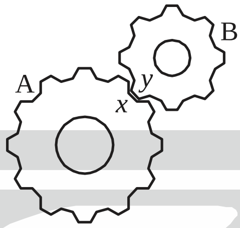
NTA-JOINT CSIR-UGC NET June 2023

PHYSICAL SCIENCES

PART - A (General Aptitude)

1. A vehicle has tyres of diameter 1 m connected by a shaft directly to gearwheel A which meshes with gear wheel B as shown in the diagram. A has 12 teeth and B has 8. If points x on A and y on B are initially in contact, they will again be in contact after the vehicle has travelled a distance (in meters)

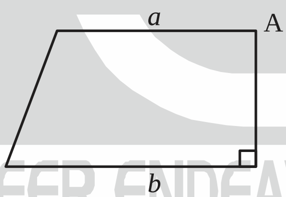


(a) 2π (b) 3π (c) 4π (d) 12π

2. A plant grows by 10% of its height every three months. If the plant's height today is 1 m, its height after one year is the closest to

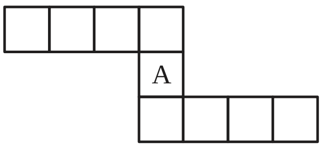
(a) 1.10 m (b) 1.21 m (c) 1.33 m (d) 1.46 m

3. At what horizontal distance from A should a vertical line be drawn so as to divide the area of the trapezium shown in the figure into two equal parts ? (a and b are lengths of the parallel sides.)



(a) $\frac{(a+b)}{4}$ (b) $\frac{(a+b)}{3}$ (c) $\frac{(a+b)}{2}$ (d) $\frac{(2a+b)}{2}$

4. The squares in the following sketch are filled with digits 1 to 9, without any repetition, such that the numbers in the two horizontal rows add up to 20 each. What number appears in the square labelled A in the vertical column?



(a) It cannot be ascertained in the absence of the sum of the numbers in the column.

(b) 3 (b) 5 (d) 7

5. How many rectangles are there in the given figure?

(a) 6 (b) 7 (c) 8 (d) 9

6. A boy has kites of which all but 9 are red, all but 9 are yellow, all but 9 are green, and all but 9 are blue. How many kites does he have?
(a) 12 (b) 15 (c) 9 (d) 18

7. In a round-robin tournament, after each team has played exactly four matches, the number of wins/ losses of 6 participating teams are as follows:

Team	Win	Loss
A	4	0
B	0	4
C	3	1
D	2	2
E	0	4
F	3	1

Which of the two teams have certainly NOT played with each other?
(a) A and B (b) C and F (c) E and D (d) B and E

8. A liar always lies and a non-liar, never. If in a group of *n* persons seated around a round-table everyone calls his/her left neighbor a liar, then
(a) all are liars.
(b) *n* must be even and every alternate person is a liar.
(c) *n* must be odd and every alternate person is a liar.
(d) *n* must be a prime.

9. Sections A, B, C and D of a class have 24, 27, 30 and 36 students, respectively. One section has boys and girls who are seated alternately in three rows, such that the first and the last positions in each row are occupied by boys. Which section could this be?
(a) A (b) B (c) C (d) D

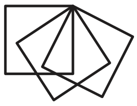
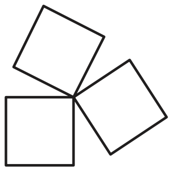
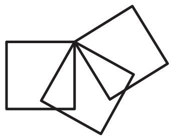
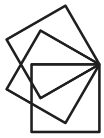
10. Tokens numbered from 1 to 25 are mixed and one token is drawn randomly. What is the probability that the number on the token drawn is divisible either by 4 or by 6?
(a) 8/25 (b) 10/25 (c) 9/25 (d) 12/25

11. I have a brother who is 4 years elder to me, and a sister who was 5 years old when my brother was born. When my sister was born, my father was 24 years old. My mother was 27 years old when I was born. How old (in years) were my father and mother, respectively, when my brother was born?
(a) 29 and 23 (b) 27 and 25 (c) 27 and 23 (d) 29 and 25

12. Starting from the top of a page and pointing downward, an ant moves according to the following commands.

Start
↓
Repeat actions in curly brackets 3 times
{ Repeat actions in the box 4 times
[Move forward 100 units
↓
Turn right by 90 degrees]
↓
Turn left by 30 degrees }

Of the following paths

A.  **B.**  **C.**  **D.** 

Which is the correct path of the ant ?

(a) A (b) B (c) C (d) D

13. In a four-digit PIN, the third digit is the product of the first two digits and the fourth digit is zero. The number of such PINs is

(a) 42 (b) 41 (c) 40 (d) 39

14. What is the product of the number of capital letters and the number of small letters of the English alphabet in the following text?

A4;={c8%$56((+B/;,.H&r]]](u];#~K@>83<??/STvx%^(d)L:/<-N347)))2;:$+}E$###[w}'",,;/89

(a) 17 (b) 37 (c) 53 (d) 63

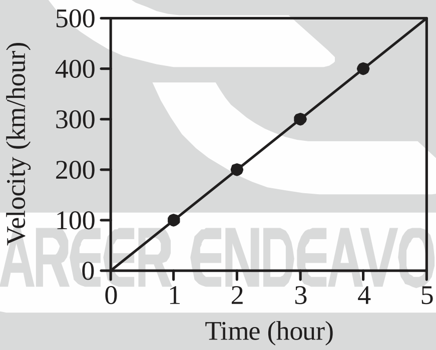
15. A beam of square cross-section is to be cut out of a wooden log. Assuming that the log is cylindrical, what approximately is the largest fraction of the wood by volume that can be fruitfully utilised as the beam?

(a) 49 % (b) 64 % (c) 71 % (d) 81 %

16. On a track of 200 m length, S runs from the starting point and R starts 20 m ahead of S at the same time. Both reach the end of the track at the same time. S runs at a uniform speed of 10 m/s. If R also runs at a uniform speed, what is R's speed (in m/s)?

(a) 9 (b) 10 (c) 12 (d) 8

17. Given plot describes the motion of an object with time.



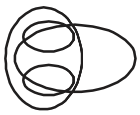
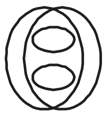
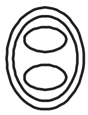
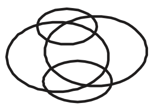
Which one of the following statements is CORRECT?

(a) The object is moving with a constant velocity.
(b) The object covers equal distance every hour.
(c) The object is accelerating.
(d) Velocity of the object doubles every hour.

18. If one letter each is drawn at random from the words CAUSE and EFFECT, the chance that they are the same is

(a) 1/30 (b) 1/11 (c) 1/10 (d) 2/11

19. The correct pictorial representation of the relations among the categories PLAYERS, FEMALE CRICKETERS, MALE FOOTBALLERS and GRADUATES is

A.  **B.**  **C.**  **D.** 

(a) A (b) B (c) C (d) D

20. After 12:00:00 the hour hand and minute hand of a clock will be perpendicular to each other for the first time at
(a) 12 : 16 : 21 (b) 12 : 15 : 00 (c) 13 : 22 : 21 (d) 12 : 48 : 08

PART - B (Physics)

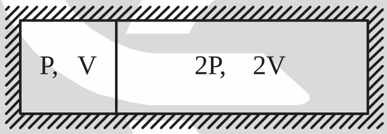
1. At $z = 0$, the function $\frac{1}{z - \sin z}$ of a complex variable z has
(a) no singularity (b) a simple pole (c) a pole of order 2 (d) a pole of order 3

2. If the expectation value of the momentum of a particle in one dimension is zero, then its (box-normalizable) wavefunction may be of the form:
(a) $\sin kx$ (b) $e^{ikx} \sin kx$ (c) $e^{ikx} \cos kx$ (d) $\sin kx + e^{ikx} \cos kx$

3. If $z = i^{i^{i^{\cdot^{\cdot^{\cdot}}}}}$ (Note that the exponent continues indefinitely), then a possible value of $\frac{1}{z} \ln z$ is
(a) $2i \ln i$ (b) $\ln i$ (c) $i \ln i$ (d) $2 \ln i$

4. The value of the integral $\int_0^\infty dx\, e^{-x^{2m}}$, where m is a positive integer, is
(a) $\Gamma\left(\frac{m+1}{2m}\right)$ (b) $\Gamma\left(\frac{m-1}{2m}\right)$ (c) $\Gamma\left(\frac{2m+1}{2m}\right)$ (d) $\Gamma\left(\frac{2m-1}{2m}\right)$

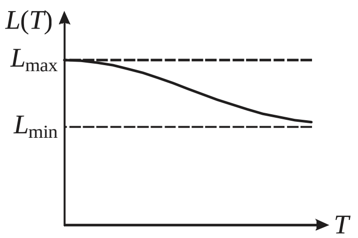
5. A thermally isolated container, filled with an ideal gas at temperature T, is divided by a partition, which is clamped initially, as shown in the figure below.

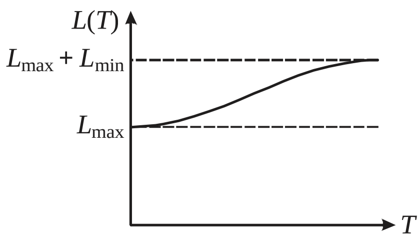


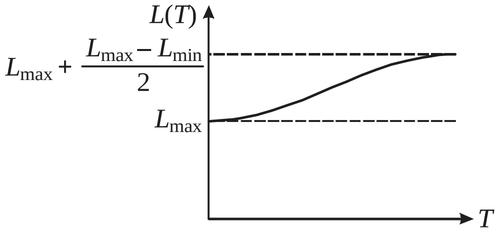
The partition does not allow the gas in the two parts to mix. It is subsequently released and allowed to move freely with negligible friction. The final pressure at equilibrium is
(a) $5P/3$ (b) $5P/4$ (c) $3P/5$ (d) $4P/5$

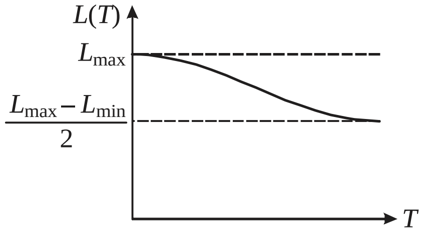
6. A particle of rest mass m is moving with a velocity $v\hat{k}$, with respect to an inertial frame S. The energy of the particle as measured by an observer S', who is moving with a uniform velocity $u\hat{i}$ with respect to S (in terms of $\gamma_u = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$ and $\gamma_v = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$) is
(a) $\gamma_u \gamma_v m(c^2 - uv)$ (b) $\gamma_u \gamma_v mc^2$ (c) $\frac{1}{2}(\gamma_u + \gamma_v) mc^2$ (d) $\frac{1}{2}(\gamma_u + \gamma_v) m(c^2 - uv)$

7. An elastic rod has a low energy state of length L_{max} and high energy state of length L_{min}. The best schematic representation of the temperature (T) dependence of the mean equilibrium length $L(T)$ of the rod, is

(a) 

(b) 

(c) 

(d) 

8. An electromagnetic wave is incident from vacuum normally on a planar surface of a non-magnetic medium. If the amplitude of the electric field of the incident wave is E_0 and that of the transmitted wave is $2E_0/3$, then neglecting any loss, the refractive index of the medium is

(a) 1.5 (b) 2.0 (c) 2.4 (d) 2.7

9. Consider the Hamiltonian $H = AI + B\sigma_x + C\sigma_y$, where A, B and C are positive constants, I is the 2×2 identity matrix and σ_x, σ_y are Pauli matrices. If the normalized eigenvector corresponding to its largest energy eigenvalue is $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ y\end{pmatrix}$, then y is

(a) $\frac{B+iC}{\sqrt{B^2+C^2}}$ (b) $\frac{A-iB}{\sqrt{A^2+B^2}}$ (c) $\frac{A-iC}{\sqrt{A^2+C^2}}$ (d) $\frac{B-iC}{\sqrt{B^2+C^2}}$

10. The periods of oscillation of a simple pendulum at the sea level and at the top of a mountain of height 6 km are T_1 and T_2, respectively. If the radius of earth is approximately 6000 km, then $\frac{(T_2 - T_1)}{T_1}$ is closest to

(a) -10^{-4} (b) -10^{-3} (c) 10^{-4} (d) 10^{-3}

11. The Lagrangian of a system described by three generalized coordinates q_1, q_2 and q_3 is

$L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2) + M\dot{q}_1\dot{q}_2 + k\dot{q}_1 q_3$, (where m, M and k are positive constants).

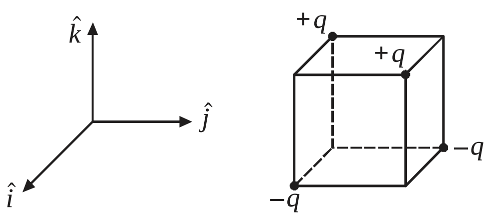
Then, as a function of time

(a) two coordinates remain constant and one evolves linearly.
(b) one coordinate remains constant, one evolves linearly and the third evolves as a quadratic function.
(c) one coordinate evolves linearly and two evolve quadratically.
(d) all three evolve linearly.

12. Two $n \times n$ invertible real matrics A and B satisfy the relation $(AB)^T = -(A^{-1}B)^{-1}$. If B is orthogonal then A must be

(a) lower triangular (b) orthogonal (c) symmetric (d) antisymmetric

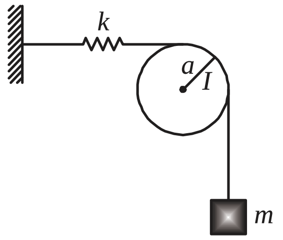
13. Two positive and two negative charges of magnitude q are placed on the alternate vertices of a cube of side a (as shown in the figure).



The electric dipole moment of this charge configuration is

(a) $-2qa\,\hat{k}$ (b) $2qa\,\hat{k}$ (c) $2qa(\hat{i}+\hat{j})$ (d) $2qa(\hat{i}-\hat{j})$

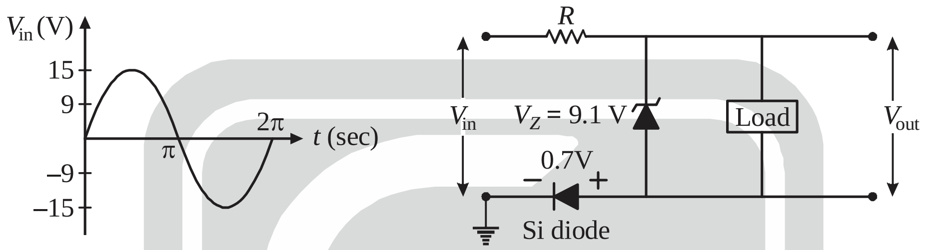
14. A wire, connected to a massless spring of spring constant k and a block of mass m, goes around a disc of radius a and moment of inertia I, as shown in the figure.



Assume that the spring remains horizontal, the pulley rotates freely and there is no slippage between the wire and the pulley. The angular frequency of small oscillations of the disc is

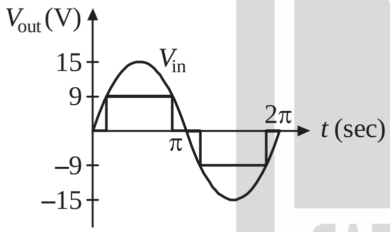
(a) $\sqrt{\dfrac{2ka^2}{ma^2+I}}$ (b) $\sqrt{\dfrac{ka^2}{ma^2+I}}$ (c) $\sqrt{\dfrac{ka^2}{ma^2+2I}}$ (d) $\sqrt{\dfrac{ka^2}{2ma^2+I}}$

15. A impedance load (network) is connected in the circuit as shown below.

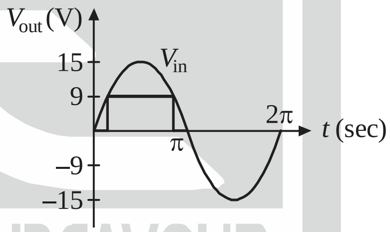


The forward voltage drop for silicon diode is 0.7 V and the Zener voltage is 9.10 V. If the input voltage (V_{in}) is sine wave with an amplitude of 15 V (as shown in the figure above), which of the following waveform qualitatively describes the output voltage (V_{out}) across the load ?

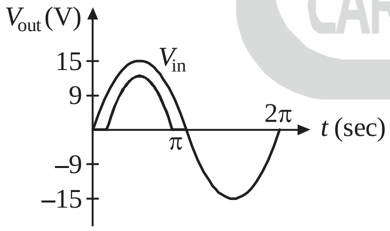
(a)



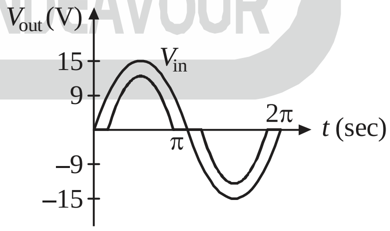
(b)



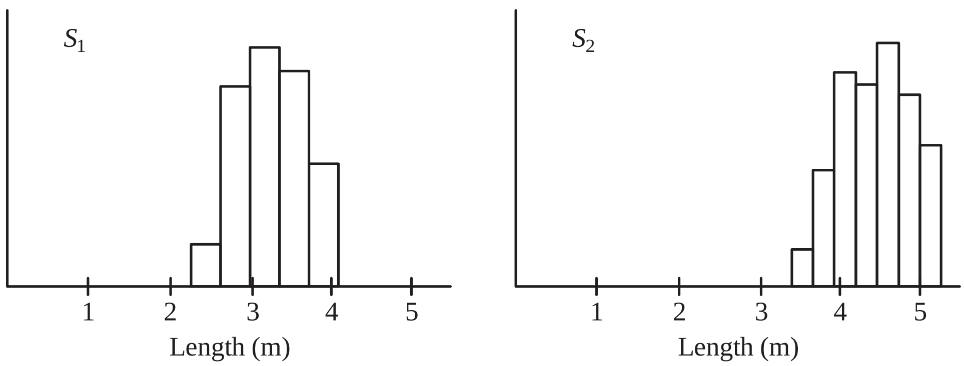
(c)

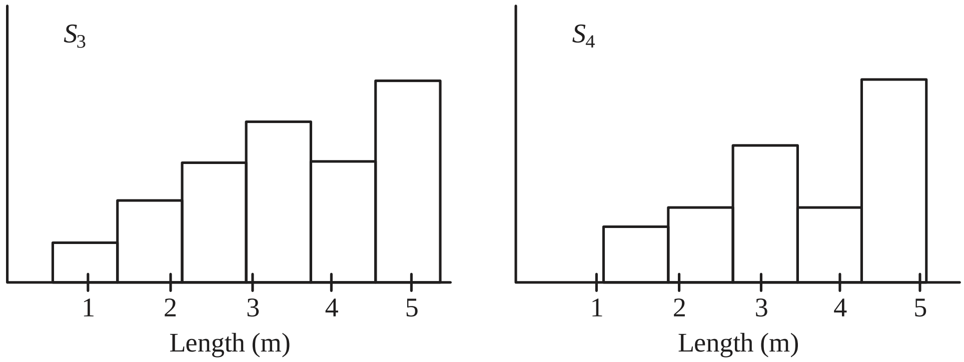


(d)



16. Four students $(S_1, S_2, S_3$ and $S_4)$ make multiple measurement on the length of a table. The binned data are plotted as histograms in the following figures.





If the length of the table, specified by the manufacturer, is 3m, the student whose measurement have the minimum have the minimum systematic error, is

(a) S_2 (b) S_1 (c) S_4 (d) S_3

17. The electric and magnetic fields in an inertial frame are $E = 3a\hat{i} - 4\hat{j}$ and $B = \frac{5a}{c}\hat{k}$, where a is a constant. A massive charged particle is released from rest. The necessary and sufficient condition that there is an inertial frame, where the trajectory of the particle is a uniform-pitched helix, is

(a) $1 < a < \sqrt{2}$ (b) $-1 < a < 1$ (c) $a^2 > 1$ (d) $a^2 > 2$

18. A walker takes steps, each of length L, randomly in the directions along east, west, north and south. After four steps its distance from the starting point is d. The probability that $d \le 3L$ is

(a) 63/64 (b) 59/64 (c) 57/64 (d) 55/64

19. The momentum space representation of the Schrödinger equation of a particle in a potential $V(\vec{r})$ is

$$\left(|p|^2 + \beta(\nabla_p^2)^2\right)\psi(p,t) = i\hbar\frac{\partial}{\partial t}\psi(p,t), \left[\text{where } (\nabla_p)_i = \frac{\partial}{\partial p_i}, \text{ and } \beta \text{ is a constant}\right].$$

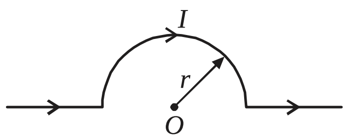
The potential is (in the following V_0 and a are constants)

(a) $V_0\, e^{-r^2/a^2}$ (b) $V_0\, e^{-r^4/a^4}$ (c) $V_0\,(r/a)^2$ (d) $V_0\,(r/a)^4$

20. If the average energy $\langle E\rangle_T$ of a quantum harmonic oscillator at a temperature T is such that $\langle E\rangle_T = 2\langle E\rangle_{T\to 0}$, then T satisfies

(a) $\coth\left(\frac{\hbar\omega}{k_BT}\right) = 2$ (b) $\coth\left(\frac{\hbar\omega}{2k_BT}\right) = 2$ (c) $\coth\left(\frac{\hbar\omega}{k_BT}\right) = 4$ (d) $\coth\left(\frac{\hbar\omega}{2k_BT}\right) = 4$

21. A part of an infinitely long wire, carrying a current I is bent into a semi-circular arc of radius r (as shown in the figure).



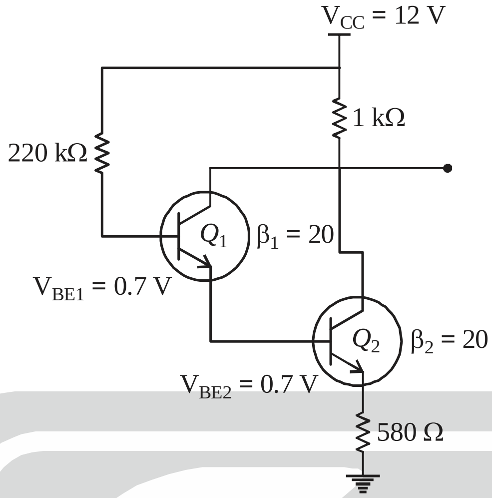
The magnetic field at the centre O of the arc is

(a) $\frac{\mu_0 I}{4r}$ (b) $\frac{\mu_0 I}{4\pi r}$ (c) $\frac{\mu_0 I}{2r}$ (d) $\frac{\mu_0 I}{2\pi r}$

22. The infinite series $\sum_{n=0}^{\infty}(n^2+3n+2)\,x^n$ evaluated at $x=\frac{1}{2}$, is

(a) 16 (b) 32 (c) 8 (d) 24

23. The figure below shows a circuit with two transistors, Q_1 and Q_2 having current gains β_1 and β_2 respectively.



The collector voltage V_C will be closest to

(a) 0.9 V (b) 2.2 V (c) 2.9 V (d) 4.2 V

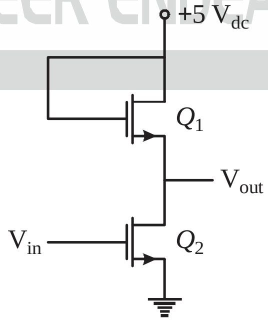
24. In terms of a complete set of orthogonal basis kets $|n\rangle$, $n=0,\pm1,\pm2,\ldots.$, the Hamiltonian is

$$H=\sum_n\left(E|n\rangle\langle n|+\varepsilon|n+1\rangle\langle n|+\varepsilon|n\rangle\langle n+1|\right)$$

where E and ε are constants. The state $|\phi\rangle=\sum_n e^{in\phi}|n\rangle$ is an eigenstate with energy

(a) $E+\varepsilon\cos\phi$ (b) $E-\varepsilon\cos\phi$ (c) $E+2\varepsilon\cos\phi$ (d) $E-2\varepsilon\cos\phi$

25. The circuit containing two *n*-channel MOSFETs shown below, works as



(a) a buffer (b) a non-inverting amplifier

(c) an inverter (d) a rectifier

PART - C (Physics)

1. At time $t = 0$, a particle is in the ground state of the Hamiltonian

$$H(t) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x \sin\frac{\omega t}{2}$$

where λ, ω and m are positive constants. To $O(\lambda^2)$, the probability that at $t = \frac{2\pi}{\omega}$, the particle would be in the first excited state of $H(t=0)$ is

(a) $\frac{9\lambda^2}{16m\hbar\omega^3}$ (b) $\frac{9\lambda^2}{8m\hbar\omega^3}$ (c) $\frac{16\lambda^2}{9m\hbar\omega^3}$ (d) $\frac{8\lambda^2}{9m\hbar\omega^3}$

2. In the absorption spectrum of H-atom, the frequency of transition from the ground state to the first excited state is ν_H. The corresponding frequency for a bound state of a positively charged muon (μ^*) and an electron is ν_μ. Using $m_\mu = 10^{-28}$ kg, $m_e = 10^{-30}$ kg and $m_p \gg m_e, m_\mu$, the value of $(\nu_\mu - \nu_H)/\nu_H$ is

(a) 0.001 (b) –0.001 (c) –0.01 (d) 0.01

3. The Raman rotational-vibrational spectrum of nitrogen molecules is observed using an incident radiation of wavenumber 12500 cm^{-1}. In the first shifted band, the wavenumbers of the observed lines (in cm^{-1}) are 10150, 10158, 10170, 10182 and 10190. The values of vibrational frequency and rotational constant (in cm^{-1}), respectively, are

(a) 2330 and 2 (b) 2350 and 2 (c) 2350 and 3 (d) 2330 and 3

4. Earth may be assumed to be an axially symmetric freely rotating rigid body. The ratio of the principal moments of inertia about the axis of symmetry and an axis perpendicular to it is 33 : 32. If T_0 is the time taken by earth to make one rotation around its axis of symmetry, then the time period of precession is closest to

(a) $33\, T_0$ (b) $33T_0/2$ (c) $32T_0$ (d) $16T_0$

5. The electronic configuration of ^{12}C is $1s^2\, 2s^2\, 2p^2$. Including LS coupling, the correct ordering of its energies is

(a) $E(^3P_2) < E(^3P_1) < E(^3P_0) < E(^1D_2)$ (b) $E(^3P_0) < E(^3P_1) < E(^3P_2) < E(^1D_2)$

(c) $E(^1D_2) < E(^3P_2) < E(^3P_1) < E(^3P_0)$ (d) $E(^3P_1) < E(^3P_0) < E(^3P_2) < E(^1D_2)$

6. A stationary magnetic dipole $m = m\hat{k}$ is placed above an infinite surface $(z = 0)$ carrying a uniform surface current density $k = k\hat{i}$. The torque on the dipole is

(a) $\frac{\mu_0}{2}mk\,\hat{i}$ (b) $-\frac{\mu_0}{2}mk\,\hat{i}$ (c) $\frac{\mu_0}{2}mk\,\hat{j}$ (d) $-\frac{\mu_0}{2}mk\,\hat{j}$

7. To first order in perturbation theory, the energy of the ground state of the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{\hbar\omega}{\sqrt{512}}\exp\left[-\frac{m\omega}{\hbar}x^2\right]$$

(Treating the third term of the Hamiltonian as a perturbation) is

(a) $\frac{15}{32}\hbar\omega$ (b) $\frac{17}{32}\hbar\omega$ (c) $\frac{19}{32}\hbar\omega$ (d) $\frac{21}{32}\hbar\omega$

8. The matrix corresponding to the differential operator $\left(1+\frac{d}{dx}\right)$ in the space of polynomials of degree at most two, in the basis spanned by $f_1 = 1,\ f_2 = x$ and $f_3 = x^2$, is

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

9. The energies of a two-level system are $\pm E$. Consider an ensemble of such non-interacting systems at a temperature T. At low temperature, the leading term in the specific heat depends on T as

(a) $\frac{1}{T^2} e^{-E/k_B T}$ (b) $\frac{1}{T^2} e^{-2E/k_B T}$ (c) $T^2 e^{-E/k_B T}$ (d) $T^2 e^{-2E/k_B T}$

10. The value of the integral $\int_{-\infty}^{\infty} \frac{\cos \alpha x}{x^2+1} dx$, for $\alpha > 0$, is

(a) πe^{α} (b) $\pi e^{-\alpha}$ (c) $\pi e^{-\alpha/2}$ (d) $\pi e^{\alpha/2}$

11. Thermal neutrons may be detected most efficiently by a

(a) ^{6}Li loaded plastic scintillator (b) Geiger-Müller counter

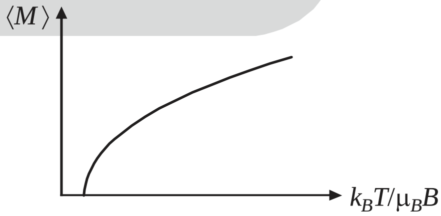
(c) inorganic scintillator CaF_2 (d) silicon detector

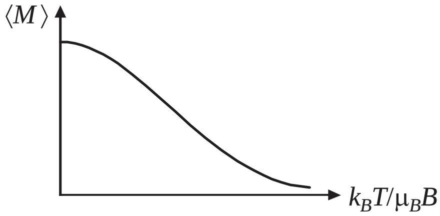
12. The Lagrangian of a system of two particles is $L = \frac{1}{2}\dot{x}_1^2 + 2\dot{x}_2^2 - \frac{1}{2}(x_1^2 + x_2^2 + x_1 x_2)$. The normal frequencies are best approximated by

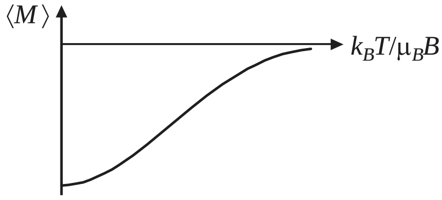
(a) 1.2 and 0.7 (b) 1.5 and 0.5 (c) 1.7 and 0.5 (d) 1.0 and 0.4

13. A paramagnetic salt with magnetic per ion $\mu_\pm = \pm\mu_B$ (where μ_B is the Bohr magneton) is in thermal equilibrium at temperature T in a constant magnetic field B. The average magnetic moment $\langle M \rangle$, as a function of $(k_B T/\mu_B B)$, is best represented by

(a) 

(b) 

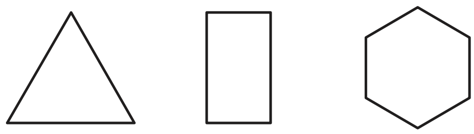
(c) 

(d) 

14. The elastic scattering process $\pi^- p \to \pi^- p$ may be a treated as a hard-sphere scattering. The mass of π^-, $m_\pi \simeq \frac{1}{6} m_p$, where $m_p \simeq 938\ \text{MeV}/c^2$ is the mass of the proton. The total scattering cross-section is closest to

(a) 0.01 milli-barn (b) 1 milli-barn (c) 0.1 barn (d) 10 barn

15. The figures (I), (II) and (III) below represent an equilateral triangle, a rectangle and a regular hexagon, respectively.



Which of these can be primitive unit cells of a Bravais lattice in two dimensions ?

(a) Only (I) and (III) but not (II) (b) Only (I) and (II) but not (III)

(c) Only (II) and (III) but not (I) (d) All of them

16. The energy/energies E of the bound state(s) of a particle of mass m in one dimension in the potential

$$V(x)=\begin{cases}\infty & ; \quad x\le 0\\ -V_0 & ; \quad 0<x<a\\ 0 & ; \quad x\ge a\end{cases}$$ (where $V_0 > a$) is/are determined by

(a) $\cot^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right)=\frac{E-V_0}{E}$ (b) $\tan^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right)=-\frac{E}{E+V_0}$

(c) $\cot^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right)=-\frac{E}{E+V_0}$ (d) $\tan^2\left(a\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\right)=\frac{E-V_0}{E}$

17. An amplifier with a voltage gain of 40 dB without feedback is used in an electronic circuit. A negative feedback with a fraction 1/40 is connected to the input of this amplifier. The net gain of the amplifier in the circuit is closest to

(a) 40 dB (b) 37 dB (c) 29 dB (d) 20 dB

18. Two small metalic objects are embedded in a weakly conducting medium of conductivity σ and dielectric constant ε. A battery connected between them leads to a potential difference V_0. It is subsequently disconnected at time $t=0$. The potential difference at later time t is

(a) $V_0 e^{-t\sigma/4\varepsilon}$ (b) $V_0 e^{-t\sigma/2\varepsilon}$ (c) $V_0 e^{-3t\sigma/4\varepsilon}$ (d) $V_0 e^{-t\sigma/\varepsilon}$

19. The tensor component of the nuclear force may be inferred from the fact that deuteron nucleus ${}^2_1\text{H}$

(a) has only one bound state with total spin S = 1.

(b) has a non-zero electric quadrupole moment in its ground state.

(c) is stable while triton ${}^3_1\text{H}$ is unstable.

(d) is the only two nucleon bound state.

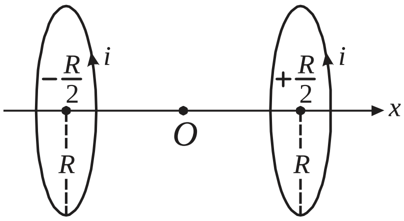
20. A receiver operating at 27° C has an input resistance of 100 Ω. The input thermal noise voltage for this receiver with a bandwidth of 100 kHz is closest to

(a) 0.4 nV (b) 0.6 pV (c) 40 mV (d) 0.4 μV

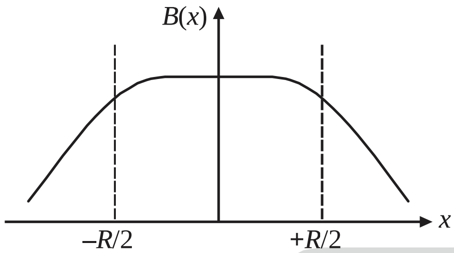
21. A bucket contains 6 red and 4 blue balls. A ball is taken out of the bucket at random and two balls of the same colour are put back. This step is repeated once more. The probability that the numbers of red and blue balls are equal at the end, is

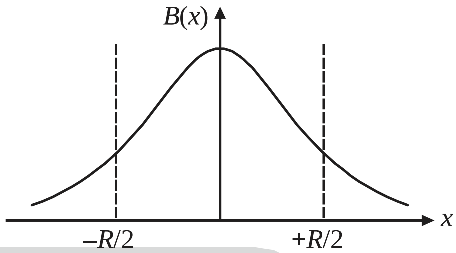
(a) 4/11 (b) 2/11 (c) 1/4 (d) 3/4

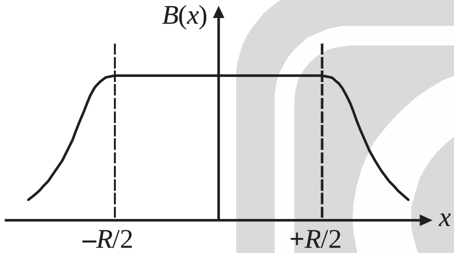
22. Two parallel conducting rings, both of radius R, are separated by a distance R. The planes of the rings are perpendicular to the line joining their centers, which of is taken to be the x-axis.

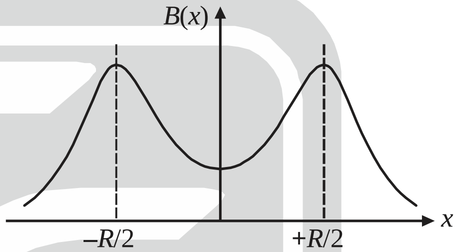


If both the rings carry the same current i along the same direction, the magnitude of the magnetic field along the x-axis is best represented by

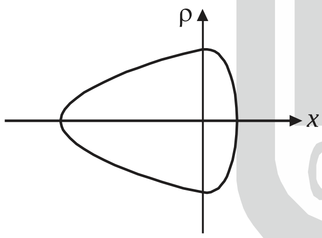
(a)


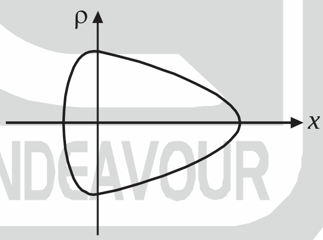
(b)


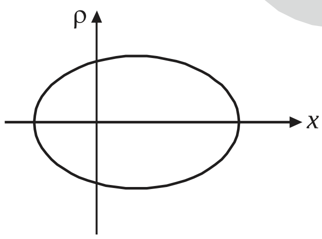
(c)


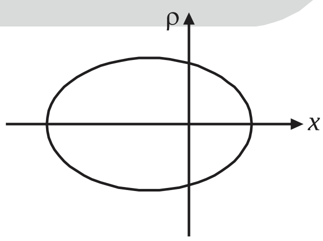
(d)


23. The Lagrangian of a particle in one dimension is $L = \frac{m}{2}\dot{x}^2 - ax^2 - V_0 e^{-10x}$, where a and V_0 are positive constants. The best qualitative representation of a trajectory in the phase space is

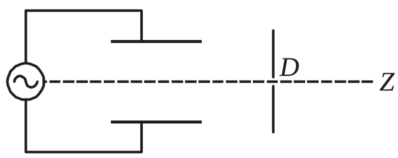
(a)


(b)


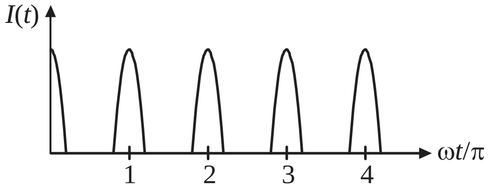
(c)


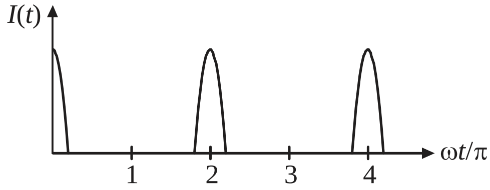
(d)


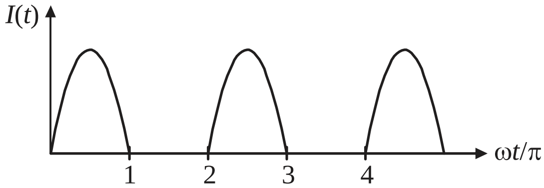
24. A high frequency voltage signal $V_i = V_m \sin \omega t$ is applied to a parallel plate deflector as shown in the figure.

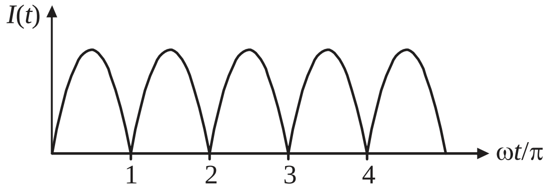


An electron beam is passing through the deflector along the central line. The best qualitative representation of the intensity $I(t)$ of the beam after it goes through the narrow circular aperature D, is

(a) 

(b) 

(c) 

(d) 

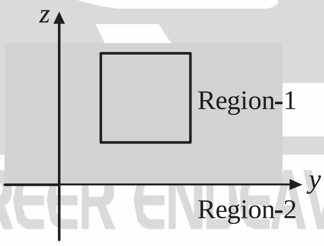
25. The Laplace transform $L[f](y)$ of the function $f(x)=\begin{cases}1 & \text{for} \quad 2n \le x \le 2n+1 \\ 0 & \text{for} \quad 2n+1 \le x \le 2n+2\end{cases}$; $(n=0,1,2,\ldots)$ is

(a) $\dfrac{e^{-y}(e^{-y}+1)}{y(e^{-2y}+1)}$ (b) $\dfrac{e^{y}-e^{-y}}{y}$ (c) $\dfrac{e^{y}+e^{-y}}{y}$ (d) $\dfrac{e^{y}(e^{y}-1)}{y(e^{2y}-1)}$

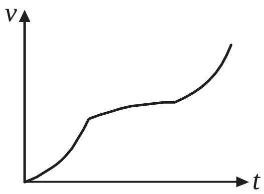
26. A system of N non-interacting particles in one-dimension, each of which is in a potential $V(x)=gx^6$ where $g>0$ is a constant and x denotes the displacement of the particle from its equilibrium position. In thermal equilibrium, the heat capacity at constant volume is

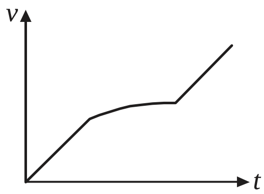
(a) $\dfrac{7}{6}Nk_B$ (b) $\dfrac{4}{3}Nk_B$ (c) $\dfrac{3}{2}Nk_B$ (d) $\dfrac{2}{3}Nk_B$

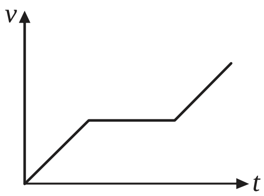
27. A square conducting loop in the yz-plane, falls downward under the gravity along the negative z-axis. Region-1, defined by $z>0$ has a uniform magnetic field $B=B_0\hat{i}$ while region-2 defined $z<0$ has no magnetic field

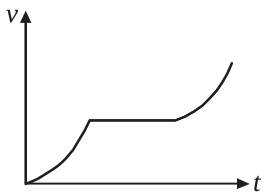


The time dependence of the speed $v(t)$ of the loop, as it starts to fall from well within the region-1 and passes into the region-2, is best represented by

(a) 

(b) 

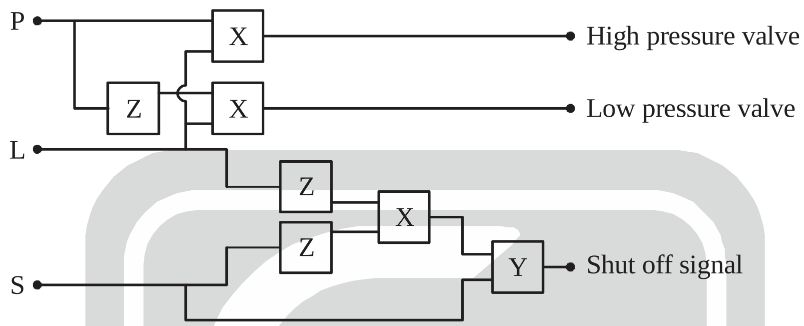
(c) 

(d) 

28. The Hamiltonian for spin-1/2 particle in a magnetic field $B = B_0\hat{k}$ is given by $H = \lambda S \cdot B$, where S is its spin (in units of $\hbar$) and λ is a constant. If the average spin density is $\langle S \rangle$ for an ensemble of such non-interacting particles, then $\frac{d}{dt}\langle S_x \rangle$

(a) $\frac{\lambda}{\hbar}B_0\langle S_x \rangle$ (b) $\frac{\lambda}{\hbar}B_0\langle S_y \rangle$ (c) $-\frac{\lambda}{\hbar}B_0\langle S_x \rangle$ (d) $-\frac{\lambda}{\hbar}B_0\langle S_y \rangle$

29. A liquid oxygen cylinder system is fitted when a level-sensor (L) and pressure-sensor (P), as shown in the figure below. The outputs of L and P are set to logic high (S = 1) when the measured values exceed the respective preset threshold values. The system can be shut off either by an operator by setting the input S to high, or when the level of oxygen in the tank falls below the threshold value.



The logic gates X, Y and X, respectively, are

(a) OR, AND and NOT
(b) AND, OR and NOT
(c) NAND, OR and NOT
(d) NOR, AND and NOT

30. The energy levels of a system, which is in equilibrium at temperature $T = \frac{1}{(k_B\beta)}$, are 0, ε and 2ε. If two identical bosons occupy these energy levels, the probability of the total energy being 3ε, is

(a) $\frac{e^{-3\beta\varepsilon}}{1+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}+e^{-4\beta\varepsilon}}$
(b) $\frac{e^{-3\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+2e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}+e^{-4\beta\varepsilon}}$
(c) $\frac{e^{-3\beta\varepsilon}}{e^{-\beta\varepsilon}+2e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}+e^{-4\beta\varepsilon}}$
(d) $\frac{e^{-3\beta\varepsilon}}{1+e^{-\beta\varepsilon}+2e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}+e^{-4\beta\varepsilon}}$

************** End of Question Paper **************