NETWORK THEORY

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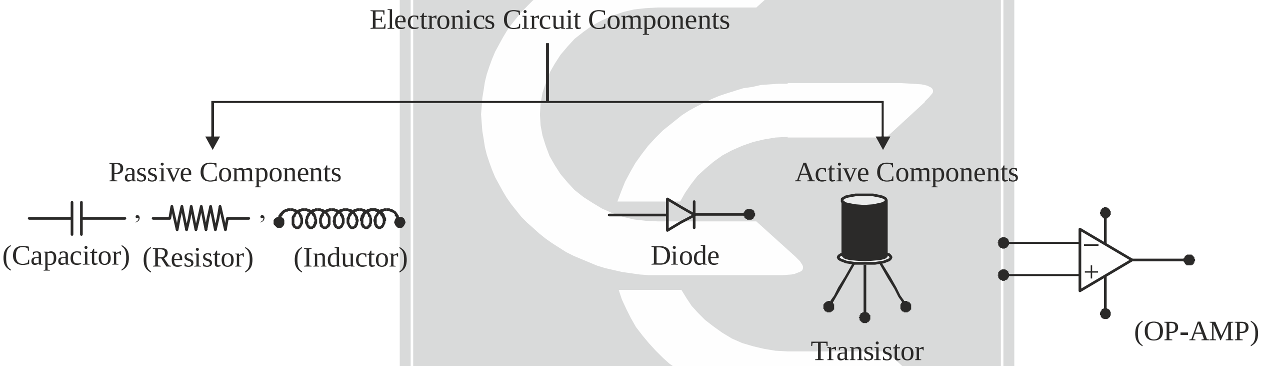
Introduction:

Nowadays electronics instruments like Computer, Phone, iPad, communication systems are playing vital roles in our everyday life. These electronics device consists of electronics circuit and circuit consists of electronics component like capacitor, resistor, inductor, semiconductor devices, voltage and current sources. To understand the operation of electronics devices it is necessary to know the current and voltage in all parts of this circuit. To analysis these circuits we frequently use the known laws of the electrical circuits. Thus, the performance analysis or the design of any electronics circuit requires a knowledge of circuit analysis.

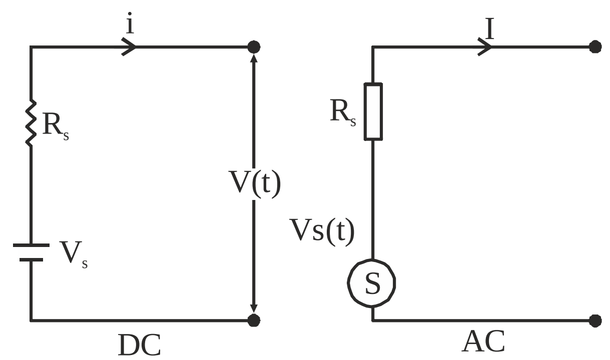
Circuit Components:

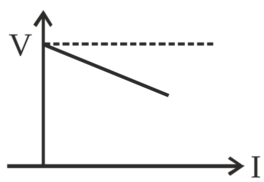


Voltage and Current Sources:

Voltage Source: Ideal voltage source delivers energy at a specified voltage (V), which is independent on current delivers by sources. The internal resistance of a voltage source is zero.

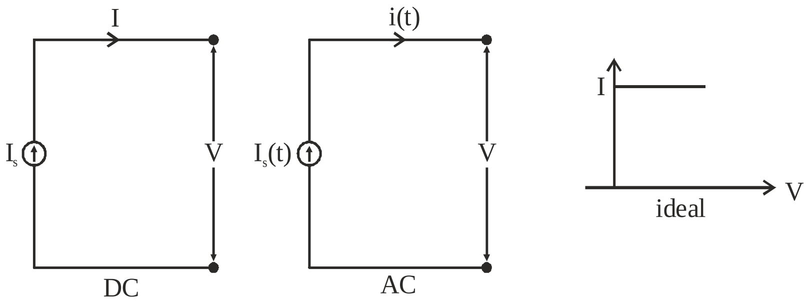
Practical Voltage Sources: It delivers energy at specified (V) which depends on current delivers by sources.



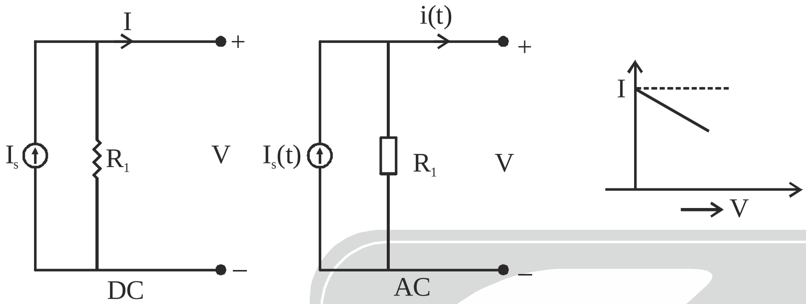


$$V(t) = Vs - IRs$$

Current Sources: Ideal current source delivers energy at a specified (I), which is independent on voltage across the source. Internal resistance of ideal current source = ∞.



Practical Current Sources: Practical current sources delivers energy at specified current I, which is dependent on voltage across the source. In real time system current source does not exist.



Network definitions:

Circuit : A circuit may be defined as a complete path for electric current flow.

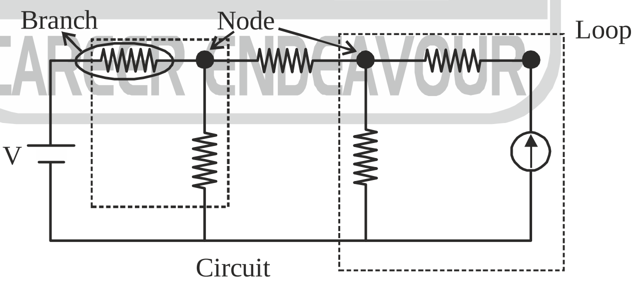
Branch : A group of circuit components having two terminals is called a branch.

Loop : A loop is any closed path formed by a number of branches in a circuit.

Node : A node is simply a common point where two or more than two components meet.

Short Circuit: If any two terminals of a network are connected by a wire of almost zero resistance, then the terminal are said to be short circuited.

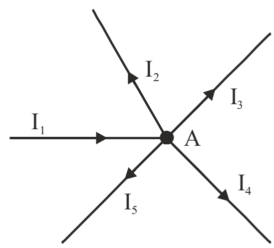
Open Circuit: If the connected path between the terminals is made open then the terminals are said to be open circuited.



Kirchhoff's current law:

The algebraic sum of the currents meeting at a junction point in a network is zero.

$$\sum_{i=1}^{n} I_i = 0$$

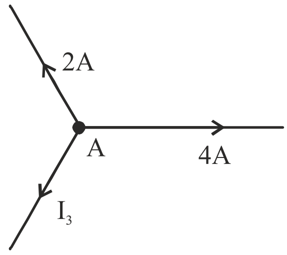


So, applying KCL at the node, we can write,

$$-I_1 + I_2 + I_3 - I_4 + I_5 = 0$$

When current enter into the junction we will take it as negative and when current flow out from the junction, we take it as a positive current.

Example: Suppose three branches are connected at the node A. Two of them current are known.



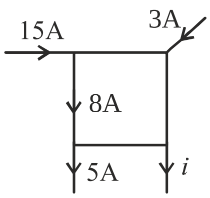
What will the current in 3rd wire?

Soln. According to Krichhoff's current law,

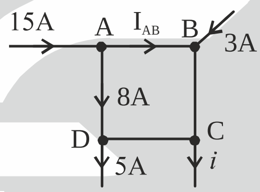
$$\Sigma I = 0 \Rightarrow 2A + 4A + I_3 = 0 \Rightarrow I_3 = -6A$$

So, current will enter to the junction.

Example: Consider the following circuit the value of current *i* is?

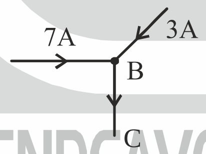


Soln. According to KCL, at the junction A,



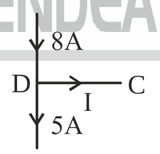
$$-15 + 8 + I_{AB} = 0 \Rightarrow I_{AB} = 7A$$

So, current will flow A to B direction.
Applying KCL at B, we can write,



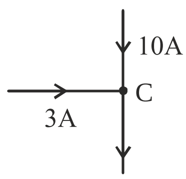
$$-7A - 3A + I_{BC} = 0 \Rightarrow I_{BC} = 10A$$

Applying KCL at D we can write



$$-8A + 5A + I_{DC} = 0 \Rightarrow I_{DC} = 3A$$

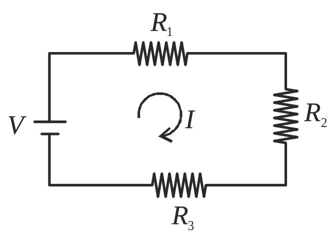
Applying KCL at C, we can write,



$$-10A - 3A + I = 0 \Rightarrow I = 13A$$

Kirchhoff's voltage law:
The algebraic sum of all voltage drops around a closed path in a network is zero.

$$-V + IR_1 + IR_2 + IR_3 = 0$$

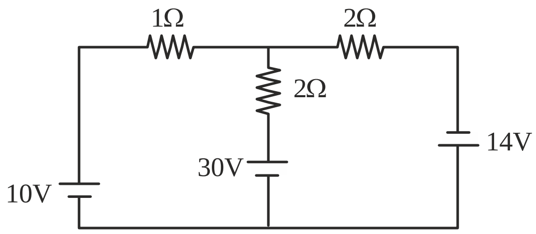


Note:

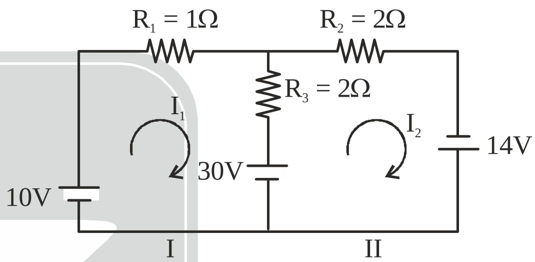
(i) If current flow out from positive terminal of voltage source, then we will take voltage negative.

(ii) If current flow out from negative terminal of votlage source, then we will take voltage positive.

Example: Find the current in each register of the given circuit.



Soln. According to KVL in Loop-I, we can write,



$$-10 + I_1(1+2) + 30 - 2I_2 = 0$$

$\Rightarrow \quad 3I_1 - 2I_2 = -20 \qquad \text{... (i)}$

And from Loop-II,

$$-30 + (I_2 - I_1)2 + 2I_2 - 14 = 0$$

$\Rightarrow \quad -2I_1 + 4I_2 - 44 = 0$

$\Rightarrow \quad I_1 - 2I_2 = -22 \qquad \text{...(ii)}$

From equation (i) and (ii), we can write,

$$2I_1 = 2 \Rightarrow I_1 = 1A$$

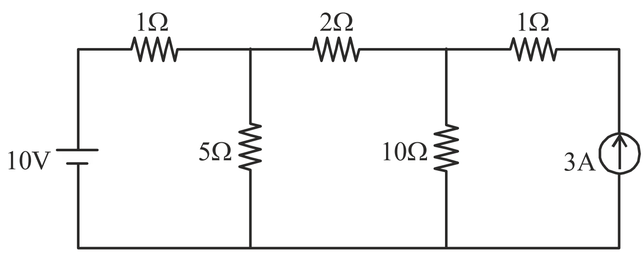
$\therefore \quad 1 - 2I_2 = -22 \Rightarrow I_2 = \dfrac{23}{2} = 11.5A$

Therefore, current through the resistance 1Ω is 1A.

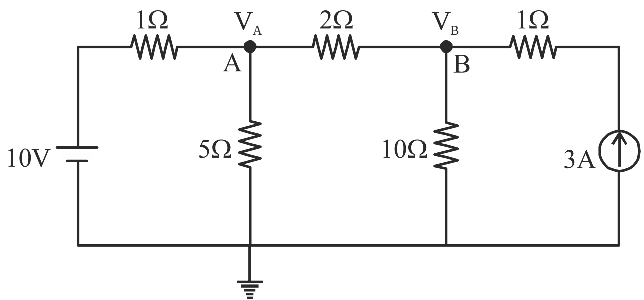
Current through the resistance, $R_3 = 2\Omega$ is 10.5A

Current through the resistance, R_2 is 11.5A

Example: Consider the cirucit is given below, find the current pass through the resistance 5Ω?



Soln. Applying nodal analysis at A, we can write,



$$\frac{V_A - 10}{1} + \frac{V_A - 0}{5} + \frac{V_A - V_B}{2} = 0$$

$$\Rightarrow \quad V_A + \frac{V_A}{5} + \frac{V_A}{2} - \frac{V_B}{2} = 10$$

$\Rightarrow \quad 17V_A - 5V_B = 100 \quad ...(i)$

And applying nodal analysis at B we can write,

$$\frac{V_B - V_A}{2} + \frac{V_B - 0}{10} - 3 = 0$$

$\Rightarrow \quad 6V_B - 5V_A = 30 \quad ...(ii)$

From equations (i) and (ii), we can write,

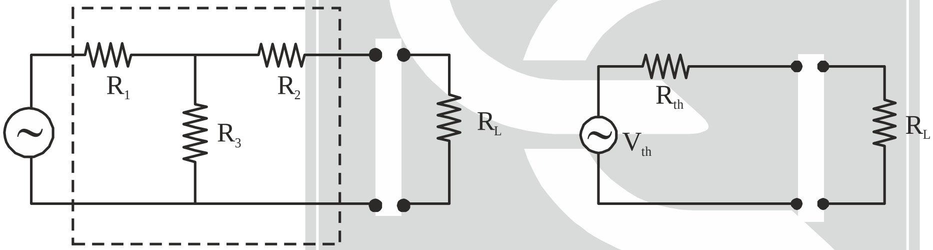
$$V_A = 9.7V$$

Therefore, current passing through the resistance 5Ω is

$$I = \frac{V_A - 0}{5\Omega} = \frac{9.7}{5}A = 1.94A$$

THEVENIN'S THEOREM

According to this theorem any two terminals of a linear network containing energy sources and impedances can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with an impedance R_{Th}, where V_{Th} is the open circuit voltage between the temrinals of the network and R_{Th} is the impedance measured between the terminals with all the energy sources replaced by their internal impedances.



Procedure for finding Thevenin equivalent circuit:

(1) Open the two terminals (i.e. remove the load resistance) between which you want to find the Thevenin equivalent circuit.

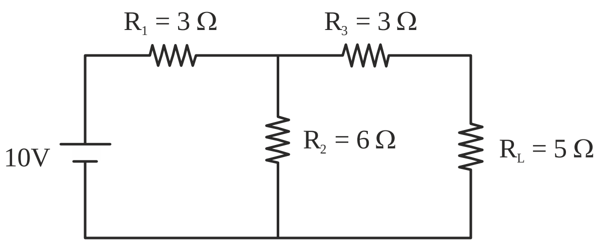
(2) Find the open-circuit voltage between the two open terminals. It is called Thevenin voltage V_{Th}.

(3) Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened. It is called Thevenin resistance R_{Th}.

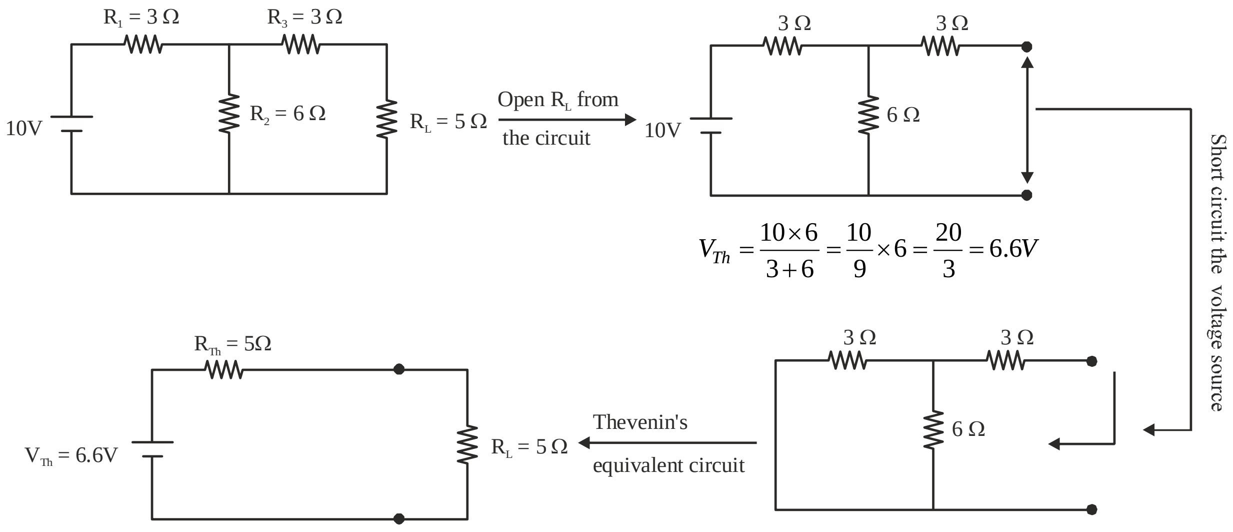
(4) Connect V_{Th} and R_{Th} in series to produce Thevenin equivalent circuit between the two terminal under the consideration.

(5) Place the load resistor removed in step(1) across the terminals of the Thevenin equivalent circuit. The load current can now be calculated using only Ohm's law and it has the same value as load current in the original circuit.

Example: Consider the circuit below, find the current through R_L using Thevenin's theorem.



Soln.

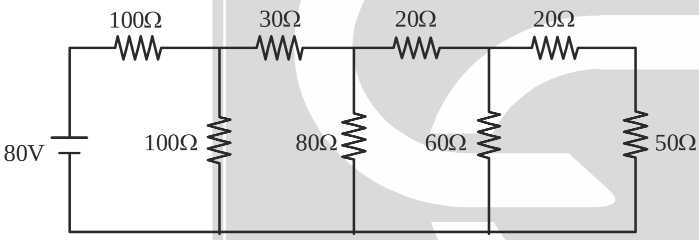


$$V_{Th} = \frac{10\times 6}{3+6} = \frac{10}{9}\times 6 = \frac{20}{3} = 6.6V$$

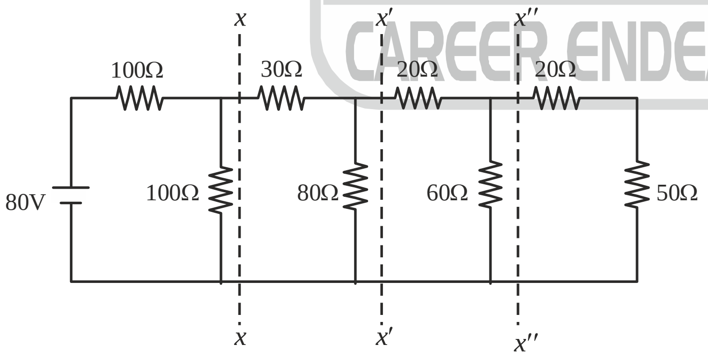
$$R_{Th} = \left[(3||6)+3\right] = 5\Omega$$

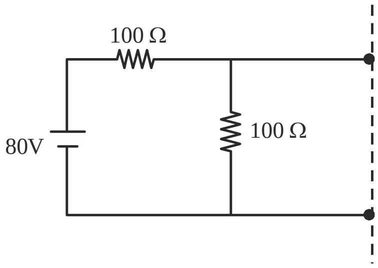
$$I = \frac{6.6}{5+5}A = 0.66\ A$$

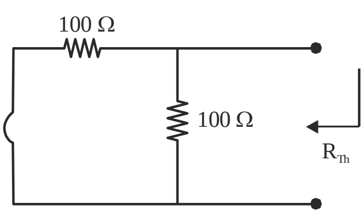
Example: Calculate the current in the $50\,\Omega$ resistor in the network shown in the figure below.



Soln. We can simplify the circuit by the repeated use of Thevenins theory, we first find out the Thevenin's equivalent circuit to the left of xx, $x'x'$, then $x''x''$.

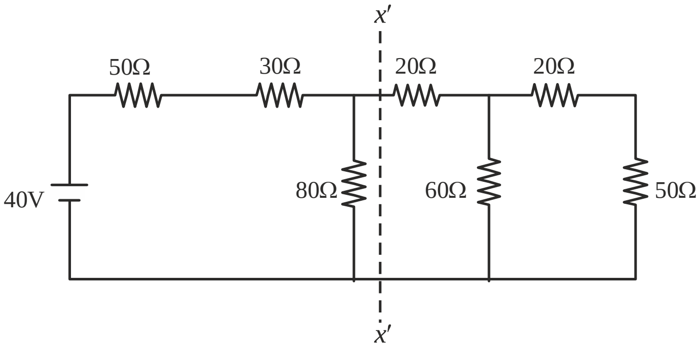




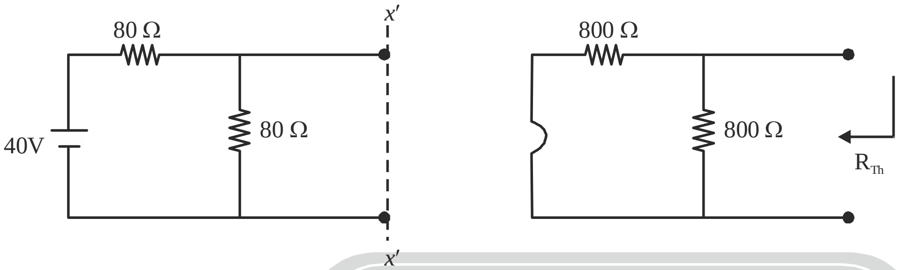


$$V_{Th} = \frac{80}{100+100}\times 100 = 40V$$

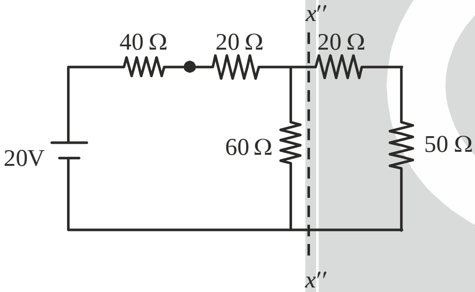
$$R_{Th} = 50\ \Omega$$



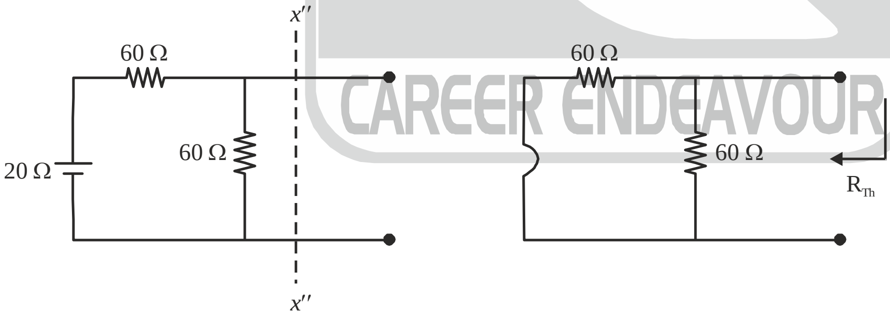
Applying Thevenin's equivalent circuit to the left of $x'x'$ is



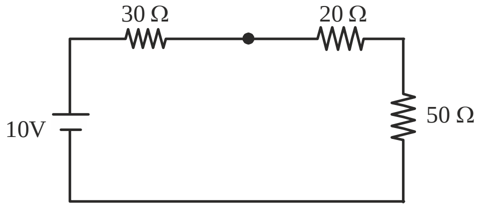
$$V_{Th} = \frac{40}{80+80} \times 80 = 20\,V \qquad R_{Th} = 40$$



Applying Thevenin's equivalent circuit to the left of $x''x''$ is



$$V_{Th} = \frac{20}{60+60} \times 60 = \frac{20}{2} = 10\,V \qquad R_{Th} = 30\,\Omega$$

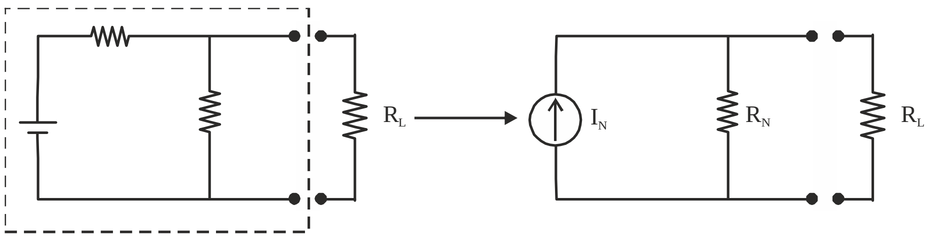


The current passing through the resistance $50\,\Omega$ is

$$I = \frac{10}{50+50} = \frac{10}{100} = 0.1\,A$$

NORTON'S THEOREM

Norton's theorem states that any network having two terminals A and B can be replaced by a current source of output I_N in parallel with a resistance R_N.



Procedure for finding Norton equivalent circuit:

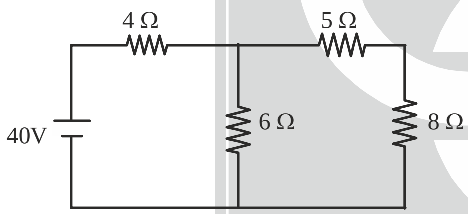
(1) Open the two terminals between which we want to find out Norton equivalent circuit.

(2) Put a short circuit across the terminal under consideration. Find the short-circuit current flowing in the short circuit. It is called Norton current I_N.

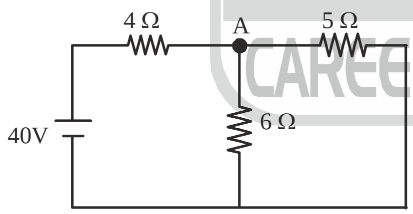
(3) Determined the resistance between two open terminals with all ideal voltage sources shorted and all ideal current sources opend. It is called Norton's resistance R_N.

(4) Connect I_N and R_N in parallel to produce Norton equivalent circuit between two terminal under consideration.

Example: Using Norton's theorem, find the current in 8Ω resistance in the network shown in figure.



Soln. **Step-I:**



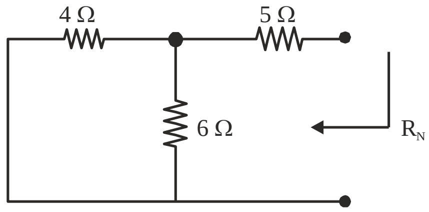
Applying Nodal analysis at the point A. We can write

$$\frac{V_A - 40}{4} + \frac{V_A - 0}{6} + \frac{V_A - 0}{5} = 0$$

$$\Rightarrow \quad V_A\left(\frac{1}{4} + \frac{1}{6} + \frac{1}{5}\right) = 10 \Rightarrow V_A\left(\frac{30+20+24}{120}\right) = 10 \Rightarrow V_A = \frac{120\times 10}{74} = 16.21$$

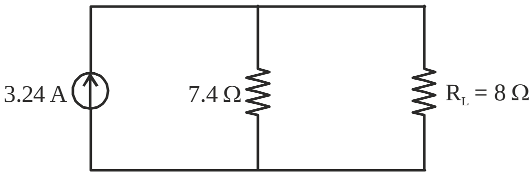
Therefore, Norton current, $I_N = \dfrac{16.4\,V}{5} = 3.24\ Amps$

Step-II: To find the Norton resistance remove the load resistance and replaced the voltage source by short circuit.



$$R_N = [(4||6)+5] = \frac{24}{10}+5 = 7.4\,\Omega$$

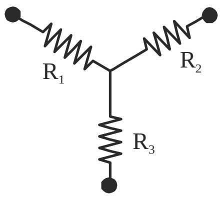
The Norton's equivalent circuit



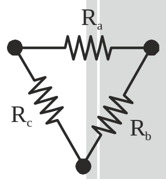
So, the current through the resistance 8Ω is

$$I = \frac{3.24\times 7.4}{(8+7.4)} = 1.55\ Amps$$

T-π transformation:



T-network



π-network

In the above two figure we have shown the T and π -network. In the network analysis sometimes it can becomes helpful to convert a T-network to π -network. Suppose we have resistance of T-network are R_1, R_2 and R_3. The resistance of the equivalent π -network are R_a, R_b and R_c, then we write,

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

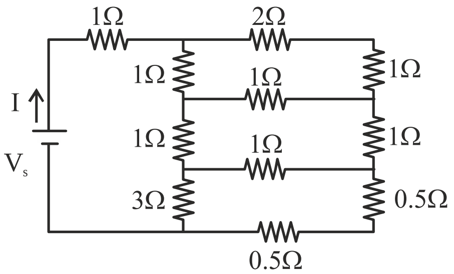
For conversion from π to T network:

$$R_1 = \frac{R_aR_c}{R_a + R_b + R_c}$$

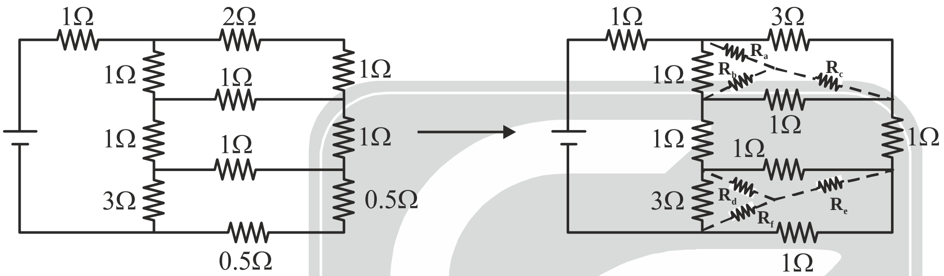
$$R_2 = \frac{R_aR_b}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_b R_b}{R_a + R_b + R_c}$$

Example: In the given circuit find the value of voltage source (V_s) that delivers 2 Amprs current through the circuit?



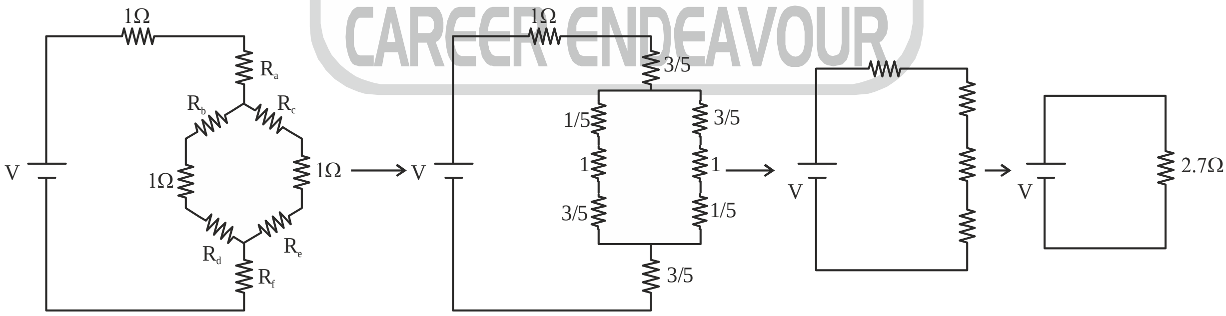
Soln.



Using π to T conversion:

$$R_a = \frac{3}{1+1+3} = \frac{3}{5}\Omega; \quad R_b = \frac{1}{5}\Omega; \quad R_c = \frac{3}{5}\Omega$$

$$R_d = \frac{3}{5}\Omega; \quad R_e = \frac{1}{5}\Omega; \quad R_f = \frac{3}{5}\Omega$$



So, current in the circuit is $I = \dfrac{V}{2.7} \Rightarrow V = 2 \times 2.7 = 5.4\,V$