Chapter 1

Simple Harmonic Motion

1.1 DESCRIPTION OF SIMPLE HARMONIC MOTION (SHM):

1.1.1 Equation of motion:

Suppose a particle of mass *m* is undergoing SHM along a line. If *x* be its displacement at any instant from the position of stable equilibrium, then restoring force *F* under small oscillation approximation may be written as

$$F = -sx \qquad \text{... (1)}$$

where *s* is the restoring force per unit displacement, called force constant.
The negative sign indicates that *F* and *x* are oppositely directed. For spring restoring force follows Hooke's law of elasticity. From Newton's law, the equation of motion is

$$m\frac{d^2x}{dt^2} = -sx \;\Rightarrow\; \frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad \text{... (2)}$$

where $\omega^2 = s/m$.
Equation (2) is the differential equation of motion of a simple harmonic oscillator.
Its general solution is given by the principle of superposition as

$$x = c_1 e^{j\omega t} + c_2 e^{-j\omega t} \qquad \text{... (3)}$$

$$\Rightarrow \quad x = c_1(\cos\omega t + j\sin\omega t) + c_2(\cos\omega t - j\sin\omega t)$$

$$= A\cos\omega t + B\sin\omega t \qquad \text{... (4)}$$

where, $A = (c_1 + c_2)$ and $B = j(c_1 - c_2)$.
Again, introducing two other constants *a* and ε by the relations $A = a\sin\varepsilon$ and $B = a\cos\varepsilon$, such that $a = \sqrt{A^2 + B^2}$ and $\tan\varepsilon = A/B$, solution (4) can be put in the form,

$$x = a\sin(\omega t + \varepsilon) \qquad \text{... (5)}$$

Putting $\varepsilon = \frac{\pi}{2} + \delta$, the solution (5) can also be expressed as

$$x = a\cos(\omega t + \delta) \qquad \text{... (6)}$$

1.1.2 Characteristics of SHM:

Let us choose the solution, $x = a\cos(\omega t + \delta)$

Here '*a*' represents the maximum value of displacement from the position of stable equilibrium. It is called the **amplitude** of SHM. The **time period *T*** is the time interval in which the motion repeats itself and is given by

$$x(t) = x(t+T)$$

$$\Rightarrow \quad a\cos(\omega t + \delta) = a\cos[\omega(t+T)+\delta] = a\cos[\omega t + \delta + \omega T]$$

$$\Rightarrow \quad a\cos[(\omega t + \delta) + 2\pi] = a\cos[(\omega t + \delta) + \omega T]$$

For equality we must have, $\omega T = 2\pi$ or, $T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{s}}$ ... (7)

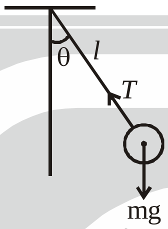
The quantity ω is called the **angular frequency**. The usual frequency ν which gives the number of oscillations per unit time is given by $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$ ... (8)

The quantity $(\omega t + \delta)$ in equation (6) is called the phase of motion and δ is called the initial phase or epoch.

Effective acceleration due to gravity (g')

When simple pendulum is kept in inertial frame its time period is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$



When simple pendulum is kept in non-inertial frame its time period is given by

$$T = 2\pi\sqrt{\frac{l}{g'}},$$

where g' is called effective gravity and is given by

$$\vec{g}' = \vec{g} - \vec{a} \quad \text{... (9)}$$

where $\vec{a}$ is acceleration of the frame.

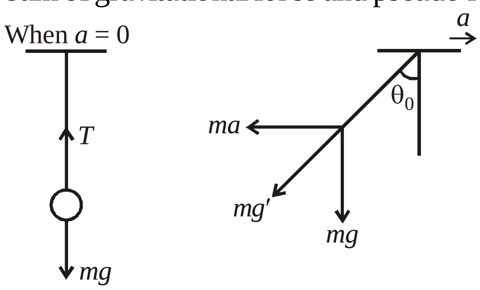
Illustration : A simple pendulum, suspended from the ceiling of a stationary cart, has a time period 2 seconds. When the cart accelerates in the horizontal direction with an acceleration of 10 m/s^2, the time period of the pendulum is (g = 10 m/s^{-2}) **[JAM-GP: 2007]**

(a) $2^{1/2}$ seconds (b) $2^{3/2}$ seconds (c) $2^{1/4}$ seconds (d) $2^{3/4}$ seconds

Soln. Forces on bob

1. Gravity downward : mg
2. Tension force : T
3. Psuedo force : – ma

Now, at equilibrium condition, sum of gravitational force and pseudo force will be balanced by tension force.



Mean position $\theta = 0°$ Mean position $\theta = \theta_0$

Here $g' = \vec{g} - \vec{a}$

$$\Rightarrow \quad g' = \sqrt{a^2 + g^2}$$

$$\text{Time period} = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}} = 2\pi\left[\sqrt{\frac{l}{g\sqrt{1+\frac{a^2}{g^2}}}}\right] = 2\pi\sqrt{\frac{l}{g}}\frac{1}{\left(1+\frac{a^2}{g^2}\right)^{1/4}} = 2\times\frac{1}{2^{1/4}} = 2^{3/4}\ \text{sec}.$$

Correct option is (d)

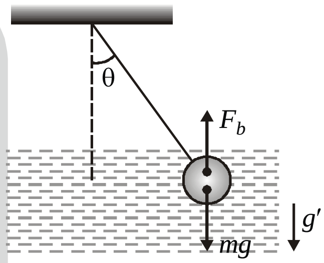
Illustration : If a pendulum bob is suspended by a string of length L inside a fluid (the bob is η times more dense than the fluid), the period of small oscillations of the pendulum will be given by

(a) $2\pi\sqrt{\frac{\eta L}{g}}$ (b) $2\pi\sqrt{\frac{\eta L}{g(\eta-1)}}$ (c) $2\pi\sqrt{\frac{(\eta-1)L}{g\eta}}$ (d) $2\pi\sqrt{\frac{L}{g\eta}}$ **[H.C.U.-2011]**

Soln. Density of bob $\rho_b = \eta\rho_\ell$; ρ_L = density of liquid (fluid)

Time period $T = 2\pi\sqrt{\ell/g'}$, where g' = effective gravity.

Assuming bob a system $Mg' = Mg - F_b = \rho_b Vg' = \rho_b Vg - \rho_L Vg$



$$\Rightarrow \quad g' = g - \frac{\rho_L}{\rho_b}g = \left(1-\frac{\rho_L}{\rho_b}\right)g = \left(1-\frac{1}{\eta}\right)g = \left(\frac{\eta-1}{\eta}\right)g$$

$$T = 2\pi\sqrt{\frac{L}{\left(\frac{\eta-1}{\eta}\right)g}} = 2\pi\sqrt{\frac{\eta L}{(\eta-1)g}}$$

Correct option is (b)

Illustration : A simple pendulum executes oscillations of period 10 s on the surface of Earth. This pendulum is taken to another planet having same mass but one fourth of Earth's size. The period of oscillations of the pendulum would now be

(a) 40 s (b) 10 s (c) 5 s (d) 2.5 s **[H.C.U.-2014]**

Soln. Time period of pendulum on earth

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Acceleration due to gravity on earth

$$g = \frac{GM_E}{R_E^2} \qquad \ldots(1)$$

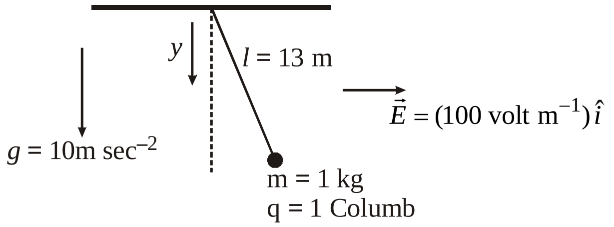
Acceleration due to gravity on planet

$$g' = \frac{GM_E}{\left(\frac{R_E}{4}\right)^2} = \frac{4GM_E}{R_E^2} = 4g \qquad \ldots(2)$$

$$T' = 2\pi\sqrt{\frac{L}{g'}} = 2\pi\sqrt{\frac{L}{4g}} = \frac{T}{2} = \frac{10}{2} = 5\,\text{sec}.$$

Correct option is (c)

Illustration : A simple pendulum has a bob of mass 1 kg and charge 1 Coulomb. It is suspended by the massless string of length 13 m. The time period of small oscillations of this pendulum is T_0. If an electric field $\vec{E} = 100\,\hat{x}$ V/m applied, the time period becomes T. What is the value of $(T_0/T)^4$? **[JEST 2017]**



Soln. Due to electric field $\vec{E}$, charge q will be pushed in horizontal direction by force

$$\vec{F}_e = q\vec{E} \qquad \text{... (1)}$$

Resultant force of $\vec{F}_e$ and weight mg will give direction of effective gravity g'. Thus,

$$m\vec{g}' = q\vec{E} + m\vec{g}$$

$$\vec{g}' = \frac{q\vec{E}}{m} + \vec{g}$$

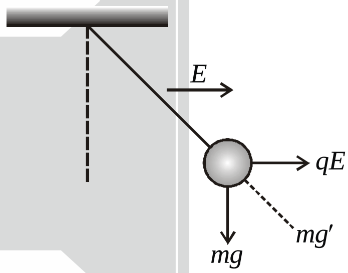
$$|\vec{g}'| = \sqrt{\left(\frac{qE}{m}\right)^2 + g^2} \qquad \text{... (2)}$$

Time period, in absence of electric field

$$T_0 = 2\pi\sqrt{\frac{L}{g}}$$

Time period, in presence of electric field

$$T = 2\pi\sqrt{\frac{L}{g'}}$$



$$\Rightarrow \quad \left(\frac{T_0}{T}\right)^4 = \left[\sqrt{\frac{g'}{g}}\right]^4 = \left(\frac{g'}{g}\right)^2$$

$$\Rightarrow \quad \left(\frac{T_0}{T}\right)^4 = \frac{\left(\frac{qE}{m}\right)^2 + g^2}{g^2} = \left(\frac{qE}{mg}\right)^2 + 1$$

Putting values, $q = 1\text{c}$, $E = 100$ V/m, $m = 1$ kg, $g = 10\ \text{m/s}^2$, we get

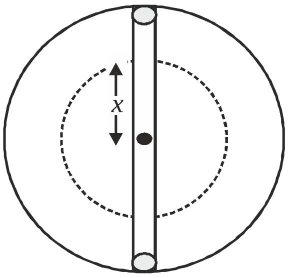
$$\left(\frac{T_0}{T}\right)^4 = \left(\frac{100}{10}\right)^2 + 1 = 101$$

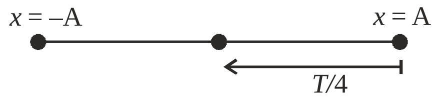
Correct answer is (101)

Illustration : A tunnel is dug from the surface of the earth through the center and opens at the other end. A ball is dropped from one end of the tunnel. The acceleration due to gravity on the earth's surface is g and the radius of the earth is R. Assuming that the earth has a constant density, the time taken by the ball to reach the center of the earth is **[DU 2017]**

(a) $\pi\sqrt{\frac{R}{g}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $\frac{\pi}{2}\sqrt{\frac{R}{g}}$ (d) $\frac{\pi}{4}\sqrt{\frac{R}{g}}$

Soln.





Let the mass of the ball is m and is at height x from the center . Gravitational force acting on the ball of mass m, is due to mass enclosed (M_{en}) in a sphere of radius *x*.

Mass enclosed within sphere of radius *x*

$$M_{en} = \rho V = \left(\frac{M_E}{4/3\pi R^3}\right)\left(\frac{4}{3}\pi x^3\right) = \frac{M_E}{R^3}x^3$$

Gravitional force due to earth

$$F = \frac{GmM_{en}}{x^2} = \frac{GmM_e x^3}{R^3x^2} = \frac{GM_E mx}{R^3} = kx$$

As the force *F* is opposite of distance x therefore, ball will perform SHM.

$$\text{Time period } T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{GM_E m}{R^3}\right)}} = 2\pi\sqrt{\frac{1}{\left(\frac{GM_E}{R^2}\right)\frac{1}{R}}} = 2\pi\sqrt{\frac{R}{g}},$$

where $g = \frac{GM_E}{R^2}$; acceleration due to gravity.

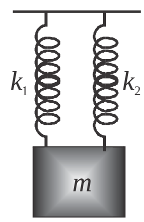
Time required to reach center (mean position) from the surface (maximum position) is equal to *T*/4, i.e.

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{R}{g}}$$

Correct option is (c)

Illustration : Two springs of force constant k_1 and k_2 are arranged in a parallel arrangement and a mass '*m*' is suspended from it. The arrangement is equivalent to a single spring of constant *k* given by **[DU 2015]**

(a) $k_1 - k_2$ (b) $k_1 + k_2$ (c) $k_1k_2/(k_1+k_2)$ (d) k_1/k_2



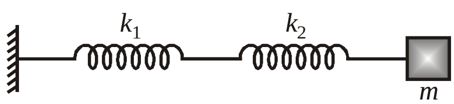
Soln. If an elongation of *x* is given to the system, then restoring force

$$F = -k_1x - k_2x = -(k_1+k_2)x = -kx$$

where, $k = k_1 + k_2 \equiv$ equivalent spring constant

Correct option is (b)

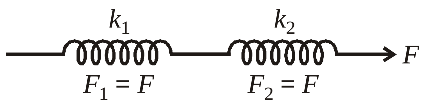
Illustration : Find the time period for the arrangement of the spring mass system having constant k_1 and k_2.



Soln. Let *x* be the displacement of mass *m* from its equilibrium position at an instant and x_1 and x_2 are the extension in the springs k_1 and k_2 respectively.

$$x = x_1 + x_2 \qquad \ldots \text{(i)}$$

Tension force at each points in a spring is equal to the external force applied at one of its end,



i.e., $F = F_1 = F_2$ ... (ii)

$-k'x = -k_1x_1 = -k_2x_2$

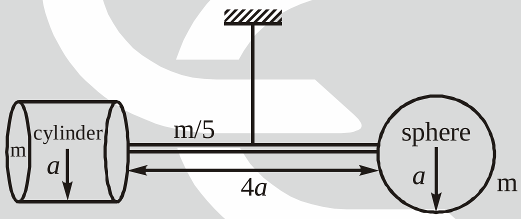
where k' is equivalent spring constant,

From equation (i)

$$\frac{F}{k'} = \frac{F}{k_1} + \frac{F}{k_2} \Rightarrow \boxed{\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}} \quad \text{... (iii)}$$

i.e., if there is same tension force in all connected springs, equivalent spring constant is given by equation (iii).

Illustration : A solid cylinder of mass *m*, radius *a* and height also *a* and a solid sphere of the same mass and the same radius are connected to the two ends of a thin rod of mass m/5 and length 4*a*. The line joining the centres of mass of the cylinder and the sphere coincides with the axis of the rod. The vertical cross-section of the assembly is shown in the figure below. The whole assembly is suspended vertically by a massless wire passing through its centre of mass. The torsional constant of the wire is 76 ma^2/sec^2. Calculate the frequency of the torsional oscillation about the suspension in the horizontal plane.



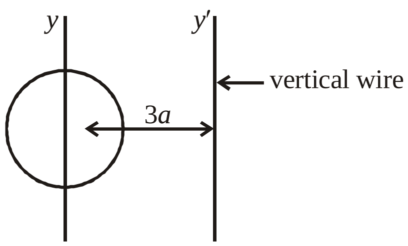
[JAM-GP: 2010]

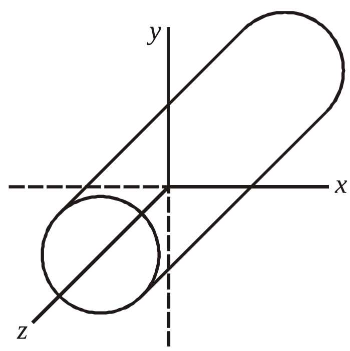
Soln. Let us calculate the moment of inertia about of cylinder and sphere, and the rod about vertical wire.

Cylinder : $I_z = I_x + I_y = \frac{1}{2}ma^2$ (Perpendicular Axes Theorem)

$$I_x = I_y = \frac{1}{4}ma^2$$

Applying Parallel Axes Theorem, moment of inertia about vertical wire of :





Cylinder : $I'_c = I_{cm} + m(3a)^2 = \frac{1}{4}ma^2 + 9ma^2 = \frac{37}{4}ma^2$.

Sphere : $I_s = I_{cm} + m.(3a)^2 = \frac{2}{5}ma^2 + 9ma^2 = \frac{47}{5}ma^2.$

Rod : $I_r = \frac{1}{12}ML^2 = \frac{1}{12}\cdot\frac{m}{5}\times 16a^2 = \frac{4}{15}ma^2.$

Moment of inertia of system about the vertical wire,

I = sum of moment of inertia of individual bodies about vertical wire

$$= \frac{37}{4}ma^2 + \frac{47}{5}ma^2 + \frac{4}{15}ma^2$$

$$I = \frac{227}{12}ma^2$$

Now, if the rod along with the cylinder and sphere is twisted by an angle of θ, a restoring torsional torque, $\tau = -k\theta$ acts and tends to brings the system back to its original position.

$$\tau = -k\theta$$

$$I\frac{d^2\theta}{dt^2} = -k\theta \Rightarrow I\frac{d^2\theta}{dt^2} + k\theta = 0$$

$$\frac{227}{12}ma^2\frac{d^2\theta}{dt^2} + 76ma^2\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{912}{227}\theta = 0$$

Angular frequency of torsional oscialltion $\omega = \sqrt{\frac{912}{227}}$ $\quad f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}\sqrt{\frac{912}{227}}.$

1.1.3 Energy in SHM:

Kinetic energy at any instant is given by $E_k = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$

$$\Rightarrow \quad E_k = \frac{1}{2}ma^2\omega^2\sin^2(\omega t + \delta) \qquad \text{... (10)}$$

Total potential energy acquired by the particle for the displacement x is

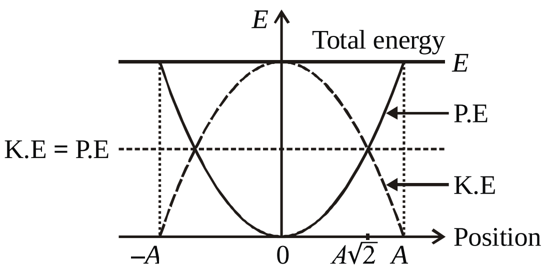
$$E_p = -\int F\cdot dx = \int_0^x sx\,dx = \frac{1}{2}sx^2$$

$$\Rightarrow E_p = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}\mathrm{m}\omega^2\mathrm{a}^2\cos^2(\omega\mathrm{t} + \delta) \qquad \text{... (11)}$$

Total instantaneous energy, $\mathrm{E} = \mathrm{E_k} + \mathrm{E_p} = \frac{1}{2}\mathrm{m}\omega^2\mathrm{a}^2$ = constant ... (12)

and it is equal to the maximum value of the kinetic energy or the potential energy. Thus in SHM energy oscillates between the kinetic and potential forms but total energy remains constant.

Energy position graph:



PE **:** Graph is parabola with vertex at $x = 0$.

KE **:** The KE curve is inverted parabola.

Illustration : Find the position in terms of amplitude when kinetic energy is equal to potential energy of a particle is SHM.

Ans. If $x = A\sin \omega t$ ... (i)

then $v = \frac{dx}{dt} = A\omega \cos \omega t$

$$= A\omega\left[1-\left(\frac{x}{A}\right)^2\right]^{1/2}$$

$$= \omega\left[A^2 - x^2\right]^{1/2} \quad \text{... (ii)}$$

Kinetic energy, $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left[A^2 - x^2\right]$ ... (iii)

Potential energy, $PE = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$,

when, $PE = KE$

$$\frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2\left[A^2 - x^2\right]$$

$$\Rightarrow \quad 2x^2 = A^2 \quad \Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}.$$

Illustration : Frequency of energy change ω from KE to PE in SHM is related with the frequency ω_0 of SHM by

(a) $\omega = 2\omega_0$ (b) $\omega = \frac{\omega_0}{2}$ (c) $\omega = \omega_0^2$ (d) $\omega = \omega_0$

Soln. Potential energy $PE = \frac{1}{2}m\omega_0^2 x^2$

If $x = A\sin \omega_0 t$ then

$$PE = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega_0 t = \frac{1}{2}m\omega_0^2 A^2\left[\frac{1-\cos 2\omega_0 t}{2}\right]$$

$$= \frac{1}{4}m\omega_0^2 A^2 - \frac{1}{4}m\omega_0^2 A^2 \cos 2\omega_0 t = c - c\cos 2\omega_0 t, \text{ where } c = \frac{1}{4}m\omega_0^2 A^2 = \text{constant}$$

$$= c - c\cos \omega_E t$$

$\Rightarrow$ frequency of energy change $\boxed{\omega_E = 2\omega_0}$.

Force Constant in terms of potential energy:

If $U(x)$ is potential energy of the oscillating system in SHM, then at equilibrium point $x = x_0$,

$$F = -\left.\frac{\partial U(x)}{\partial x}\right|_{x_0} = 0.$$

Solving this, we get mean position x_0. The force constant of the oscillating system can be given by

$$K = \left.\frac{d^2U(x)}{dx^2}\right|_{x_0}$$

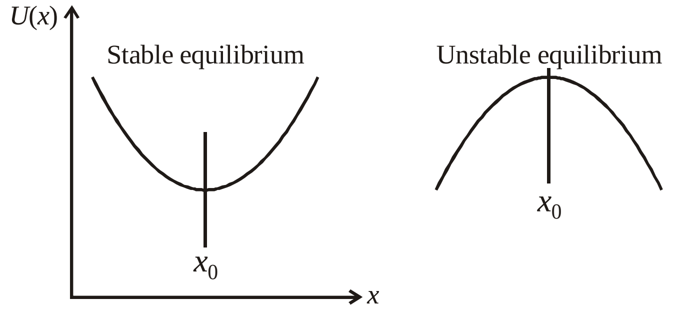


Illustration : A particle of mass '*m*' is moving in a potential

$$V(x) = \frac{1}{2}m\omega_0^2 x^2 + \frac{a}{2mx^2}$$

where ω_0 and 'a' are positive constants. The angular frequency of small oscillations for the simple harmonic motion of the particle about a stable minimum of the potential V(x) is:

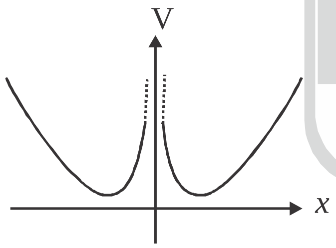
(a) $\sqrt{2}\omega_0$ (b) $2\omega_0$ (c) $4\omega_0$ (d) $4\sqrt{2}\omega_0$

[IIT-JAM : 2011]

Soln. **(b)** $V(x) = \frac{1}{2}m\omega_0^2 x^2 + \frac{a}{2mx^2}$ **;** $\frac{\partial V}{\partial x} = m\omega_0^2 x - \frac{a}{mx^3}$ **;** $\frac{\partial^2 V}{\partial x^2} = m\omega_0^2 + \frac{3a}{mx^4}$

At minimum potential $\frac{\partial V}{\partial x} = 0\left(\text{at } x = x_0\right)$

$$m\omega_0^2 x_0 = \frac{a}{mx_0^3} \Rightarrow x_0 = \left(\frac{a}{m^2\omega_0^2}\right)^{1/4}\text{, equilibrium position}$$



Force constant, $k = \left.\frac{\partial^2 V}{\partial x^2}\right|_{x=x_0} = m\omega_0^2 + \frac{3a}{mx_0^4} = m\omega_0^2 + \frac{3m^2\omega_0^2 x_0^4}{mx_0^4} = 4m\omega_0^2$

$$\therefore \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4m\omega_0^2}{m}} \Rightarrow \boxed{\omega = 2\omega_0}$$

1.1.4 Equation of motion from principle of conservation of energy:

Total energy of the oscillator is $\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}sx^2 = E$ (constant) ... (13)

$$\Rightarrow m\frac{dx}{dt}.\frac{d^2x}{dt^2} + sx\frac{dx}{dt} = 0 \qquad \Rightarrow \left[m\frac{d^2x}{dt^2} + sx\right]\frac{dx}{dt} = 0$$

Since dx/dt cannot be zero for all values of t, $m\frac{d^2x}{dt^2} + sx = 0$ ... (14)

which is the equation of motion.

1. 2 REPRESENTATION OF SHM BY ROTATING VECTOR:

$\overrightarrow{OB}$ is a vector of constant magnitude a which rotates with constant angular velocity ω in anticlockwise sense. At time t = 0 the vector coincides with the line OA which makes an angle δ with x - axis. The x–component of the rotating vector is $x = a\cos(\omega t + \delta)$

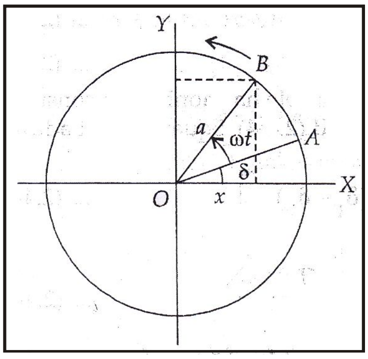


Figure **1.1**

which represents a SHM. The y-component of the rotating vector is $y = a\sin(\omega t + \delta)$ which also represents a SHM. From the above treatment it is clear that SHM can be viewed as a projection of circular motion on any fixed diameter of the circle.

1.3 SUPERPOSITION OF COLLINEAR SHMs:

The principle of superposition according to which resultant displacement due to a number of sources is given by the algebraic sum of the displacements caused by the individual sources.

1.3.1 SHMs of same frequency acting along the same direction but having different amplitudes and phases:

Let two SHMs be represented by

$$x_1 = a_1\cos(\omega t + \delta_1) \quad \text{... (15)}$$

$$x_2 = a_2\cos(\omega t + \delta_2) \quad \text{... (16)}$$

where a_1 and a_2 are the amplitudes, δ_1 and δ_2 are the initial phase angles of the two SHMs of same angular frequency ω.

By the superposition principle the resultant displacement is given by

$$x = x_1 + x_2 = a_1\cos(\omega t + \delta_1) + a_2\cos(\omega t + \delta_2)$$

$$= (a_1\cos\delta_1 + a_2\cos\delta_2)\cos\omega t - (a_1\sin\delta_1 + a_2\sin\delta_2)\sin\omega t$$

Putting $a_1\cos\delta_1 + a_2\cos\delta_2 = A\cos\phi$

$a_1\sin\delta_1 + a_2\sin\delta_2 = A\sin\phi$

we get $x = A\cos(\omega t + \phi)$ ... (16)

It shows that the resultant motion is also simple harmonic, amplitude of which is given by

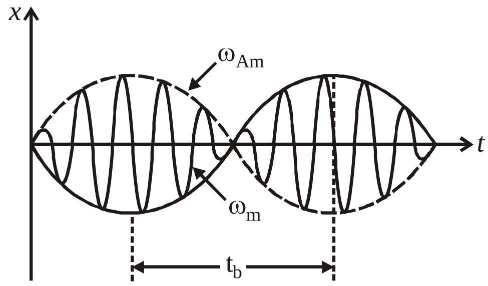
$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\delta_1 - \delta_2) \quad \text{... (17)}$$

And $\tan\phi = \dfrac{a_1\sin\delta_1 + a_2\sin\delta_2}{a_1\cos\delta_1 + a_2\cos\delta_2}$ ... (18)

1.3.2 Two SHMs of slightly different frequencies acting along the same direction:

Let us consider two SHMs having slightly different angular frequencies ω_1 and ω_2.

$$x_1 = A_0 \sin\omega_1 t$$
$$x_2 = A_0 \sin\omega_2 t$$
$$x = x_1 + x_2 = 2A_0\left[\sin\left(\frac{\omega_1+\omega_2}{2}\right)t \cos\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$



Let $\dfrac{\omega_1+\omega_2}{2} = \omega_m$ and $\dfrac{\omega_1-\omega_2}{2} = \omega_{Am}$

$$\boxed{x = 2A_0 \sin\omega_m t \cos\omega_{Am} t}$$
$$x = (2A_0 \cos\omega_{Am} t) \sin\omega_m t$$
$$x = A\sin\omega_m t$$

where $A = (2A_0 \cos\omega_{Am} t)$, if $\omega_1 \sim \omega_2,\ \omega_m \to 0$

amplitude $A = 2A_0 \to$ nearly constant.

Then the result oscillation is nearly SHM with frequency ω_{Am} and amplitude $2A_0$.

For maximum amplitude

$$\Rightarrow \cos\omega_{Am} t = \pm 1$$
$$\Rightarrow \omega_{Am} t = n\pi$$
$$\Rightarrow t = \frac{2n\pi}{(\omega_2-\omega_1)} = \frac{2n\pi}{2\pi(\nu_2-\nu_1)} = \frac{n}{\nu_2-\nu_1} = 0, \frac{1}{\nu_2-\nu_1}, \frac{2}{\nu_2-\nu_1}, \frac{3}{\nu_2-\nu_1}, \ldots\ldots$$

time difference between subsequent maxima, $\Delta t = \dfrac{1}{\nu_2-\nu_1}$.

Similarly,

time difference between subsequent minima, $\Delta t = \dfrac{1}{\nu_2-\nu_1}$.

Beat time : It is time interval between successive maxima.

$$t_b = \frac{1}{\nu_2-\nu_1}$$

Beat frequency : $\mathbf{\dfrac{1}{t_b} = |\nu_2 - \nu_1|}$

Illustration : The superposition of two harmonic oscillator oscillating in same direction result motion of a point given by $x = a\cos 2.1t \cos 50t$

Calculate (i) individual frequency, (ii) Beat frequency

Soln. (i) If $x_1 = A\cos\omega_1 t$ and $x_2 = A\cos\omega_2 t$, then

$$x = x_1 + x_2$$
$$= 2A\cos\left(\frac{\omega_2-\omega_1}{2}\right)t \cos\left(\frac{\omega_2+\omega_1}{2}\right)t$$

By comparing, $\dfrac{\omega_2-\omega_1}{2} = 2.1 \Rightarrow \omega_2-\omega_1 = 4.2$ and $\dfrac{\omega_2+\omega_1}{2} = 50.0 \Rightarrow \omega_2+\omega_1 = 100.0$