CSIR-UGC–NET/JRF– DEC - 2019
PHYSICAL SCIENCES

PART - B

1. The energy eigenvalues of a particle of mass *m*, confined to a rigid one-dimensional box of width *L*, are $E_n(n=1,2,....)$. If the walls of the box are moved very slowly toward each other, the rate of change of time-dependent energy dE_2/dt of the first excited state is

(a) $\frac{E_1}{L}\frac{dL}{dt}$ (b) $\frac{2E_2}{L}\frac{dL}{dt}$ (c) $-\frac{2E_2}{L}\frac{dL}{dt}$ (d) $-\frac{E_1}{L}\frac{dL}{dt}$

2. A box contains 5 white and 4 black balls. Two balls are picked together at random from the box. What is the probability that these two balls are of different colours ?

(a) 1/2 (b) 5/18 (c) 1/3 (d) 5/9

3. Consider the set of polynomials $\left\{x(t)=a_0+a_1t+\cdots+a_{n-1}t^{n-1}\right\}$ in *t* of degrees less than *n*, such that $x(0)=0$ and $x(1)=1$. This set

(a) constitutes a vector space of dimension *n*
(b) constitutes a vector space of dimension *n* – 1
(c) constitutes a vector space of dimension *n* – 2
(d) does not constitute a vector space

4. A mole of gas at initial temperature T_i comes into contact with a heat reservoir at temperature T_f and the system is allowed to reach equilibrium at constant volume. If the specific heat of the gas is $C_V=\alpha T$, where α is a constant, the total change in entropy is

(a) Zero (b) $\alpha\left(T_f-T_i\right)+\frac{\alpha}{2T_f}\left(T_f-T_i\right)^2$

(c) $\alpha\left(T_f-T_i\right)$ (d) $\alpha\left(T_f-T_i\right)+\frac{\alpha}{2T_f}\left(T_f^2-T_i^2\right)$

5. The values of *a* and *b* for which the force $\vec{F}=(axy+z^3)\hat{i}+x^2\hat{j}+bxz^2\hat{k}$ is conservative are

(a) $a=2,\ b=3$ (b) $a=1,\ b=3$ (c) $a=2,\ b=6$ (d) $a=3,\ b=2$

6. Which of the following terms, when added to the Lagrangian $L(x,y,\dot{x},\dot{y})$ of a system with two degrees of freedom, will not change the equations of motion ?

(a) $x\ddot{x}-y\ddot{y}$ (b) $x\ddot{y}-y\ddot{x}$ (c) $x\dot{y}-y\dot{x}$ (d) $y\dot{x}^2+x\dot{y}^2$

7. The Hamiltonian of two interacting particles, one with spin-1 and the other with spin-$\frac{1}{2}$, is given by

$$H=AS_1\cdot S_2+B(S_{1x}+S_{2x}),$$

where S_1 and S_2 denote the spin operators of the first and second particles, respectively, and *A* and *B* are positive constants. The largest eigenvalue of this Hamiltonian is

(a) $\frac{1}{2}(A\hbar^2+3B\hbar)$ (b) $3A\hbar^2+B\hbar$ (c) $\frac{1}{2}(3A\hbar^2+B\hbar)$ (d) $A\hbar^2+3B\hbar$

8. Two spin-$\frac{1}{2}$ fermions of mass *m* are confined to move in a one-dimensional infinite potential well of width *L*. If the particles are known to be in a spin triplet state, the ground state energy of the system (in units of $\frac{\hbar^2\pi^2}{2mL^2}$) is

(a) 8 (b) 2 (c) 3 (d) 5

9. Consider black body radiation in thermal equilibrium contained in a two-dimensional box. The dependence of the energy density on the temperature *T* is

(a) T^3 (b) T (c) T^2 (d) T^4

10. An ideal Carnot engine extracts 100 J from a heat source and dumps 40 J to a heat sink at 300 K. The temperature of the heat source is

(a) 600 K (b) 700 K (c) 750 K (d) 650 K

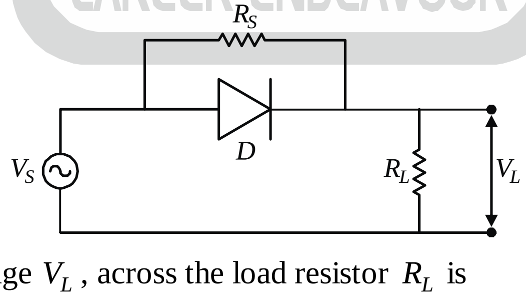
11. The *yz*-plane at $x = 0$ carries a uniform surface charge density σ. A unit point charge is moved from a point $(\delta, 0, 0)$ on one side of the plane to a point $(-\delta, 0, 0)$ on the other side. If δ is an infinitesimally small positive number, the work done in moving the charge is

(a) 0 (b) $\frac{\sigma}{\varepsilon_0}\delta$ (c) $-\frac{\sigma}{\varepsilon_0}\delta$ (d) $\frac{2\sigma}{\varepsilon_0}\delta$

12. The angular frequency of oscillation of a quantum harmonic oscillator in two dimensions is ω. It is contact with an external heat bath at temperature *T*, its partition function is (in the following $\beta = \frac{1}{k_B T}$)

(a) $\frac{e^{2\beta\hbar\omega}}{\left(e^{2\beta\hbar\omega}-1\right)^2}$ (b) $\frac{e^{\beta\hbar\omega}}{\left(e^{\beta\hbar\omega}-1\right)^2}$ (c) $\frac{e^{\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$ (d) $\frac{e^{2\beta\hbar\omega}}{e^{2\beta\hbar\omega}-1}$

13. In the circuit below, *D* is an ideal diode, the source voltage $V_S = V_0 \sin\omega t$ is a unit amplitude sine wave and $R_S = R_L$.



The average output voltage V_L, across the load resistor R_L is

(a) $\frac{1}{2\pi}V_0$ (b) $\frac{3}{2\pi}V_0$ (c) $3V_0$ (d) V_0

14. The electric field of an electromagnetic wave is $\vec{E} = \hat{i}\sqrt{2}\sin(kz - \omega t)\ \text{Vm}^{-1}$. The average flow of energy per unit area per unit time, due to this wave, is

(a) $27\times10^{4}\ \text{W/m}^2$ (b) $27\times10^{-4}\ \text{W/m}^2$ (c) $27\times10^{-2}\ \text{W/m}^2$ (d) $27\times10^{2}\ \text{W/m}^2$

15. A particle of mass *m* is confined to a box of unit length in one dimension. It is described by the

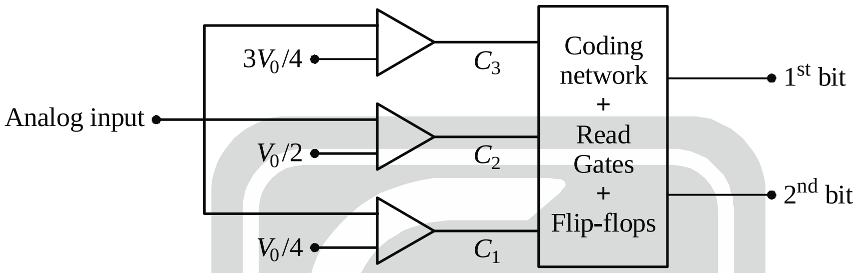
wavefunction $\psi(x)=\sqrt{8/5}\sin\pi x(1+\cos\pi x)$ for $0\le x\le 1$, and zero outside this interval. The expectation value of energy in this state is

(a) $\frac{4\pi^2}{3m}\hbar^2$ (b) $\frac{4\pi^2}{5m}\hbar^2$ (c) $\frac{2\pi^2}{5m}\hbar^2$ (d) $\frac{8\pi^2}{5m}\hbar^2$

16. If the rank of an $n\times n$ matrix A is m, where m and n are positive integers with $1\le m\le n$, then the rank of the matrix A^2 is

(a) m (b) $m-1$ (c) $2m$ (d) $m-2$

17. The figure below shows a 2-bit simultaneous analog-to-digital (A/D) converter operating in the voltage range 0 to V_0. The output of the comparators are C_1, C_2 and C_3 with the reference inputs $\frac{V_0}{4}, \frac{V_0}{2}$ and $\frac{3V_0}{4}$, respectively.



The logic expression for the output corresponding to the less significance bit is

(a) $C_1C_2C_3$ (b) $C_2\bar{C}_3+\bar{C}_1$ (c) $C_1\bar{C}_2+C_3$ (d) $C_2\bar{C}_3+C_2$

18. A positively charged particle is placed at the origin (with zero initial velocity) in the presence of a constant electric and a constant magnetic field along the positive z and x-directions, respectively. At large times, the overall motion of the particle is adrift along the

(a) positive y-direction (b) negative z-direction (c) positive z-direction
(d) negative y-direction

19. Let C be the circle of radius $\frac{\pi}{4}$, centered at $z=\frac{1}{4}$ in the complex z-plane that is traversed counter-clockwise. The value of the contour integral $\oint_C \frac{z^2}{\sin^2 4z}dz$ is

(a) 0 (b) $i\pi^2/4$ (c) $i\pi^2/16$ (d) $i\pi/4$

20. A student measures the displacement x from the equilibrium of a stretched spring and reports it be 100 μm with a 1% error. The spring constant k is known to be 10 N/m with 0.5 % error. The percentage error in the estimate of the potential energy $V=\frac{1}{2}kx^2$ is

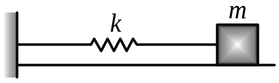
(a) 0.8 % (b) 2.5 % (c) 1.5 % (d) 3.0 %

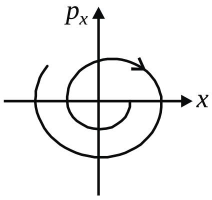
21. A ball, initially at rest, is dropped from a height h above the floor bounces again and again vertically.

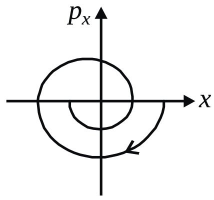
If the co-efficient of restitution between the ball and the floor is 0.5, the total distance travelled by the ball before it comes to rest is

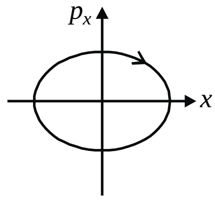
(a) $8h/3$ (b) $5h/3$ (c) $3h$ (d) $2h$

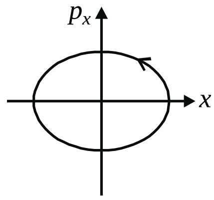
22. A block of mass m, attached to a spring, oscillates horizontally on a surface. The co-efficient of friction between the block and the surface is μ. Which of the following trajectories best describes the motion of the block in the phase space (xp_x-plane)?



(a) 

(b) 

(c) 

(d) 

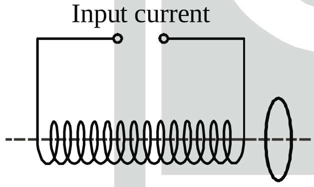
23. The normalized wavefunction of a particle in three-dimensions is given by

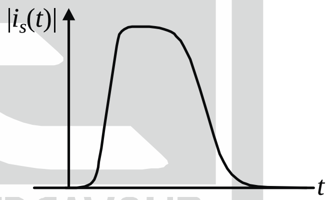
$$\psi(x, y, z) = N z \exp\left[-a(x^2 + y^2 + z^2)\right]$$

where a is a positive constant and N is a normalization constant. If L is the angular momentum operator, the eigenvalues, of L^2 and L_z, respectively, are

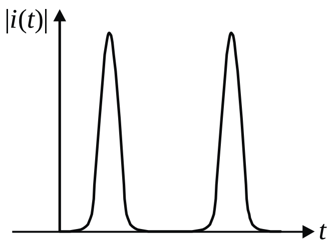
(a) $2\hbar^2$ and $\hbar$ (b) $\hbar^2$ and 0 (c) $2\hbar^2$ and 0 (d) $\frac{3}{4}\hbar^2$ and $\frac{1}{2}\hbar$

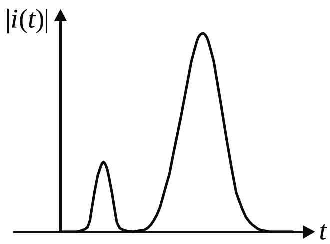
24. A circular conducting wire loop is placed close to a solenoid as shown in the figure below. Also shown is the current through the solenoid as a function of time.

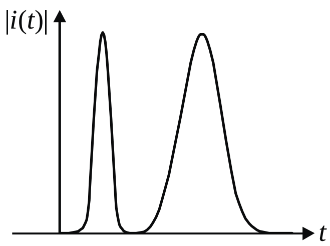


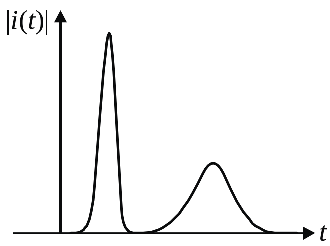


The magnitude $|i(t)|$ of the induced current in the wire loop, as a function of time t, is best represented as

(a) 

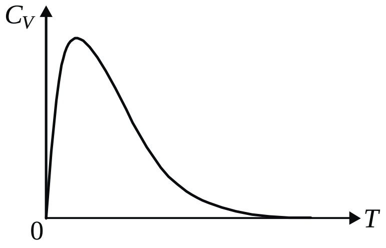
(b) 

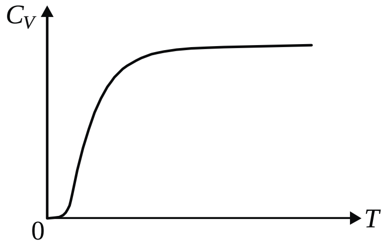
(c) 

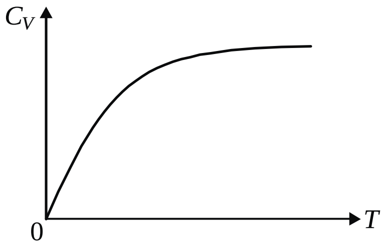
(d) 

25. The energies available to a three-state system are 0, E and $2E$, where $E > 0$. Which of the following

graphs best represents the temperature dependence of the specific heat ?

(a) 

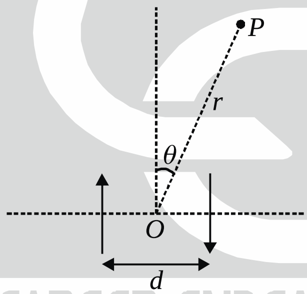
(b) 

(c) 

(d) 

PART - C

1. The phase difference between two small oscillating electric dipoles, separated by a distance d, is π. If the wavelength of the radiation is λ, the condition for constructive interference between the two dipolar radiations at a point P when $r \gg d$ (symbols are as shown in the figure, and n is an integer) is



(a) $d\sin\theta = \left(n + \frac{1}{2}\right)\lambda$ (b) $d\sin\theta = n\lambda$

(c) $d\cos\theta = n\lambda$ (d) $d\cos\theta = \left(n + \frac{1}{2}\right)\lambda$

2. The Hamiltonian of two particles, each of mass m, is

$$H(q_1, p_1; q_2, p_2) = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + k\left(q_1^2 + q_2^2 + \frac{1}{4}q_1 q_2\right), \text{ where } k > 0 \text{ is a constant.}$$

The value of the partition function $Z(\beta) = \int_{-\infty}^{\infty} dq_1 \int_{-\infty}^{\infty} dp_1 \int_{-\infty}^{\infty} dq_2 \int_{-\infty}^{\infty} dp_2\, e^{-\beta H(q_1, p_1; q_2, p_2)}$ is

(a) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{16}{15}}$ (b) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{15}{16}}$ (c) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{63}{64}}$ (d) $\frac{2m\pi^2}{k\beta^2}\sqrt{\frac{64}{63}}$

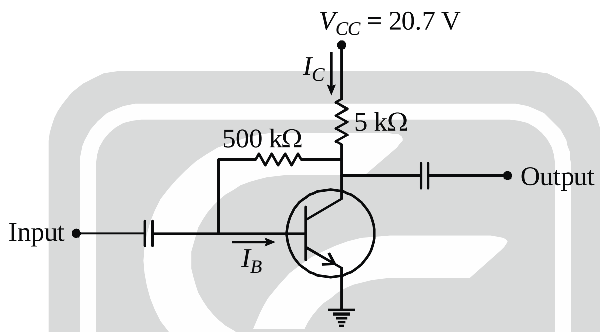
3. The generator of the infinitesimal canonical transformation $q \to q' = (1+\varepsilon)q$ and $p \to p' = (1-\varepsilon)p$ is

(a) $q + p$ (b) qp (c) $\frac{1}{2}(q^2 - p^2)$ (d) $\frac{1}{2}(q^2 + p^2)$

4. Following a nuclear explosion, a shock wave propagates radially outwards. Let E be the energy released in the explosion, and ρ be the mass density of the ambient air. Ignoring the temperature of the ambient air, using dimensional analysis, the functional dependence of the radius R of the shock front on E, ρ and the time t is

(a) $\left(\frac{Et^2}{\rho}\right)^{1/5}$ (b) $\left(\frac{\rho}{Et^2}\right)^{1/5}$ (c) $\frac{Et^2}{\rho}$ (d) $E\rho t^2$

5. In a collector feedback circuit shown in the figure below, the base emitter voltage $V_{BE} = 0.7\,\text{V}$ and current gain $\beta = \frac{I_C}{I_B} = 100$ for the transistor



The value of the base current I_B is

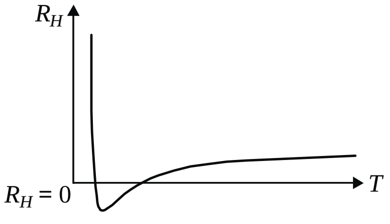
(a) 20 μA (b) 40 μA (c) 10 μA (d) 100 μA

6. The Hall co-efficient for a semiconductor having both types of carriers is given as:

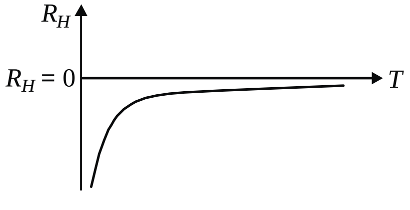
$$R_H = \frac{p\mu_p^2 - n\mu_n^2}{|e|\left(p\mu_p + n\mu_n\right)^2}$$

where p and n are the carrier densities of the holes and electrons, μ_p and μ_n are their respective mobilities. For a p-type semiconductor in which the mobility of holes is less than that of electrons, which of the following graphs best describes the variation of the Hall coefficient with temperature ?

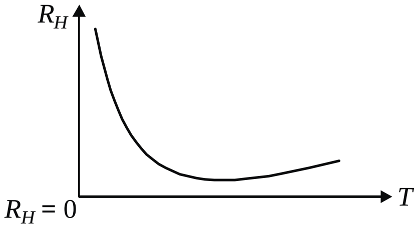
(a)



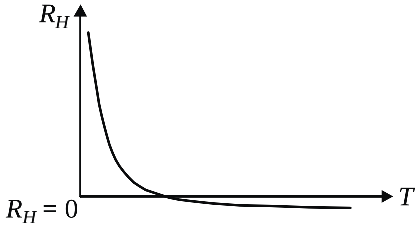
(b)



(c)



(d)



7. Let the normalized eigenstates of the Hamiltonian ,

$$H = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

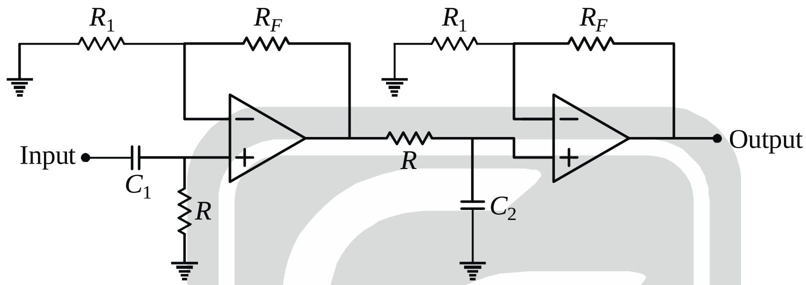
be $|\psi_1\rangle, |\psi_2\rangle$ and $|\psi_3\rangle$. The expectation value $\langle H\rangle$ and the variance of H in the state $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|\psi_1\rangle + |\psi_2\rangle - i|\psi_3\rangle\right)$ are

(a) $\frac{4}{3}$ and $\frac{1}{3}$ (b) $\frac{4}{3}$ and $\frac{2}{3}$ (c) 2 and $\frac{2}{3}$ (d) 2 and 1

8. The outermost shell of an atom of an element is $3d^3$. The spectral symbol for the ground state is

(a) $^4F_{3/2}$ (b) $^4F_{9/2}$ (c) $^4D_{7/2}$ (d) $^4D_{1/2}$

9. In the circuit diagram of a band pass filter shown below, $R = 10\ \text{k}\Omega$.



In order to get a lower cut-off frequency of 150 Hz and an upper cut-off frequency of 10 kHz, the appropriate values of C_1 and C_2 respectively are

(a) 0.1 μF and 1.5 nF (b) 0.3 μF and 5.0 nF (c) 1.5 nF and 0.1 μF (d) 5.0 nF and 0.3 μF

10. The function $f(t)$ is a periodic function of period 2π. In the range $(-\pi, \pi)$, it equals e^{-t}. If $f(t) = \sum_{-\infty}^{\infty} c_n e^{int}$ denotes its Fourier series expansion, the sum $\sum_{-\infty}^{\infty} |c_n|^2$ is

(a) 1 (b) $\frac{1}{2\pi}$ (c) $\frac{1}{2\pi}\cosh(2\pi)$ (d) $\frac{1}{2\pi}\sinh(2\pi)$

11. An alternating current $I(t) = I_0 \cos(\omega t)$ flows through a circular wire loop of radius R, lying in the xy-plane, and centered at the origin. The electric field $\vec{E}(\vec{r}, t)$ and the magnetic field $\vec{B}(\vec{r}, t)$ are measured at a point $\vec{r}$ such that $r \gg \frac{c}{\omega} \gg R$, where $r = |\vec{r}|$. Which one of the following statements is correct ?

(a) The time-averaged $|\vec{E}(\vec{r}, t)| \propto \frac{1}{r^2}$.

(b) The time-averaged $|\vec{E}(\vec{r}, t)| \propto \omega^2$.

(c) The time-averaged $|\vec{B}(\vec{r}, t)|$ as a function of the polar angle θ has a minimum at $\theta = \frac{\pi}{2}$.

(d) $|\vec{B}(\vec{r}, t)|$ is along the azimuthal direction.

12. In a spectrum resulting from Raman scattering, let I_R denote the intensity of Rayleigh scattering and I_S and I_{AS} denote the most intense Stokes line and the most intense anti-Stokes line, respectively. The correct order of these intensities is

(a) $I_S > I_R > I_{AS}$ (b) $I_R > I_S > I_{AS}$ (c) $I_{AS} > I_R > I_S$ (d) $I_R > I_{AS} > I_S$

13. The positive zero of the polynomial $f(x) = x^2 - 4$ is determined using Newton-Raphson method, using an initial guess $x = 1$. Let the estimate, after two iterations, be $x^{(2)}$. The percentage error $\left|\frac{x^{(2)} - 2}{2}\right| \times 100\%$ is

(a) 7.5 % (b) 5.0 % (c) 1.0 % (d) 2.5 %

14. The Bethe-Weizsäecker formula for the binding energy (in MeV) of a nucleus of atomic number Z and mass number A is

$$15.8A - 18.3A^{2/3} - 0.714\frac{Z(Z-1)}{A^{1/3}} - 23.2\frac{(A-2Z)^2}{A}$$

The ratio Z/A for the most stable isobar of a $A = 64$ nucleus, is nearest to

(a) 0.30 (b) 0.35 (c) 0.45 (d) 0.50

15. The strong nuclear force between a neutron and a proton in a zero orbital angular momentum state is denoted by $F_{np}(r)$, where r is the separation between them. Similarly, $F_{nn}(r)$ and $F_{pp}(r)$ denote the forces between a pair of neutrons and protons, respectively, in zero orbital momentum state. Which of the following is true on average if the inter-nucleon distance is $0.2\,\text{fm} < r < 2\,\text{fm}$?

(a) F_{np} is attractive for triplet spin state, and F_{nn}, F_{pp} are always repulsive.
(b) F_{nn} and F_{np} are always attractive and F_{pp} is repulsive in this triplet spin state.
(c) F_{pp} and F_{np} are always attractive and F_{nn} is always repulsive.
(d) All three forces are always attractive.

16. In the AC Josephson effect, a supercurrent flows across two superconductors separated by a thin insulating layer and kept at an electric potential difference ΔV. The angular frequency of the resultant supercurrent is given by

(a) $\frac{2e\Delta V}{\hbar}$ (b) $\frac{e\Delta V}{\hbar}$ (c) $\frac{e\Delta V}{\pi\hbar}$ (d) $\frac{e\Delta V}{2\pi\hbar}$

17. Let $\hat{x}$ and $\hat{p}$ denote position and momentum operators obeying the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. If $|x\rangle$ denotes an eigenstate of $\hat{x}$ corresponding to the eigenvalue x, then $e^{ia\hat{p}/\hbar}|x\rangle$ is

(a) an eigenstate of $\hat{x}$ corresponding to the eigenvalue x.
(b) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x+a)$.
(c) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x-a)$.
(d) not an eigenstate of $\hat{x}$.

18. The wavefunction of a particle of mass m, constrained to move on a circle of unit radius centered at the origin in the xy-plane, is described by $\psi(\phi) = A\cos^2\phi$, where ϕ is the azimuthal angle. All the possible outcomes of measurements of the z-component of the angular momentum L_z in this state, in units of $\hbar$, are

(a) ± 1 and 0 (b) ± 1 (c) ± 2 (d) ± 2 and 0

19. The fixed points of the time evolution of a one-variable dynamical system described by $y_{t+1} = 1 - 2y_t^2$ are 0.5 and –1. The fixed points 0.5 and –1 are
(a) Both stable
(b) Both unstable
(c) Unstable and stable, respectively
(d) Stable and unstable, respectively

20. A metallic wave guide of square cross-section of side L is excited by an electromagnetic wave of wave number k. The group velocity of the TE_{11} mode is
(a) $\dfrac{ckL}{\sqrt{k^2L^2+\pi^2}}$
(b) $\dfrac{c}{kL}\sqrt{k^2L^2-2\pi^2}$
(c) $\dfrac{c}{kL}\sqrt{k^2L^2-\pi^2}$
(d) $\dfrac{ckL}{\sqrt{k^2L^2+2\pi^2}}$

21. For a crystal, let ϕ denote the energy required to create a pair of vacancy and interstitial defects. If n pairs of such defects are formed, and $n \ll N, N'$, where N and N' are, respectively, the total number of lattice and interstitial sites, then n is approximately
(a) $\sqrt{NN'}\, e^{-\phi/(2k_BT)}$
(b) $\sqrt{NN'}\, e^{-\phi/(k_BT)}$
(c) $\frac{1}{2}(N+N')e^{-\phi/(2k_BT)}$
(d) $\frac{1}{2}(N+N')e^{-\phi/(k_BT)}$

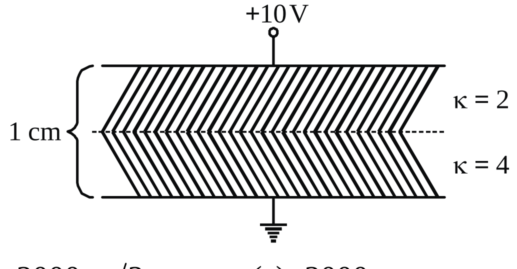
22. Assume that the noise spectral density, at any given frequency, in a current amplifier is independent of frequency. The bandwidth of measurement is changed from 1 Hz to 10 Hz. The ratio A/B of the RMS noise current before (A) and after (B) the bandwidth modification is
(a) $1/10$
(b) $1/\sqrt{10}$
(c) $\sqrt{10}$
(d) 10

23. A particle hops randomly from a site to its nearest neighbour in each step on a square lattice of unit lattice constant. The probability of hopping to the positive x-direction is 0.3, to the negative x-direction is 0.2, to the positive y-direction is 0.2 and to the negative y-direction is 0.3. If a particle starts from the origin, its mean position after N steps is
(a) $\frac{1}{10}N(-\hat{i}+\hat{j})$
(b) $\frac{1}{10}N(\hat{i}-\hat{j})$
(c) $N(0.3\hat{i}-0.2\hat{j})$
(d) $N(0.2\hat{i}-0.3\hat{j})$

24. A negative muon, which has a mass nearly 200 times that of an electron, replaces an electron in a Li atom. The lowest ionization energy for the muonic Li atom is approximately
(a) the same as that of He
(b) the same as that of normal Li
(c) 200 times larger than that of normal Li
(d) the same as that of normal Be

25. For T much less than the Debye temperature of copper, the temperature dependence of the specific heat at constant volume of copper, is given by (in the following a and b are positive constants)
(a) aT^3
(b) $aT + bT^3$
(c) $aT^2 + bT^3$
(d) $\exp\left(-\dfrac{a}{k_BT}\right)$

26. A parallel plate capacitor, with 1 cm separation between the plates, has two layers of dielectric with dielectric constants $\kappa = 2$ and $\kappa = 4$, as shown in the figure below. If a potential difference of 10 **V** is applied between the plates, the magnitude of the bound surface charge density (in units of C/m^2) at the junction of the dielectrics is



(a) $250\varepsilon_0$
(b) $2000\varepsilon_0/3$
(c) $2000\varepsilon_0$
(d) $200\varepsilon_0/3$

27. The pressure p of a gas depends on the number density ρ of particles and the temperature T as

$$p = k_B T\rho - B_2\rho^2 + B_3\rho^3,$$ where B_2 and B_3 are positive constants.

Let T_c, ρ_c and p_c denote the critical temperature, critical number density and critical pressure, respectively. The ratio $\rho_c k_B T_c / p_c$ is equal to

(a) 1/3 (b) 3 (c) 8/3 (d) 4

28. The mean kinetic energy per atom in a sodium vapour lamp is 0.33 eV. Given that the mass of sodium atom is approximately 22.5×10^9 eV, the ratio of the Doppler width of an optical line to its central frequency is

(a) 7×10^{-7} (b) 6×10^{-6} (c) 5×10^{-5} (d) 4×10^{-4}

29. The Hamiltonian of a system with two degrees of freedom is $H = q_1 p_1 - q_2 p_2 + aq_1^2$, where $a > 0$ is a constant. The function $q_1 q_2 + \lambda p_1 p_2$ is a constant of motion only if λ is

(a) 0 (b) 1 (c) $-a$ (d) a

30. Which of the following decay processes is allowed ?

(a) $K^0 \to \mu^+ + \mu^-$ (b) $\mu^- \to e^- + \gamma$ (c) $n \to p + \pi^-$ (d) $n \to \pi^+ + \pi^-$