ (c) $0.5 \times 10^{-8} \text{ sec}$ (d) $1.2 \times 10^{-8} \text{ sec}$
3. A parallel plate capacitor with rectangular plates of length l , breadth b and plate separation d , is held vertically on the surface of a dielectric liquid of dielectric constant κ and density ρ as shown in the figure. The length and breadth are large enough for edge effects to be neglected.



The plates of the capacitor are kept at a constant voltage difference V . Ignoring effects of surface tension, the height h upto which the liquid level rises inside the capacitor, is

(a) $\frac{V^2 \epsilon_0 (\kappa - 1)}{\rho g b d}$ (b) $\frac{V^2 \epsilon_0 (\kappa - 1)}{2 \rho g b^2}$ (c) $\frac{V^2 \epsilon_0 (\kappa - 1)}{2 \rho g d^2}$ (d) $\frac{V^2 \epsilon_0 (\kappa - 1)}{\rho g d^2}$

4. The electric field due to a uniformly charged infinite line along the z -axis, as observed in the rest frame S of the line charge, is $\vec{E}(\vec{r}) = \frac{\lambda}{2\pi\epsilon_0} \frac{x\hat{i} + y\hat{j}}{(x^2 + y^2)}$. In a frame M moving with a constant speed v

with respect to S along the z -direction, the electric field \vec{E}' is (in the following

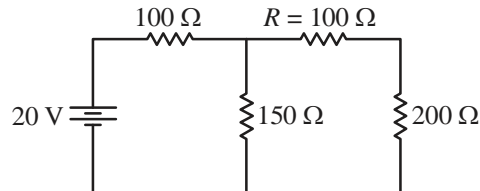
$\beta = v/c$ and $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$)

- (a) $E'_x = E_x$ and $E'_y = E_y$ (b) $E'_x = \beta\gamma E_x$ and $E'_y = \beta\gamma E_y$
- (c) $E'_x = \frac{E_x}{\gamma}$ and $E'_y = \frac{E_y}{\gamma}$ (d) $E'_x = \gamma E_x$ and $E'_y = \gamma E_y$

5. The Hamiltonian of a system of 3 spins is $H = J(S_1S_2 + S_2S_3)$, where $S_i = \pm 1$ for $i = 1, 2, 3$. Its canonical partition function, at temperature T , is

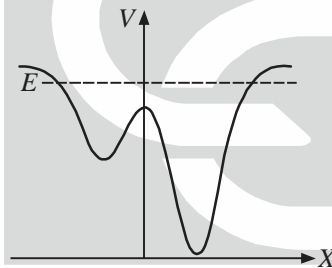
(a) $2\left(2\sinh\frac{J}{k_B T}\right)^2$ (b) $2\left(2\cosh\frac{J}{k_B T}\right)^2$ (c) $2\left(2\cosh\frac{J}{k_B T}\right)$ (d) $2\left(2\cosh\frac{J}{k_B T}\right)^3$

6. Two voltmeters A and B with internal resistances $2\text{ M}\Omega$ and $0.1\text{ k}\Omega$ are used to measure the voltage drops V_A and V_B , respectively, across the resistor R in the circuit shown below.

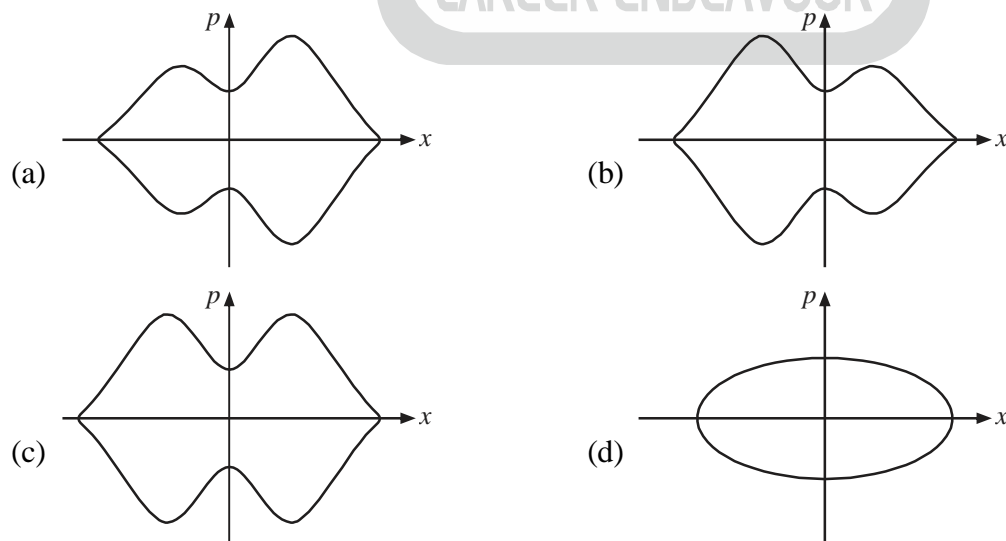


The ratio V_A/V_B is

- (a) 0.58 (b) 1.73 (c) 1 (d) 2
7. Falling drops of rain break up and coalesce with each other and finally achieve an approximately spherical shape in the steady state. The radius of such a drop scales with the surface tension σ as
- (a) $1/\sqrt{\sigma}$ (b) $\sqrt{\sigma}$ (c) σ (d) σ^2
8. Consider a particle with total energy E moving in one dimension in a potential $V(x)$ as shown in the figure below.

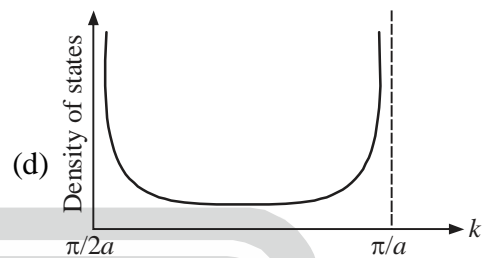
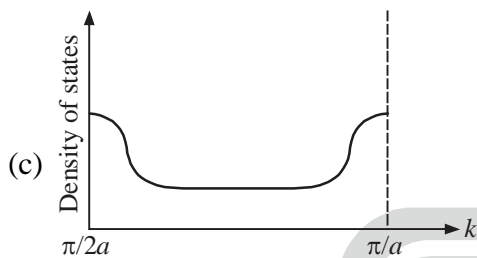
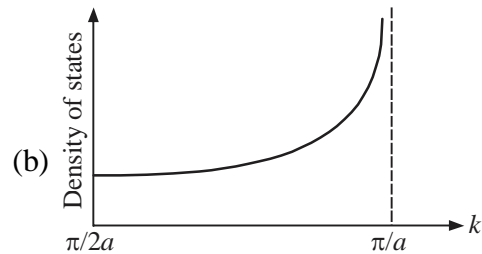
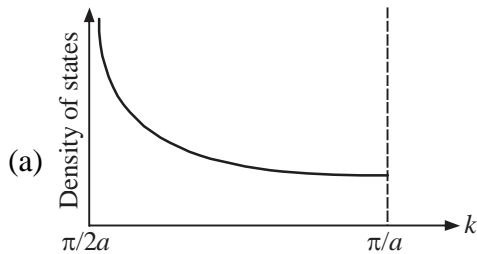


Which of the following figures best represents the orbit of the particle in the phase space ?

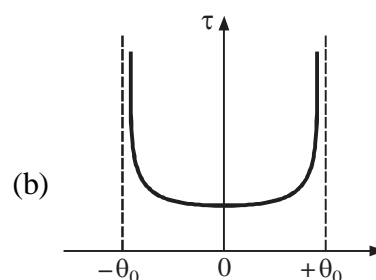
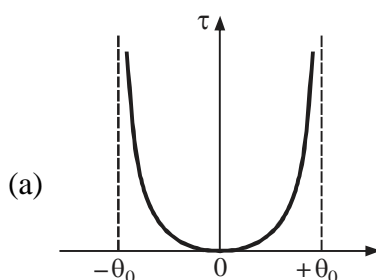


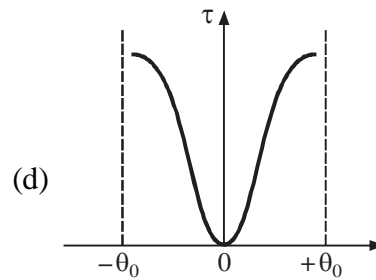
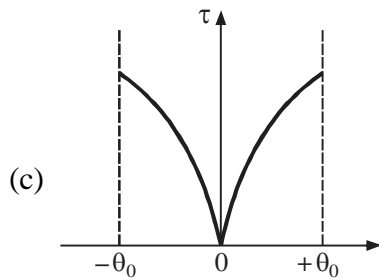


9. A tight binding model of electrons in one dimension has the dispersion relation $\varepsilon(k) = -2t(1 - \cos ka)$, where $t > 0$, a is the lattice constant and $-\frac{\pi}{a} < k < \frac{\pi}{a}$. Which of the following figures best represents the density of states over the range $\frac{\pi}{2a} \leq k < \frac{\pi}{a}$?



10. A quantum particle in a one-dimensional infinite potential well, with boundaries at 0 and a , is perturbed by adding $H' = \varepsilon \delta\left(x - \frac{a}{2}\right)$ to the initial Hamiltonian. The correction to the energies of the ground and the first excited states (to first order in ε) are respectively
- (a) 0 and 0 (b) $\frac{2\varepsilon}{a}$ and 0 (c) 0 and $\frac{2\varepsilon}{a}$ (d) $\frac{2\varepsilon}{a}$ and $\frac{2\varepsilon}{a}$
11. If we take the nuclear spin I into account, the total angular momentum is $\vec{F} = \vec{L} + \vec{S} + \vec{I}$, where \vec{L} and \vec{S} are the orbital and spin angular momenta of the electron. The Hamiltonian of the hydrogen atom is corrected by the additional interaction $\lambda \vec{I} \cdot (\vec{L} + \vec{S})$, where $\lambda > 0$ is a constant. The total angular momentum quantum number F of the p -orbital state with the lowest energy is
- (a) 0 (b) 1 (c) 1/2 (d) 3/2
12. A spacecraft of mass $m = 1000$ kg has a fully reflecting sail that is oriented perpendicular to the direction of the sun. The sun radiates 10^{26} W and has a mass $M = 10^{30}$ kg. Ignoring the effect of the planets, for the gravitational pull of the sun to balance the radiation pressure on the sail, the area of the sail will be
- (a) 10^2 m² (b) 10^4 m² (c) 10^8 m² (d) 10^6 m²
13. A pendulum executes small oscillations between angles $+\theta_0$ and $-\theta_0$. If $\tau(\theta)d\theta$ is the time spent between θ and $\theta + d\theta$, then $\tau(\theta)$ is best represented by

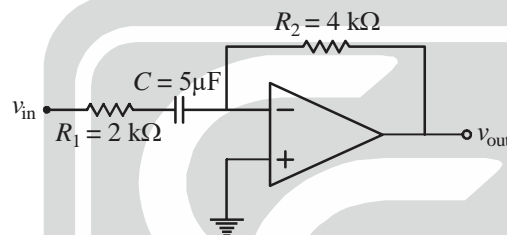




14. A particle with incoming wave vector \vec{k} , after being scattered by the potential $V(r) = \frac{c}{r^2}$, goes out with wave vector \vec{k}' . The differential scattering cross-section, calculated in the first Born approximation, depends on $q = |\vec{k} - \vec{k}'|$, as

(a) $\frac{1}{q^2}$ (b) $\frac{1}{q^4}$ (c) $\frac{1}{q}$ (d) $\frac{1}{q^{3/2}}$

15. In the circuit shown below, the gain of the op-amp in the middle of its bandwidth is 10^5 . A sinusoidal voltage with angular frequency $\omega = 100$ rad/sec is applied to the input of the op-amp.



The phase difference between the input and the output voltage is

(a) $\frac{5\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

16. A certain two-dimensional solid crystallises to a square monoatomic lattice with lattice constant 'a'. Each atom can contribute an integer number of free conduction electrons. The minimum number of electrons each atom must contribute such that the free electron Fermi circle at zero temperature encloses the first Brillouin zone completely, is

(a) 3 (b) 1 (c) 4 (d) 2

17. The absorption lines arising from pure rotational effects of HCl are observed at 83.03 cm^{-1} , 103.73 cm^{-1} , 124.30 cm^{-1} , 145.03 cm^{-1} and 165.51 cm^{-1} . The moment of inertia of the HCl molecule is

$\left[\text{Take } \frac{\hbar}{2\pi c} = 5.6 \times 10^{-44} \text{ kg-m} \right]$

(a) $1.1 \times 10^{-48} \text{ kg-m}^2$ (b) $2.8 \times 10^{-47} \text{ kg-m}^2$ (c) $2.8 \times 10^{-48} \text{ kg-m}^2$ (d) $1.1 \times 10^{-42} \text{ kg-m}^2$

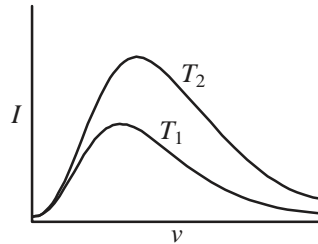
18. A rod pivoted at one end is rotating clockwise 25 times a second in a plane. A video camera which records at a rate of 30 frames per second is used to film the motion. To someone watching the video, the apparent motion of the rod will seem to be

- (a) 10 rotations per second in the clockwise direction.
 (b) 10 rotations per second in the anticlockwise direction.
 (c) 5 rotations per second in the clockwise direction.
 (d) 5 rotations per second in the anticlockwise direction.

24. The energy density I of a black body radiation at temperature T is given by the Planck's distribution function

$$I(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\left(e^{\frac{h\nu}{k_B T}} - 1\right)},$$

where ν is the frequency. The function $I(\nu, T)$ for two different temperatures T_1 and T_2 are shown below.



If the two curves coincide when $I(\nu, T)\nu^a$ is plotted against ν^b/T , then the values of a and b are, respectively.

- (a) 2 and 1 (b) -2 and 2 (c) 3 and -1 (d) -3 and 1
25. The Green's function for the differential equation $\frac{d^2x}{dt^2} + x = f(t)$, satisfying the initial conditions $x(0) = \frac{dx}{dt}(0) = 0$, is $G(t, \tau) = \begin{cases} 0 & \text{for } 0 < t < \tau \\ \sin(t - \tau) & \text{for } t > \tau \end{cases}$. The solution of the differential equation when the source $f(t) = \theta(t)$ (the Heaviside step function) is
- (a) $\sin t$ (b) $1 - \sin t$ (c) $1 - \cos t$ (d) $\cos^2 t - 1$
26. The solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} = e^y$, with the boundary conditions $y(0) = 0$ and $y'(0) = -1$ is
- (a) $-\ln\left(\frac{x^2}{2} + x + 1\right)$ (b) $-x \ln(e + x)$ (c) $-xe^{-x^2}$ (d) $-x(x + 1)e^{-x}$
27. Spin 1/2 fermions of mass m and $4m$ are in a harmonic potential $V(x) = \frac{1}{2}kx^2$. Which configuration of 4 such particles has the lowest value of the ground state energy ?
- (a) 4 particles of mass m
 (b) 4 particles of mass $4m$
 (c) 1 particle of mass m and 3 particles of mass $4m$
 (d) 2 particles of mass m and 2 particles of mass $4m$



28. The binding energy B of a nucleus is approximated by a formula:

$$B = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (A - 2Z)^2 A^{-1}$$

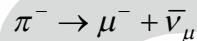
where Z is the atomic number and A is the mass number of the nucleus. If $\frac{a_4}{a_3} \simeq 30$, the atomic number Z for naturally stable isobars (constant value of A) is

(a) $\frac{30A}{60 + A^{2/3}}$ (b) $\frac{30A}{30 + A^{2/3}}$ (c) $\frac{60A}{120 + A^{2/3}}$ (d) $\frac{120A}{60 + A^{2/3}}$

29. The velocity $v(x)$ of a particle moving in one dimension is given by $v(x) = v_0 \sin(\pi x/x_0)$, where v_0 and x_0 are positive constants of appropriate dimensions. If the particle is initially at $x/x_0 = \varepsilon$, where $|\varepsilon| \ll 1$, then, in the long time, it

- (a) executes an oscillatory motion around $x = 0$.
 (b) tends towards $x = 0$.
 (c) tends towards $x = x_0$.
 (d) executes an oscillatory motion around $x = x_0$.

30. Charged pions π^- decay to muons μ^- and anti-muon neutrinos $\bar{\nu}_\mu$:



Take the rest masses of a muon and a pion to be 105 MeV and 140 MeV, respectively. The probability that the measurement of the muon spin along the direction of its momentum is positive, is closest to

- (a) 0.5 (b) 0.75 (c) 1 (d) 0

