(iv) η is one only when $\frac{T_C}{T_H}$ is zero. i.e. when $T_C = 0K$ or $T_H = \infty K$. Both these temperature are impossible to obtain. Therefore, the efficiency of an engine can never be one or 100%. That is heat can't be transformed completely into work.

(v) For all reversible cycle operating between the same source and sink temperature, the efficiency is the same.

Comparison of efficiencies of reversible and irreversible engine.

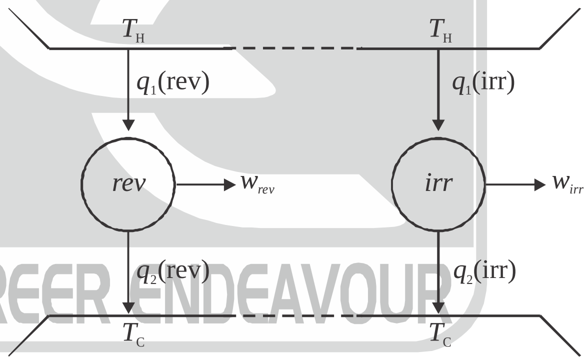
The efficiency of a reversible Carnot cycle is the theoretically possible maximum value which an engine can have. Since the various processes of this type of engine are to be carried out reversibly, therefore, such type of an engine does not have any realistic basis because reversible processes are idealized concepts which can never be realized. A real heat engine, which is irreversible in nature, will have efficiency smaller than the reversible heat engine.

Let us have two cycles, one operating reversibly and the other irreversibly. Let both of them operate between the same two temperature T_C and T_H and involve ideal gas as the working substance. These two cycles along with q values, are shown in the figure below.

(A) Isothermal expansion form volume V_1 to V_2. The expressions for the work involved are

$$-w_1(rev) = q_1(rev) = RT_H \ln\frac{V_2}{V_1}$$

And $$-w_1(irr) = q_1(irr) = p_{opp}(V_2 - V_1)$$



Since we know that $|w_1(rev)| > |w_1(irr)|$, therefore,

$$q_1(rev) > q_1(irr)$$

(B) Isothermal compression from volume V_3 to V_4. The expressions for the work involved are

$$-w_3(rev) = q_2(rev) = RT_C \ln\frac{V_4}{V_3}$$

$$-w_3(irr) = q_2(irr) = P'_{ext}(V_4 - V_3)$$

Now, since in the irreversible process, more work is done as compared to that in the reversible process, we have

$$w_3(irr) > w_3(rev)$$

It follows that

$$|q_2(irr)| > |q_2(rev)|$$

Now the efficiencies of the two cycles are

$$\eta(rev) = \frac{q_1(rev) + q_2(rev)}{q_1(rev)} = 1 - \frac{|q_2(rev)|}{q_1(rev)}$$

$$\eta(irr) = \frac{q_1(irr) + q_2(irr)}{q_1(irr)} = 1 - \frac{|q_2(irr)|}{q_1(irr)}$$

Now since $q_1(rev) > q_1(irr)$ and $|q_2(rev)| < |q_2(irr)|$, therefore, it follows that

$$\frac{|q_2(rev)|}{q_1(rev)} < \frac{|q_2(irr)|}{q_1(irr)} \text{ or } \left\{1 - \frac{|q_2(rev)|}{q_1(rev)}\right\} > \left\{1 - \frac{|q_2(irr)|}{q_1(irr)}\right\}$$

i.e. $\eta(rev) > \eta(irr)$

Basic Conclusion from Efficiency of a Carnot Cycle :

For a reversible Carnot cycle operating between two temperatures T_H and T_C, the efficiency is given as

$$\eta = \frac{q_1 + q_2}{q_1} = \frac{T_H - T_C}{T_H}$$

where q_1 and q_2 are the heats exchanged with the thermal reservoirs at temperatures T_H and T_C, respectively. Rewriting the above expression, we have

Or, $1 + \frac{q_2}{q_1} = 1 - \frac{T_C}{T_H}$ or $\frac{q_2}{q_1} = -\frac{T_C}{T_H}$

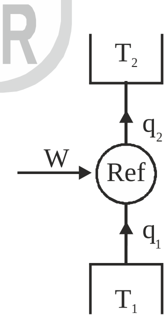
Or, $\frac{q_1}{T_H} + \frac{q_2}{T_C} = 0$

that is, the sum of the ratios of the heat involved and the corresponding temperature is zero for a Carnot cycle.

Carnot Refrigerator:

It is the reverse of Carnot engine i.e. the energy flow from low temperature body to a high temperature body by providing energy in the form of work to the system. It is the energy transfer device therefore, the ratio of its output to input is represented by coefficient of performance which can be greater than 1.

In case of Carnot refrigerator system absorbed heat from low temperature body and transfer it to the high temeprature body. In carnot engine heat is input work is output. In refrigerator heat is output and work is input.



Co-efficient of performance (β) of Carnot refrigerator:

It is defined as the ratio of heat transferred from a lower temperature to a higher temperature to the work done on the system, i.e. $\beta = \frac{|q_C|}{w}$

The lesser the work done the more efficient the operation and greater the coefficient of performance.

$$\beta = \frac{|q_C|}{|q_h| - |q_C|} = \frac{T_C}{T_H - T_C}$$

At $T_C \to 0K$, $\beta = 0$

$$\therefore \qquad w = \frac{|q_C|}{\beta} = \frac{(q_C)}{0} = \infty$$

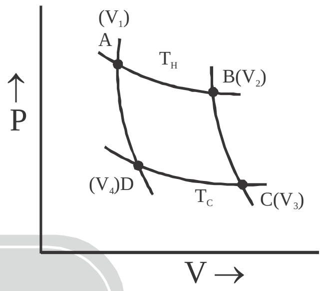
Thus as the temperature of a system is lowered the amount of work required to lower the temperature further increases rapidly and approaches infinity as the zero kelvin temperature is attained.

Efficiency of Carnot cycle $(\eta) = 1 - \frac{T_C}{T_H}$

For adiabatic curve $TV^{\gamma-1}$

$$\therefore \qquad T_H V_2^{(\gamma-1)} = T_C V_3^{(\gamma-1)}$$

$$\frac{T_C}{T_H} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \Rightarrow \frac{T_C}{T_H} = \frac{1}{\left(\frac{V_2}{V_3}\right)^{\gamma-1}} = \frac{1}{(\rho)^{\gamma-1}}$$



$$\therefore \qquad \eta = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1} \quad \left\{\text{where, } \rho = \frac{V_3}{V_2} = \frac{V_4}{V_1}\right\}$$

Relation between η and β :

$$\because \qquad \beta = \frac{T_C}{T_H - T_C} \qquad T_H > T_C$$

Again. $$\eta = \frac{T_H - T_C}{T_H} \Rightarrow \frac{1}{\eta} = \frac{T_H}{T_H - T_C} \Rightarrow \frac{1}{\eta} - 1 = \frac{T_H}{T_H - T_C} - 1 \Rightarrow \frac{1-\eta}{\eta} = \frac{T_H - T_H + 1}{T_H - T_C} = \frac{T_C}{T_H - T_C}$$

$$\beta = \frac{1-\eta}{\eta} \quad \text{or} \quad \eta = \frac{1}{\beta + 1}$$

Problem-1: A certain engine which operates in a Carnot cycle absorbs 4 kJ at 527ºC how much work is done on the engine per cycle and how much heat is evolved at 127ºC in each cycle?

Soln. The efficiency of the Carnot cycle is given by

$$\eta = \frac{T_H - T_C}{T_H} = \frac{q_1 + q_2}{q_1}$$

Thus, $-\frac{T_C}{T_H} = \frac{q_2}{q_1}$ and hence $q_2 = -\left(\frac{T_C}{T_H}\right) q_1$

Thus, the heat evolved in the present case is

$$q_2 = -\left(\frac{400K}{800K}\right)(4kJ) = -2kJ$$

and the work done on the engine is

$$w = -(q_1 + q_2) = -4 + 2 = -2kJ$$

The negative sign indicates that the work is actually done by the engine.

Problem-2: What % of T_2 should be T_1 for a 10% efficiency?

Soln. $\eta = 1 - \frac{T_C}{T_H} \Rightarrow \frac{10}{100} = 1 - \frac{T_C}{T_H}$

$\Rightarrow \quad \frac{T_C}{T_H} = 1 - \frac{10}{100} = \frac{90}{100} = 0.9$

$T_C = 0.9T_H$

$\Rightarrow \quad T_C\% = 90\% T_H$

Problem-3: Calculate the maximum efficiency of an engine operating between 110ºC and 25ºC.

Soln. Maximum efficiency of an engine working is

$$\eta = (T_H - T_C)/T_H = (383K - 298K)/383K = 0.222 = 22.2\%$$

Problem-4: Heat supplied to a Carnot engine is 1897.8 kJ. How much useful work can be done by the engine which operates between 0ºC and 100ºC?

Soln. $T_H = 100 + 273 = 373K; T_C = 0 + 273 = 273K; q_1 = 1897.8\ kJ$

$$\eta = \frac{T_H - T_C}{T_H} = \frac{373 - 273}{373} = 0.268$$

$$\eta = \frac{w}{q_1} \Rightarrow w = 0.268 \times 1897.8 = 508.7\text{ kJ}$$

Problem-5: A Carnot cycle operates on a temperature difference of 200K. One-third of the heat absorbed from the source at T_2 is discharged as waste heat to the sink at T_1. The cycle does 400J of work. Calculate q_1, q_2, T_1 and T_2.

Soln. Given, $q_1 = Q$

$q_2 = \frac{Q}{3}$

$w = -400J$ and $\Delta T = 200K$

$\because \quad \eta = 1 - \frac{|q_2|}{q_1} = 1 - \frac{Q}{3Q} \quad \Rightarrow \eta = \frac{2}{3}$

$\because \quad \eta = \frac{T_H - T_C}{T_H}$ or $\frac{2}{3} = \frac{200}{T_H}$ or $T_H = 300$

$\Delta T = T_H - T_C$

$200 = 300 - T_C$

$\therefore \quad T_C = 100$

Since, $\eta = \frac{|w|}{q_1} \Rightarrow \frac{2}{3} = \frac{400}{q_1} \Rightarrow q_1 = 600$

$\therefore \quad q_2 = \frac{200}{3} = 200\ J$

Problem-6: A Carnot engine operates between 600K and 800K and absorbs 2, 000 calories from the source per cycle. The work done (in Cal) per cycle is

(a) 1, 000 (b) 500 (c) 666 (d) 2, 000

Soln. Since, $\eta = \dfrac{T_H - T_C}{T_H}$

$$\eta = \frac{800-600}{800} = \frac{200}{800} = \frac{1}{4} \qquad \Rightarrow \eta = \frac{|w|}{q_1}$$

$$\frac{\eta}{4} = \frac{|w|}{2000} \text{ or } w = 500\ cal$$

Correct option is (b)

Problem-7: The coefficient of performance of a perfect refrigerator working reversibly between the temperatures T_c and T_h is given by

(a) $\dfrac{T_c - T_h}{T_c}$ (b) $\dfrac{T_h - T_c}{T_c}$ (c) $\dfrac{T_c}{T_h - T_c}$ (d) $\dfrac{T_h}{T_h - T_c}$

Soln. The ratio of heat transferred from a lower temperature to a higher temperature to the work done on the machine to cause this removal, i.e. $\beta = \dfrac{|q_C|}{w}$

The less the work done the more efficient the operation and greater the coefficient of performance.

$\because \quad w_{irr} > w_{rev}, \quad \therefore \quad \beta_{rev} > \beta_{irr}$

$$\beta = \frac{|q_C|}{|q_h| - |q_C|} = \frac{T_c}{T_h - T_c}$$

Correct option is (c)

Problem-8: In a carnot engine 200 cal heat is given to the sink by a reservoir at 27°C. If temperature of the source is 57°C, then find:

(i) How much heat flows from the source.

(ii) Efficiency of the engine

(iii) Work done by the reservoir.

Soln : q_2 = 200 cal, q_1 = ?

T_C = 27°C = (27 + 273) = 300K

T_H = 57°C = (57 + 273) = 330K; η = ? and w = ?

(i) $\dfrac{q_1}{T_H} = \dfrac{q_2}{T_C}$

$$\Rightarrow \frac{q_1}{330K} = \frac{200cal}{300K} \Rightarrow \frac{q_1}{11} = \frac{200}{10}\text{cal} \qquad \Rightarrow q_1 = 220 \text{ cal}$$

(ii) $\eta = 1 - \dfrac{q_2}{q_1} \Rightarrow \eta = 1 - \dfrac{200}{220} \Rightarrow \eta = 1 - \dfrac{10}{11} \Rightarrow \eta = \dfrac{1}{11} = 0.091$

(iii) $|w| = q_1 - q_2$ = (220 – 200)cal = 20cal

Problem-9: Temperature of the sink of a carnot engine is 7°C and efficiency is 50%. Calculate change in temperature of the source to increase the efficiency of the same engine 70%.

Soln. $\eta = 1 - \dfrac{T_{sink}}{T_{source}}$ ... (i)

Case-I: η = 0.5, T_{sink} = 7°C = 280K

from equation (i), $0.5 = 1 - \dfrac{280K}{T_{source}}$

$$\frac{280K}{T_{source}} = 1 - 0.5 = 0.5 \Rightarrow T_{source} = \frac{280}{0.5} = 560K$$

Case-II: $\eta = 0.7 = \frac{7}{10}$, $T_{sink} = 7°C = 280K$

from equation (i)

$$\Rightarrow \quad \frac{7}{10} = 1 - \frac{280K}{T_{source}} \quad \text{or} \quad \frac{280K}{T_{source}} = 1 - \frac{7}{10} = \frac{3}{10}$$

$$\Rightarrow \quad T_{source} = \frac{280K \times 10}{3 \times 1} \quad \text{or } T_{source} = 933.33K$$

Therefore, change in temperature of the source = (933.33 – 560)K = 373.33K

Problem-10: The efficiency of a carnot's cycle is 1/6. If on reducing the temperature of the sink by 75K, efficiency becomes $\frac{1}{3}$. Calculate the initial and final temperature between which the cycle is working.

Soln: We have, $\eta = 1 - \frac{T_C}{T_H}$ or $\frac{1}{6} = 1 - \frac{T_C}{T_H} \Rightarrow \frac{T_C}{T_H} = 1 - \frac{1}{6} \Rightarrow \frac{T_C}{T_H} = \frac{5}{6}$

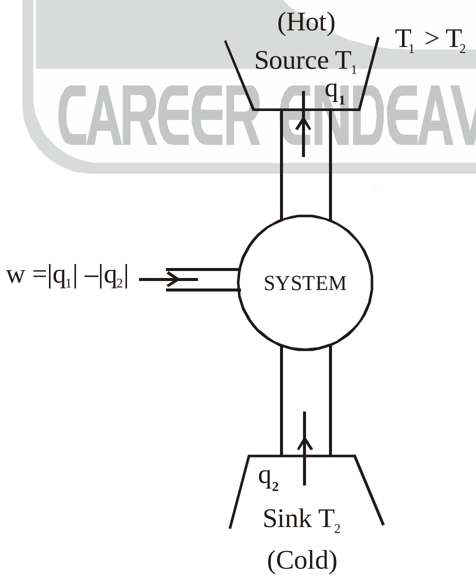
Now, $\eta = 1 - \frac{T_C - 75}{T_H} \Rightarrow \frac{1}{3} = 1 - \frac{T_C}{T_H} + \frac{75}{T_H} \Rightarrow \frac{1}{3} = 1 - \frac{5}{6} + \frac{75}{T_H}$ [Using equation (i)]

$$\Rightarrow \quad \frac{1}{3} = \frac{1}{6} + \frac{75}{T_H} \Rightarrow \frac{75}{T_H} = \frac{1}{3} - \frac{1}{6} \Rightarrow \frac{75}{T_H} = \frac{1}{6} \Rightarrow T_H = 450 \text{ k}$$

(ii) $\Rightarrow T_C = \frac{5}{6} \times 450 \text{ K} = 5 \times 75 \text{ K} = 375 \text{ K}$

Problem-11: A Carnot refrigerator takes heat from water at 0ºC and discard it to a room at 27ºC. 1 kg of water is to be changed into ice at 0°C. How many calories of heat is discarded to the room? Calculate the work done by the refrigerator and coefficient of performance. [Given: $\Delta H_{fussion} = 80 \text{ cal g}^{-1}$]

Soln:



Here $T_1 = 27°C = 300K$, $T_2 = 0°C = 273K$

For the process, $H_2O(\ell) \longrightarrow H_2O(s) \quad \Delta H = 80 \text{ cal g}^{-1}$

$$\therefore \quad Q_2 = 1 \times 10^3 \text{ g} \times 80 \frac{\text{Cal}}{\text{g}} \Rightarrow Q_2 = 8 \times 10^4 \text{ Cal}$$

We have, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow Q_1 = 8 \times 10^4 \text{ cal} \times \frac{300}{273}$

$\therefore$ Work done = $Q_1 - Q_2 = 8 \times 10^4 \times \frac{300}{273}$ Cal – 8×10^4 Cal = 7.9×10^3 cal

and $\beta = \frac{T_2}{T_1 - T_2} = \frac{273}{27} = 10.11$

Problem-12: A Carnot engine operates at 55% efficiency. If the temperature of reject steam is 105ºC, then the absolute temperature of input steam is ___________K.

Soln. Efficiency of carnot engine,

$$\eta = 1 - \frac{T_C}{T_H} \quad \text{... (1)}$$

Given, $\eta = 55\% = 0.55$

And $T_1 = 105 + 273 = 378K$

From equation (1),

$$0.55 = 1 - \frac{378}{T_H}K \quad \Rightarrow \; 0.55 - 1 = -\frac{378}{T_H}K$$

$$\Rightarrow \quad -0.45 = -\frac{378}{T_H} \Rightarrow T_H = \frac{378}{0.45} = 840K$$

Problem-13: A heat engine operates between 1000 K and 600 K. The heat discharged into the cold sink in a reversible process when 5 kJ of heat is supplied by the hot source, is

(a) 2 kJ (b) 2.5 kJ (c) 3 kJ (d) 5.5 kJ

Soln. $\frac{q_1}{q_2} = \frac{T_H}{T_C} \Rightarrow \frac{5kJ}{q_2} = \frac{1000\,K}{600\,K}$

$\Rightarrow \quad q_2 = 3kJ$

Correct option is (b)

Problem-14: Suppose the coldest reservoir we have at hand is at 10ºC. If we want a heat engine that is at least 90% efficient, the minimum temperature required for the hot reservoir is

(a) 1800K (b) 2880K

(c) 2800K (d) 2830K

Soln. $T_C = 10°C = 283K$, $\eta = 90\%$, $T_H = ?$

$$\eta = \frac{T_H - T_C}{T_H} \Rightarrow 0.9 = \frac{T_H - 283}{T_H} = 1 - \frac{283}{T_H} \Rightarrow T_H = 2830K$$

Correct option is (d)

Problem-15: A heat engine operates between 1000 K and 600 K. The heat discharged into the cold sink in a reversible process when 5 kJ of heat is supplied by the hot source, is

(a) 2 kJ (b) 2.5 kJ (c) 3 kJ (d) 5.5 kJ

Soln. $T_H = 1000K$, $T_1 = 600K$, $q_1 = 5kJ$, $q_2 = ?$

$$\because \frac{q_1}{q_2} = \frac{T_H}{T_C} \Rightarrow q_2 = 5 \times \frac{600}{1000} = 3\text{kJ}$$

Correct option is (c)

Problem-16: Liquid He boils at about –269ºC and liquid H_2 boils at about –253ºC. The efficiency of a reversible engine operating between heat reservoirs at these temperatures

(a) 20% (b) 80% (c) 10% (d) 90%

Soln. $T_C = -269°C = -269 + 273 = 4K$

$T_H = -253°C = -253 + 273 = 20K$

The efficiency of a reversible engine .

$$\eta = \frac{T_H - T_C}{T_H} = \frac{20-4}{20} = 80\%$$

Correct option is (b)

ENTROPY

For Carnot cycle,

$$\oint \frac{dq_{rev}}{T} = \frac{q_1}{T_h} + 0 + \frac{q_2}{T_c} + 0 = 0$$

$\Rightarrow$ $\frac{dq_{rev}}{T}$ is a state function which is called the change in entropy

$\therefore$ $\frac{dq_{rev}}{T} = dS$ or $\oint dS = 0$

The entropy is a function of the independent variables which are used to define the state of a system. It is an extensive function. The change in the value of the entropy in going from one state to another, is independent of the path.

The unit of entropy is JK^{-1} or cal K^{-1}.

$\oint \frac{dq(irr)}{T}$ for an irreversible cyclic process :

If there is any irreversibility at any stage of a cycle, the net work obtained $|w_{net}|$ in the cycle is less than the maximum work obtainable from the reversible cycle operating between the same two temperatures. Consequently, the efficiency of an irreversible cycle is always less than the efficiency of the corresponding reversible cycle. It follows that

$$\frac{|w_{total}|}{q_1} < \frac{T_h - T_c}{T_h} \quad \text{or} \quad \frac{q_1 + q_2}{q_1} < \frac{T_h - T_c}{T_h}$$

Or, $\frac{q_2}{q_1} < -\frac{T_c}{T_h}$ or $\frac{q_1}{T_h} + \frac{q_2}{T_c} < 0$

It shows that for irreversible cyclic process,

$$\oint \frac{dq(irr)}{T} = \frac{q_1}{T_h} + 0 + \frac{q_2}{T_c} + 0 < 0$$

Hence, $\oint \frac{dq(irr)}{T} < 0$

Spontaneous (irreversible) process and non-spontaneous (irreversible) process:

The process which proceed by themselves and bring the system closer to the equilibrium are called spontaneous process. All natural process are spontaneous.

The process which can't proceed by themselves are called non-spontaneous process. These require external force and brings the system away from equilibrium.

- In spontaneous processes entropy of universe is increasing i.e. unavailable energy goes on increasing and the workable energy is continuously decreasing. A spontaneous process comes to an end at equilibrium, and since at equilibrium the entropy is maximum, it can be remarked that nature is trying to attain the state of maximum entropy.
- Entropy is an extensive property.
- Entropy of reaction: $m_1A + m_2B \longrightarrow n_1C + n_2D$

$$\Delta S_{(\text{reaction})} = \sum^{\text{Product}} n\,S - \sum^{\text{Reactant}} m\,S$$

S = absolute entropies of respective substances.
n and m = moles of products and reactants respectively

$$\Delta S_{(\text{reaction})} = \left[n_1S_{(C)} + n_2S_{(D)}\right] - \left[m_1S_{(A)} + m_2S_{(B)}\right]$$

Entropy changes of universe for reversible process:

Let us consider an isothermal reversible process. In this the system absorbs heat q from surroundings at temperature T. This results in an increase in the entropy of system given by.

$$\Delta S_{\text{system}} = \frac{+q}{T}$$

$$\Delta S_{\text{surroundings}} = \frac{-q}{T}$$

Total entropy of the process = $\Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$

$$\Delta S_{\text{universe}} = \frac{q}{T} + \left(\frac{-q}{T}\right)$$

$\therefore \quad \Delta S_{\text{universe}} = 0$

Entropy change of universe for an irreversible processes:

Let q amount of heat is transferred from system which is at higher temperature (T_2) to the surroundings which is at lower temperature (T_1)

$\therefore$ Decrease in entropy of system $= \frac{-q}{T_2}$

Incease in entropy of surroundings $= \frac{+q}{T_1}$

$\therefore$ Total entropy change $= \Delta S_{\text{system}} + \Delta S_{\text{surrounding}}$

$$\Delta S_{\text{universel}} = \frac{-q}{T_2} + \frac{q}{T_1} = q\left[\frac{1}{T_1} - \frac{1}{T_2}\right] = q\left[\frac{T_2 - T_1}{T_1.T_2}\right]$$

$\because \quad T_2 - T_1 > 0$

$\therefore \quad \Delta S_{\text{universe}} > 0$