2 System of Linear Equation

Consider the following system of m linear equation in n unknowns over a field F

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$$

$a_{ij}, b_i \in F$, $i = 1, 2, ..., m,\ j = 1, 2, ..., n$

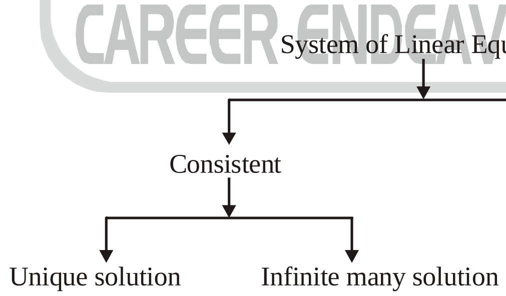
This system can be written as $Ax = b$

Where $A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$

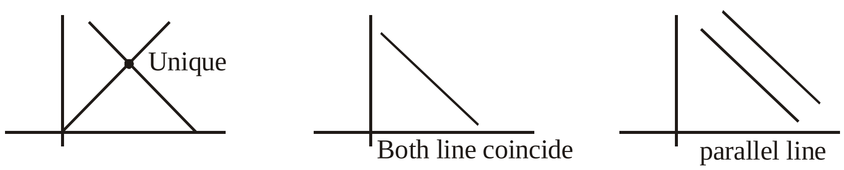
The $m \times n$ matrix A is called the coefficient matrix

(a) If $b = 0$ then the system is called homogeneous otherwise it is called non homogeneous.

(b) A system of linear equations is called consistent if there exist $x_1 \in F^n$ such that $Ax_1 = b$ otherwise it is called inconsistent.



Eg.



(c) The $m \times (n + 1)$ matrix $[A:b]$ is called the augmented matrix of the system.

(d) Two system $Ax = b$ and $A'x = b'; A, A' \in F^{m \times n}$, $b, b' \in F^m$ are said to be equivalent if they have the same set of solutions.

Homogeneous System :

Consider a homogeneous system $Ax = 0$ ... (1)

Clearly $x_1 = x_2 ... = x_n = 0$ *i.e* $x = 0$ is a solution of (1). Therefore it is always a consistent system and this solution is called trivial solution of (1).

Again if x_1 and x_2 are two solution of (1)

Then their linear combination $k_1x_1 + k_2x_2$, where $k_1, k_2 \in F$ is also a solution of (1).

Therefore the collection of all the solutions of the system of equations $Ax = 0$ form a subspace of the *n*-dimensional vector space.

Theorem: The number of linear independent solution of *m* homogeneous linear equations in *n* variables, $Ax = 0$ is $(n-r)$, where *r* is the rank of the matrix *A*. Hence dimension of solution space is *n-r.*

Consider the system $Ax = 0$

$$\text{Where } A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}_{m\times n} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$$

Case –I : If $n \le m$

(a) If Rank (*A*) = *n*, then the system has only trivial solution.

(b) If *r* = rank (*A*) < *n*, then the system has infinite number of solutions and the dimension of solution space is *n-r*.

Case -II If $m < n$

(a) If Rank (*A*) = *m*, then the solution space is of *n-m* dimension.

(b) If $r = Rank(A) < m$, then the system has infinite number of solutions and the dimension of solution space is *n-r*.

Non- Homogeneous system :

Consider the system $Ax = b$

$$\text{Where } A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}, \; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Theorem:

The system of equation $Ax = b$ is consistent if and only if the coefficient matrix *A* and the augmented matrix [*A*:*b*] are of the same rank.

Case - I If $m = n$

(a) If Rank (*A*) = *m* = *n* then the system is always consistent and has unique solution.

(b) If $r = Rank(A) < m = n$ then the system has solution if *b* can be written as linear combination of columns of *A* and has infinite solutions.

If *b* can not be written as linear combination of columns of *A* then the system has no solution

Case -II If $m < n$

(a) If $Rank(A) = m < n$, then the system always have infinite number of solutions.

(b) If $Rank(A) < m < n$ then the system has infinite number of solutions if *b* can be written as linear combinations of columns of A otherwise it does not have any solution.

Case-III if $n < m$

(a) If $Rank(A) = n < m$, then the system has unique solution if b can be written as linear combination of columns of A.

(b) If $Rank(A) < n < m$, then the system has infinite number of solutions if b can be written as linear combination of columns of A otherwise it is inconsistent.

SOLVED PROBLEMS

1. Let $x+y+z=0, \quad x-y-z=0$ then the number of solution of this system of equation is

(a) unique (b) infinitely many

(c) finitely many but greater than 2 (d) none of these

[(SCQ) CSIR-NET/JRF : June-2010]

Soln. We have

$x+y+z=0$ ...(i)

$x-y-z=0$ ...(ii)

Solving these we get $x=0$ and $y+z=0 \Rightarrow y=-z$ and $x=0$. Hence, there is infinitely many solution.

Correct option is (b).

2. Consider the system of equations $AX=0,\ BX=0$ where A and B are $n\times n$ matrices and X is a $n\times 1$ matrix. Which of the following statements are true. **[HCU-2010]**

(i) $\det(A)=\det(B)$ implies that the two systems have the same solutions

(ii) The two systems have the same solutions implies $\det(A)=\det(B)$

(iii) $\det(A)=0\neq\det(B)$ implies that the two systems can have different solutions

(a) All are true (b) (i) is true

(c) (iii) is true (d) (i) and (ii) are true

Soln.(i) Consider $A=\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}_{3\times3}$ and $B=\begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}_{3\times3}$

Clearly $\det(A)=\det(B)=0$

Now, $AX=0 \Rightarrow \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow\begin{bmatrix}x_1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\Rightarrow x_1=0$

$\Rightarrow$ Solution set of $AX=0$ is $\{(0,x_2,x_3)\mid x_2,x_3\in\mathbb{R}\}$

Now $BX=0 \Rightarrow \begin{bmatrix}0&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow\begin{bmatrix}0\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow x_2=x_3=0$

$\Rightarrow$ Solution set for $BX = 0$ is $\{(x_1, 0, 0) \mid x_1 \in \mathbb{R}\}$

Hence (i) is incorrect.

(ii) If $\det(A) = 3$ and $\det(B) = 4$.

$AX=0$ and $BX = 0$ has unique trivial solution. Hence solutions of the system $AX = 0$ and $BX = 0$ are the same, but $\det(A) \neq \det(B)$

Hence (ii) is incorrect.

(iii) If $\det(A) = 0$ and $\det(B) \neq 0$ then the system $AX = 0$ have more than one solution and the system $BX = 0$ always have unique trivial solution. Thus the two systems have different solutions.

Hence (iii) is correct

Correct option is (c)

3. Let A be a 4×4 real matrix. Which of the following 4 conditions is not equivalent to the other 3?

(a) The matrix A is invertible **[HCU-2011]**

(b) The system of equations $Ax = 0$ has only trivial solution

(c) Any two distinct rows u and v of A are linearly independent

(d) The system of equations $Ax = b$ has a unique solution $\forall b \in \mathbb{R}^4$

Soln. Consider $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

Clearly any two rows of A are linearly independent. But $\det(A) = 0$

$\Rightarrow A$ is not invertible

$\Rightarrow Ax = 0$ has more than one solution

Correct option is (c)

4. The system of equations $6x_1 - 2x_2 + 2\alpha x_3 = 1$ and $3x_1 - x_2 + x_3 = 5$ has no solution if α is equal to

(a) -5 (b) -1 (c) 1 (d) 5 **[HCU-2016]**

Soln. Consider $[A:b] = \begin{bmatrix} 6 & -2 & 2\alpha & : & 1 \\ 3 & -1 & 1 & : & 5 \end{bmatrix}$

Apply $R_2 \to R_2 - \frac{1}{2}R_1$, we have

$$[A:b] = \begin{bmatrix} 6 & -2 & 2\alpha & : & 1 \\ 0 & 0 & 1-\alpha & : & \frac{9}{2} \end{bmatrix}$$

The system has no solution if $1 - \alpha = 0 \Rightarrow \alpha = 1$

Correct option is (c)

5. Let A be a 5×5 real matrix. Suppose 0 is one of eigenvalues of A. Which of the following statement is true?

(a) System $AX = 0$ has unique solution (b) System $AX = C$ has unique solution for any C

(c) $AX = 0$ has a non-trivial solution (d) none of the above **[HCU-2018]**

Sol. If 0 is one of the eigen value of A

$\Rightarrow \det(A) = 0 \Rightarrow AX = 0$ has a non trivial solution

Correct option is (c)

6. The system of equations $2x-y+3z=9,\ x+y+z=6,\ x-y+z=2$ has **[B.H.U.-2012]**

(a) a unique non-zero solution (b) infinitely many solutions

(c) no solution (d) zero solution

Sol. Consider $[A:b]=\begin{bmatrix}1 & -1 & 1 & : & 2\\ 1 & 1 & 1 & : & 6\\ 2 & -1 & 3 & : & 9\end{bmatrix}$

Apply $R_2 \to R_2 - R_1; R_3 \to R_3 - 2R_1,$ we have

$$[A:b]=\begin{bmatrix}1 & -1 & 1 & : & 2\\ 0 & 2 & 0 & : & 4\\ 0 & 1 & 1 & : & 5\end{bmatrix}$$

$$R_2 \to \frac{1}{2}R_2$$

$$[A:b]=\begin{bmatrix}1 & -1 & 1 & : & 2\\ 0 & 1 & 0 & : & 2\\ 0 & 1 & 1 & : & 5\end{bmatrix}$$

Apply $R_1 \to R_1 + R_2$ *and* $R_3 \to R_3 - R_2,$ we have

$$[A:b]=\begin{bmatrix}1 & 0 & 1 & : & 4\\ 0 & 1 & 0 & : & 2\\ 0 & 0 & 1 & : & 3\end{bmatrix}$$

$\Rightarrow$ Rank (A) = Rank$([A:b])$ and also A is invertible.

$\Rightarrow$ The system has a unique non zero solution.

Correct option is (a)

7. Let $P=\begin{pmatrix}1 & -2 & 1\\ 2 & 1 & 1\\ 0 & 5 & -1\end{pmatrix}$ be a 3×3 matrix over $\mathbb{R}$. Then for a given vector $Y=\begin{pmatrix}y_1\\ y_2\\ y_3\end{pmatrix}\in\mathbb{R}^{3\times1}$, the vector space of all 3×1 matrices over $\mathbb{R}$, the system $PX=Y$ has a solution if **[CUCET-2016]**

(a) $y_1-y_2+y_3=0$ (b) $2y_1-y_2+y_3=0$

(c) $y_1+y_2-y_3=0$ (d) $2y_1+y_2-y_3=0$

Soln. Consider $[A:b]=\begin{bmatrix}1 & -2 & 1 & : & y_1\\ 2 & 1 & 1 & : & y_2\\ 0 & 5 & -1 & : & y_3\end{bmatrix}$

Apply, $R_2 \to R_2 - 2R_1$, we have

$$[A:b]=\begin{bmatrix}1 & -2 & 1 & : & y_1\\ 0 & 5 & -1 & : & y_2-2y_1\\ 0 & 5 & -1 & : & y_3\end{bmatrix}$$

Apply, $R_3 \to R_3 - R_2$, we have

$$[A:b]=\begin{bmatrix}1 & -2 & 1 & : & y_1\\ 0 & +5 & -1 & : & y_2-2y_1\\ 0 & 0 & 0 & : & y_3-y_2+2y_1\end{bmatrix}$$

Thus the system has a solution if $2y_1 - y_2 + y_3 = 0$

Correct option is (b)

8. Following system of linear equations **[CUCET-2017]**

$x+4y+3z=0$

$x+3y+4z=0$

$x+2y+5z=0$ does have

(a) no solution (b) infinitely many solutions

(c) more than one but finitely many solutions (d) exactly one solution.

Soln. Consider $A=\begin{bmatrix}1 & 4 & 3\\ 1 & 3 & 4\\ 1 & 2 & 5\end{bmatrix}$

Apply, $R_2 \to R_2 - R_1; R_3 \to R_3 - R_1$, we have $A=\begin{bmatrix}1 & 4 & 3\\ 0 & -1 & 1\\ 0 & -2 & 2\end{bmatrix}$

Apply, $R_3 \to R_3 - 2R_2$, we have $A=\begin{bmatrix}1 & 4 & 3\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix}$

$\Rightarrow$ Rank(A) = 2 < 3

$\Rightarrow$ The system has infinitely many solution.

Correct option is (b)

9. Consider the following system of linear equations **[HCU-2012]**

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{15}x_5 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{25}x_5 = b_2$$
$$\vdots \qquad \vdots$$
$$a_{81}x_1 + a_{82}x_2 + \dots + a_{85}x_5 = b_8$$

A vector $(\lambda_1, \lambda_2, \dots, \lambda_5) \in \mathbb{R}^5$ is said to be a solution of the system if $x_i = \lambda_i, i = 1, 2\dots, 5$ satisfies all the equations. Then

(a) If the system of equations has only finitely many solutions then it has exactly one solution

(b) If all the b_i's are zero then the set of solutions of the system is a subspace of $\mathbb{R}^5$.

(c) A system of 8 equations in 5 unknowns is always consistent

(d) If the system of equations has a unique solution then the rank of the matrix $[a_{ij}]$ must be 5.

Soln. (a) If the field is inifinite then we come across only three cases which are

(i) No solution (ii) Infinite solution (iii) Unique solution

Thus if the system of equations has only finitely many solutions. Then it has to be unique. Thus option (a) is correct.

(b) If the system $AX = b$, $A = [a_{ij}]_{m \times n}$ is homogeneous then the solution set is subspace of $F^{n.}$

$\Rightarrow$ option (b) is correct.

(c) Consider the system $AX = b$, where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{8\times 5}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Clearly Rank$(A) \neq$ Rank$([A:b])$

$\Rightarrow$ The system $AX = b$ has no solution

$\Rightarrow$ Option (c) is incorrect

(d) If the system $AX = b$, $A = [a_{ij}]_{m\times n}$ has unique solution then rank of A is always n.

$\Rightarrow$ Option is (d) is correct.

Correct option are (a), (b), (d)

10. Consider a linear system of equations $A\vec{x} = \vec{b}$ where A is a 3×3 matrix and $\vec{b} \neq 0$. Suppose the rank of the matrix of coefficients $A = (a_{ij})$ is equal to 2 then **[HCU-2015]**

(a) there definitely exists a solution of the system of equations

(b) there exists a non-zero column vector $\vec{v}$ in $\mathbb{R}^3$ such that $A\vec{v} = \vec{0}$

(c) if there exists a solution to the system of equations $A\vec{x} = \vec{b}$ then at least one equation is a linear combination of the other two equations

(d) $\det A = 0$

Soln. (a) Consider a system $A\vec{x} = \vec{b}$, where $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Clearly $\rho(A) \neq \rho([A:b])$

$\Rightarrow$ The system has no solution

$\Rightarrow$ option (a) is incorrect

(b) If $\rho(A) < 3$ then there always exist a non zero vector v such that $Av = 0$.

$\Rightarrow$ Option (b) is correct

(c) Option (c) is correct [By property of system of linear equation's]

(d) If $\rho(A) < 3$

$\Rightarrow$ $\det(A) = 0$

$\Rightarrow$ option (d) is correct

Correct option is (b), (c), (d)

11. Consider the system of simultaneous equations

$$\begin{aligned} 2x-2y-2z &= a_1 \\ -2x+2y-3z &= a_2 \\ 4x-4y+5z &= a_3 \end{aligned}$$

[NBHM-2007]

Write down the condition to be satisfied by a_1, a_2, a_3 for this system NOT to have a solution.

Soln. Consider $[A:b]=\begin{bmatrix} 2 & -2 & -2 & : & a_1 \\ -2 & 2 & -3 & : & a_2 \\ 4 & -4 & 5 & : & a_3 \end{bmatrix}$

Apply, $R_2 \to R_2+R_1,\ R_3 \to R_3-2R_1$, we have

$$[A:b]\sim\begin{bmatrix} 2 & -2 & -2 & : & a_1 \\ 0 & 0 & -5 & : & a_1+a_2 \\ 0 & 0 & 9 & : & a_3-2a_1 \end{bmatrix}$$

$R_3 \to \frac{9}{5}R_2+R_3$, We have

$$[A:b]\sim\begin{bmatrix} 2 & -2 & -2 & : & a_1 \\ 0 & 0 & -5 & : & a_1+a_2 \\ 0 & 0 & 0 & : & (a_3-2a_1)+\frac{9}{5}(a_1+a_2) \end{bmatrix}$$

The system has no solution if $(a_3-2a_1)+\frac{9}{5}(a_1+a_2)\neq 0$

$\Rightarrow\ 5a_3-10a_1+9a_2+9a_1\neq 0 \Rightarrow\ -a_1+9a_2+5a_3\neq 0$

12. If the system of equation $x-ky-z=0, kx-y-z=0$ and $x+y-z=0$ has a non zero solution, then the possible value of k are

(a) –1, 2 (b) 0, 1 (c) 1, 2 (d) –1, 1 **[IIT-JEE : 2011]**

Soln. Given that, system of equation is

$$\begin{aligned} x-ky-z&=0 \\ kx-y-z&=0 \\ x+y-z&=0 \end{aligned}$$

Matrix representation is $A=\begin{bmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$

Given system has non zero solution $\Rightarrow |A|=0$

$\Rightarrow\ 1(1+1)+k(-k+1)-1(k+1)=0$

$\Rightarrow\ 2-k^2+\cancel{k}-\cancel{k}-1=0 \Rightarrow\ -k^2+1=0$

$\Rightarrow\ k^2=1 \Rightarrow\ \boxed{k=\pm 1}$

Correct option is (d)

13. The equation true or false verify **[TIFR-2012]**

$$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = 1$$

$$x_1 + \frac{1}{4}x_2 + \frac{1}{9}x_3 = 1$$

$$x_1 + \frac{1}{8}x_2 + \frac{1}{27}x_3 = 1$$

has no solution

Soln. After solving these we get $\boxed{\rho(A) = \rho(A:B) = 3}$, hence unique solution

Statement is false.

14. The equation $x_1 + 2x_2 + 3x_3 = 1, \quad x_1 + 4x_2 + 9x_3 = 1, \quad x_1 + 8x_2 + 27x_3 = 1$ have **[TIFR-2010]**

(a) only one solution (b) two solution
(c) infinitely many solution (d) no solution

Soln. $\boxed{\rho(A) = \rho(A:B)}$ and also A is invertible

Correct option is (a)

15. Let A be a 5×4 matrix with real entries such that $A\underline{x} = \underline{0}$ iff $\underline{x} = \underline{0}$ where $\underline{x}$ is 4×1 vector and $\underline{0}$ is null vector. Then rank of A is **[(SCQ) CSIR-NET/JRF : Dec. 2013]**

(a) 4 (b) 5 (c) 2 (d) 1

Soln. $Ax = 0$ has trivial solution only If $\rho(A)$ = the number of columns = 4

Correct option is (a).

16. Let A be a 5×4 matrix with real enteries such that the space of all solution of the linear system. $Ax^t = [1, 2, 3, 4, 5]^t$ is given by $\left\{[1+2s, 2+3s, 3+4s, 4+5s]^t : s \in \mathbb{R}\right\}$. Here M^t denote the transpose of a matrix M) then the rank of A is equal to **[(SCQ) CSIR-NET/JRF : Dec-2011]**

(a) 4 (b) 3 (c) 2 (d) 1

Soln. Solution set of $Ax^t = [1\ 2\ 3\ 4\ 5]^t$ is given by

$$S = \left\{[1+2s, 2+3s, 3+4s, 4+5s]^t : s \in \mathbb{R}\right\} = \left\{(1, 2, 3, 4) + s(2, 3, 4, 5) : s \in \mathbb{R}\right\} \Rightarrow \dim S = 2$$

$\therefore$ The number of L.I. solution of the given non homogeneous system of equation is 2.

Hence, $n - r + 1 = 2$

$$\Rightarrow 4 - r + 1 = 2 \qquad \Rightarrow \cancel{-}r = \cancel{-}3 \qquad \Rightarrow \boxed{r = 3}$$

Correct option is (b).

17. $[A]_{m\times n}\ x = [b]_{n\times 1}$ be a system of equation then which of the following is true?

(a) If $m = n$ then the given system of equation has always unique solution **[(SCQ) CSIR-NET/JRF : Dec-2011]**
(b) If $m < n$ then the given system of equation has no solution
(c) If $m < n$ then the given system of equation has infinite or no solution
(d) If $m \le n$ then the given system of equation has no solution

Soln. We know that, if number of equation is less than number of unknowns then system of linear equation have either no or infinite solutions. As, rank $[A:B] <$ number of unknowns and if rank $[A:B]$ = rank $[A] \Rightarrow$ infinite solution. rank $[A:B] \ne$ rank $[A] \Rightarrow$ no solution.

Correct option is (c).

18. Tick true or false

A is 3×4 matix of rank 3. Then system of equations $Ax=b$ has exactly one solution. **[TIFR-2011]**

Soln. $n-r=4-3=1$ so it has infinite number of solution.

Statement is false.

UNSOLVED PROBLEMS

SIMULTANEOUS LINEAR SYSTEM

In all the following, you show consistency and inconsistency. If consistent find solution on it.

(1) $x+2y-3z=-4$
$2x+3y+2z=2$
$3x-3y-4z=1$

(2) $x+y+z=6$
$x+2z=7$
$3x+y+z=12$

(3) $5x-7y=2$
$7x-5y=3$

(4) $4x-2y=3$
$6x-3y=5$

(5) $x-2y+z=0$
$x+y-z=0$
$3x+6y-5z=0$

(6) $4x+3z=8$
$2x-z=2$
$3x+2y=5$

(7) $x+y+z=5$
$x+3y+3z=9$
$x+2y+\alpha z=\beta$

Find α,β so that it have infinite many solution.

(8) $x+y+z=3$
$x+2y+3z=4$
$x+4y+kz=6$

will not have unique solution for k equal to

(a) 0 (b) 5 (c) 6 (d) 7

(9) $x+2y+z=6$
$2x+y+2z=6$
$x+y+z=5$

(10) $2x+3y=4$
$x+y+z=4$
$x+2y-z=a$

for what value of a system has solution

(11) $4x+2y=7$
$2x+y=6$

(12) $-x+5y=-1$
$x-y=2$
$x+3y=3$

(13) $2x_1-x_2+3x_3=1$
$3x_1-2x_2+5x_3=2$
$-x_1-4x_2+x_3=3$

(14) $x_1+x_2+2x_3=1$
$x_1+2x_2+3x_3=2$
$x_1+4x_2+ax_3=4$

has unique solution. Find = ?