**Example:** Three charges $q, -q,$ and $2q$ are placed at $(a, a, a)$ $(a, -a, a)$ and $(a, -a, -a)$ respectively. Calculate net force on $2q$.

**Soln.** $2q$ will experience force due to two other charges.

Let $q_1 = q,\ q_2 = -q,\ q_3 = 2q$

$$\vec{r}_1 = a\left(\hat{i}+\hat{j}+\hat{k}\right),\ \vec{r}_2 = a\left(\hat{i}-\hat{j}+\hat{k}\right),\ \vec{r}_3 = a\left(\hat{i}-\hat{j}-\hat{k}\right)$$

Force on $2q$

$$\vec{F} = \vec{F}_{31} + \vec{F}_{32}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_1}{r_{31}^3}\vec{r}_{31} + \frac{q_3 q_2}{r_{32}^3}\vec{r}_{32}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{\left|\vec{r}_3-\vec{r}_1\right|^3}(\vec{r}_3-\vec{r}_1) - \frac{2q^2}{\left|\vec{r}_3-\vec{r}_2\right|^2}(\vec{r}_3-\vec{r}_2)\right] = \frac{q^2}{2\pi\varepsilon_0}\left[\frac{a(-2\hat{j}-2\hat{k})}{16\sqrt{2}a^3} - \frac{a(-2\hat{k})}{8a^3}\right]$$

$$= \frac{q^2}{16\pi\varepsilon_0 a^2}\left[-\frac{1}{\sqrt{2}}\left(\hat{j}+\hat{k}-2\sqrt{2}\hat{k}\right)\right]$$

$$= \frac{-q^2}{16\sqrt{2}\pi\varepsilon_0 a^2}\left[\hat{j}+(1-2\sqrt{2})\hat{k}\right] = \frac{q^2}{16\sqrt{2}\pi\varepsilon_0 a^2}\left[\left(2\sqrt{2}-1\right)\hat{k}-\hat{j}\right].$$

## Electric field:

If we keep a charge in the space, the region within which other charges feel force is called electric field. And the force that acts on the unit charge at a particular point is called electric field intensity $\left(\vec{E}\right)$ at that point.

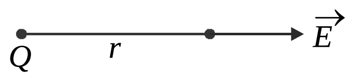
If $\vec{F}$ be force on charge $q$ due to some other charge then field at location of $q$ due to other charge is

$$\vec{E} = \frac{\vec{F}}{q}$$



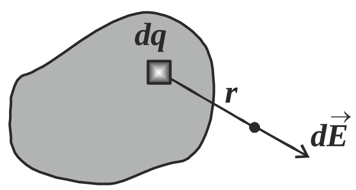
Field due to a point charge is given

$$\therefore \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$




If $\vec{r}_q$ be position vector of $q$ and $\vec{r}_p$ is position vector of a point $p$ where field is to be calculated, then

$$\vec{E}\left(\vec{r}=\vec{r}_p\right) = \frac{q\left(\vec{r}_p-\vec{r}_q\right)}{4\pi\epsilon_0\left|\vec{r}_p-\vec{r}_q\right|^3}$$

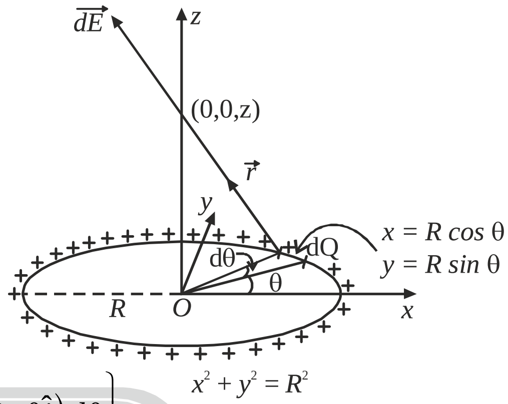



For continuous charge distribution,

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^3}\vec{r}$$

**Electric field due to uniformly charged ring at axial point:**

Let R be the radius of ring and Q be total charge on the ring. We are assuming that Q is uniformly distributed. Electric field at (0, 0, z) due to elementary portion shown in figure is




$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\lambda\, dl\, \vec{r}}{4\pi\varepsilon_0 r^3}$$

Here $\lambda = \frac{Q}{2\pi R}$, $dl = Rd\theta$

$$r = \sqrt{R^2 + z^2}, \quad \vec{r} = z\hat{k} - \vec{R} = z\hat{k} - R\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$$

$$\therefore \quad d\vec{E} = \frac{Q\left[z\hat{k} - R\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)\right]d\theta}{8\pi^2\varepsilon_0\left(R^2 + z^2\right)^{3/2}}$$

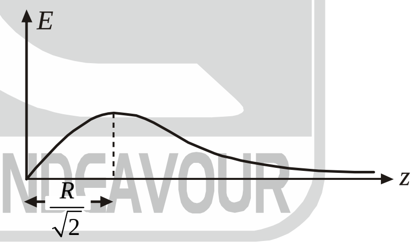
$$\therefore \quad \vec{E} = \int d\vec{E} = \frac{Q}{8\pi^2\varepsilon_0\left(R^2 + z^2\right)^{3/2}}\left\{z\hat{k}\int_0^{2\pi} d\theta - R\int_0^{2\pi}\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)d\theta\right\}$$

$$\therefore \quad \vec{E} = \frac{Q}{8\pi^2\varepsilon_0\left(R^2 + z^2\right)^{3/2}}\left\{z\hat{k}\cdot 2\pi - R(0)\right\}$$

or $\boxed{\vec{E} = \frac{Qz}{4\pi\varepsilon_0\left(R^2 + z^2\right)^{3/2}}\hat{k}}$ i.e. in axial direction

for $z << R$ (near center), $z^2 + R^2 \approx R^2$

$$\boxed{E = \frac{Qz}{4\pi\varepsilon_0 R^3}}$$




for $z >> R$ (for from center), $z^2 + R^2 \sim z^2$

$\boxed{E = \frac{Q}{4\pi\varepsilon_0 z^2}}$ (like a point charge)

electric field varies with axial distance as shown in figure

To find the position where field is extremum, use $\frac{dE}{dz} = 0$

or $\frac{d}{dz}\left[\frac{z}{\left(R^2 + z^2\right)^{3/2}}\right] = 0 \Rightarrow \frac{1.\left(R^2 + z^2\right)^{3/2} - z.\frac{3}{2}\left(R^2 + z^2\right)^{1/2} 2z}{\left(R^2 + z^2\right)^{3/2}} = 0$

$$\left(R^2 + z^2\right)^{1/2}\left[R^2 + z^2 - 3z^2\right] = 0$$

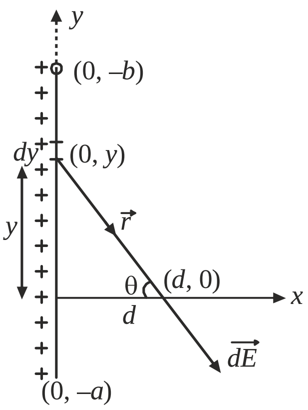
or $\left(R^2 + z^2\right)^{1/2}\left(R^2 - 2z^2\right) = 0$

$$\therefore \quad z = \pm\frac{R}{\sqrt{2}}$$

## Electric field due to straight wire :

Consider a uniformly charged wire of length $l$ and charge density $\lambda$.
Let us find electric field due to this wire at a distance '$d$' from it
Field due to elementary portion

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\lambda dy \vec{r}}{4\pi\varepsilon_0 r^3}$$




Here $\vec{r} = d\hat{i} - y\hat{j}$

$$r = \sqrt{d^2 + y^2}$$

$$\therefore \quad d\vec{E} = \frac{\lambda dy\left(d\hat{i} - y\hat{j}\right)}{4\pi\varepsilon_0\left(\sqrt{d^2 + y}\right)^3}$$

$$\therefore \quad \vec{E} = \int d\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\left\{d\hat{i}\int\frac{dy}{\left(d^2+y^2\right)^{3/2}} - \hat{j}\int y\frac{dy}{\left(d^2+y^2\right)^{3/2}}\right\}$$

Let the ends of wire are at $(0, -a)$ and $(0, b)$

$$\therefore \quad \vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\left\{d\hat{i}\int_{-a}^{b}\frac{dy}{\left(d^2+y^2\right)^{3/2}} - \hat{j}\int_{-a}^{b}\frac{y\,dy}{\left(d^2+y^2\right)^{3/2}}\right\}$$

To integrate put $y = d\tan\theta, dy = d\sec^2\theta\, d\theta$

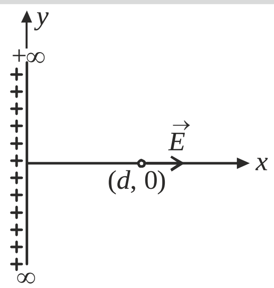
$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{\hat{i}}{d}[\sin\theta] + \frac{\hat{j}}{d}[\cos\theta]\right\} = \frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{\hat{i}}{d}\left[\frac{y}{\sqrt{y^2+d^2}}\right]_{-a}^{b} + \frac{\hat{j}}{d}\left[\frac{d}{\sqrt{y^2+d^2}}\right]_{-a}^{b}\right\}$$

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 d}\left[\hat{i}\left(\frac{b}{\sqrt{b^2+d^2}} + \frac{a}{\sqrt{a^2+d^2}}\right) + \hat{j}\left(\frac{d}{\sqrt{b^2+d^2}} - \frac{d}{\sqrt{a^2+d^2}}\right)\right]$$

**Special Cases.**

- Field near mid point of infinite wire

$a \to \infty, b \to +\infty$

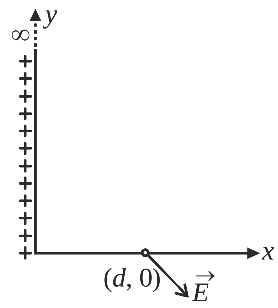



$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 d}\left\{\hat{i}\left(1+1\right) + \hat{j}\left(0-0\right)\right\}$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 d}\hat{i}$$

- Field near end of infinite wire

$a = 0,\; b \to \infty$ ,




$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 d}\left[\hat{i}\left(1+0\right) + \hat{j}\left(0-1\right)\right]$$

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 d}\left(\hat{i} - \hat{j}\right)$$

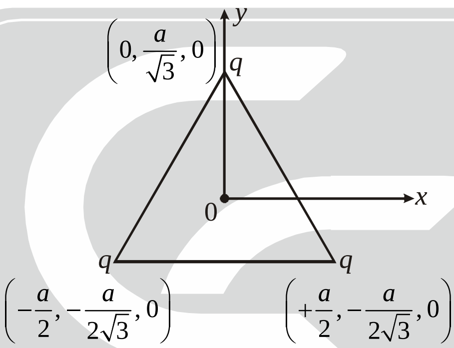
**Example :** Two charges $q$ and $-q$ are placed at $-(a,0,0)$ and $(a,0,0)$. Calculate electric field at point $(x, y, z)$.

**Soln.** Position vector of charges $q$ and $-q$ respectively are $\vec{r}_1 = -a\hat{i}$, $\vec{r}_2 = a\hat{i}$, position vector of space point where we have to find field is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Therefore, $\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3} + \frac{-q}{4\pi\varepsilon_0}\frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3}$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(x+a)\hat{i} + y\hat{j} + z\hat{k}}{\left((x+a)^2 + y^2 + z^2\right)^{3/2}} - \frac{(x-a)\hat{i} + y\hat{j} + z\hat{k}}{\left[(x-a)^2 + y^2 + z^2\right]^{3/2}}\right]$$

**Example:** Three point charges each equal to $q$ are placed at vertices of an equilateral triangle of side '$a$'. Calculate electric field at a distance $a\sqrt{2/3}$ above the centroide of the triangle.




**Soln.** Let us choose a coordinate system as shown in the figure.
Therefore, coordinates of charges become

$$\vec{r}_1 = \left(0, \frac{a}{\sqrt{3}}, 0\right),\ \vec{r}_2 = \left(\frac{a}{2}, \frac{-a}{2\sqrt{3}}, 0\right),\ \vec{r}_3 = \left(\frac{-a}{2}, \frac{-a}{2\sqrt{3}}, 0\right)$$

Coordinate of the point where, we have to find electric field is $\vec{r} = \left(0, 0, \sqrt{\frac{2}{3}}a\right)$

Therefore, $\vec{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{r-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3} + \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3} + \frac{q}{4\pi\varepsilon_0}\frac{\vec{r}-\vec{r}_3}{|\vec{r}-\vec{r}_3|^3}$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\sqrt{\frac{2}{3}}a\hat{k} - \frac{a}{\sqrt{3}}\hat{j}}{\left(\frac{2}{3}a^2 + \frac{1}{3}a^2\right)^{3/2}} + \frac{\sqrt{\frac{2}{3}}a\hat{k} - \frac{a}{2}\hat{i} + \frac{a}{2\sqrt{3}}\hat{j}}{\left(\frac{2}{3}a^2 + \frac{a^2}{4} + \frac{a^2}{12}\right)^{3/2}} + \frac{\sqrt{\frac{2}{3}}a\hat{k} + \frac{a}{2}\hat{i} + \frac{a}{2\sqrt{3}}\hat{j}}{\left(\frac{2}{3}a^2 + \frac{a^2}{4} + \frac{a^2}{12}\right)^{3/2}}\right]$$

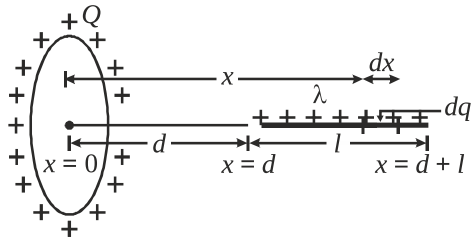
$$\vec{E} = \frac{q}{4\pi\varepsilon_0 a^3}\left[\sqrt{\frac{2}{3}}a\hat{k} - \frac{a}{\sqrt{3}}\hat{j} + \sqrt{\frac{2}{3}}a\hat{k} - \frac{a}{2}\hat{i} + \frac{a}{2\sqrt{3}}\hat{j} + \sqrt{\frac{2}{3}}a\hat{k} + \frac{a}{2}\hat{i} + \frac{a}{2\sqrt{3}}\hat{j}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 a^3}\left[3\sqrt{\frac{2}{3}}a\hat{k}\right] \Rightarrow \vec{E} = \frac{\sqrt{6}q}{4\pi\varepsilon_0 a^2}\hat{k}.$$

**Exaxmple:** A thin wire of length $l$ and uniform charge density $\lambda$ is placed along the axis of a ring of radius $R$ and uniform total charge $q$. If distance between the nearer end of wire and center of ring is '$d$'. Calculate force on the wire.

**Soln.** The two objects are not point charges, therefore we cannot get force directly from coulomb's law. If $\vec{E}$ be field of ring near elementary charge $dq$ of wire then force on wire is

$$\vec{F} = \int \vec{E}\, dq$$




$$= \int_d^{d+l} \frac{Qx\hat{i}}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}} \cdot \lambda\, dx$$

$$= \frac{Q\lambda\hat{i}}{4\pi\varepsilon_0} \int_d^{d+l} \frac{xdx}{(R^2+x^2)^{3/2}}$$

Substitute, $t^2 = R^2 + x^2 \Rightarrow tdt = xdx$

$$\int \frac{xdx}{(R^2+x^2)^{3/2}} = \frac{tdt}{t^3} = \left[-\frac{1}{t}\right]$$

$$\therefore \qquad \vec{F} = \frac{Q\lambda\hat{i}}{4\pi\varepsilon_0 R^2}\left[-\frac{1}{\sqrt{R^2+x^2}}\right]_d^{d+l}$$

$$\vec{F} = \frac{Q\lambda\hat{i}}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{R^2+d^2}} - \frac{1}{\sqrt{R^2+(d+l)^2}}\right].$$

**Example:** A thin conducting ring of radius $r$ has an electric charge $+Q$. What would be the increase in the tension of the wire, if a point charge $+q$ is placed at the centre of the ring ?

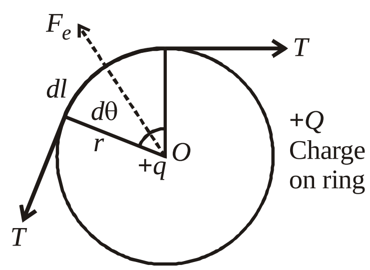
**Soln.** The situation is shown in figure. Charge on a small element $dl$ of the ring $dQ = \frac{Q}{2\pi r} dl$, outward electric force on this element $F_e = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q\,dl}{2\pi r}\right)\left(\frac{q}{r^2}\right)$.

Let the tension be increased by $T$, to balance this force $F_e$. The increase in tension is given by

$$= 2T\sin\left(\tfrac{d\theta}{2}\right)$$

$$\cong 2T\left(\tfrac{d\theta}{2}\right)$$

$$\cong T\left(\tfrac{dl}{r}\right) \qquad (\because\ dl = r\,d\theta)$$




Hence, $T\frac{dl}{r} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q\,dl}{2\pi r}\right)\left(\frac{q}{r^2}\right)$

Therefore, $T = \frac{Qq}{8\pi^2\varepsilon_0 r^2}$.

## Electric line of force:

An electric line of force is an imaginary curve drawn in such a way that tangent to this curve at any point gives the direction of electric field at the point.

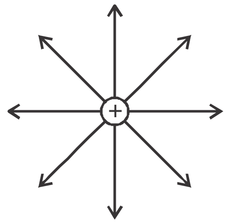


**Figure (1)**

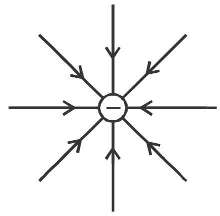


**Figure (2)**

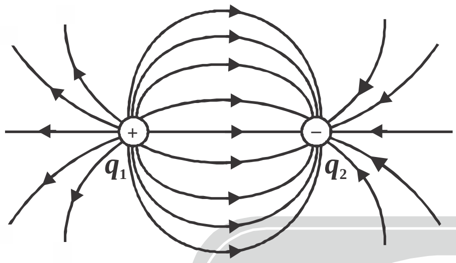



**Figure (3)**

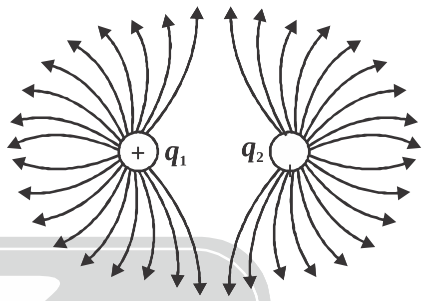



**Figure (4)**

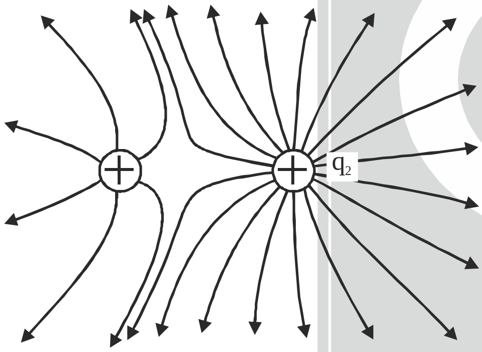



Figure (5)

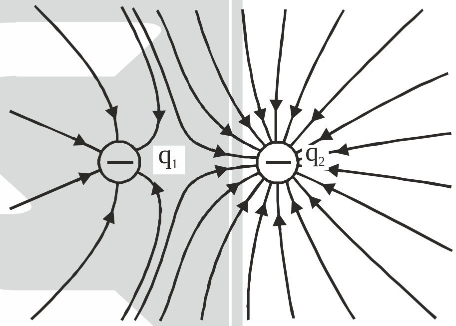



Figure (6)



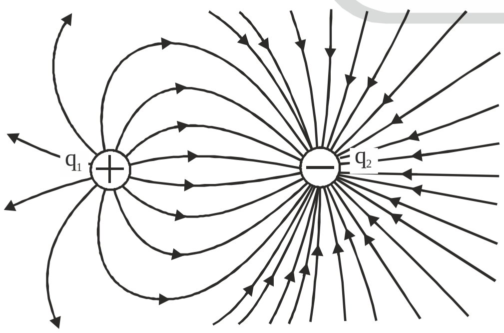



Figure (7)

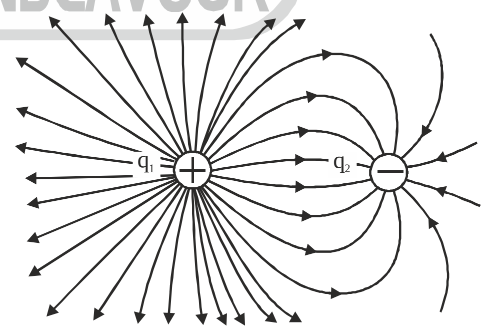



Figure (8)

(1) For single positive charge

(2) For single negative charge

(3) $q_1 = q_2$ and $q_1$ positive and $q_2$ negative.

(4) $q_1 = q_2$ and $q_1$ = positive, $q_2$ = positive

(5) $q_1 < q_2$ and both is positive

(6) $q_1 < q_2$ and both is negative

(7) $q_1 < q_2$ and $q_1$= positive, $q_2$ = negative

(8) $q_1 > q_2$ and $q_1$ is positive and $q_2$ is negative.