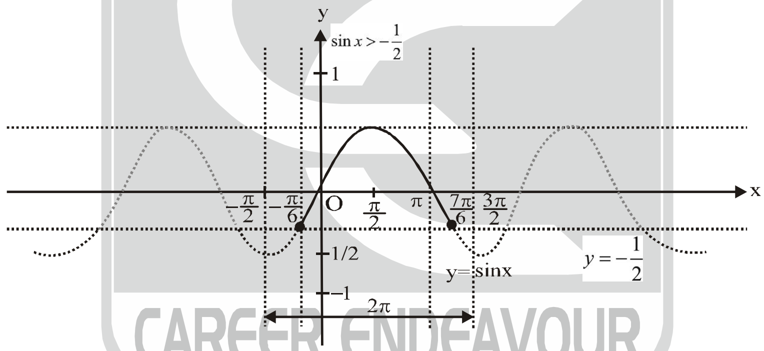
	Function	Domain	Range
1.	$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
2.	$\cos^{-1} x$	$[-1,1]$	$[0, \pi]$
3.	$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
4.	$\operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] = \{0\}$
5.	$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
6.	$\cot^{-1} x$	R	$(0, \pi)$

1.3 Trigonomertic Inequalities:

Example: Solve the inequality; $\sin x > -\frac{1}{2}$

Solution: $y = \sin x$ and $y = -\frac{1}{2}$



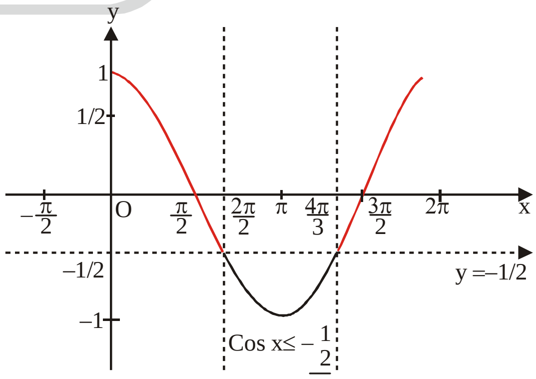
Thus on generalising above solution:

$$2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{7\pi}{6}; n \in \mathbb{Z}$$

Example: Solve the inequality $\cos x \le -\frac{1}{2}$

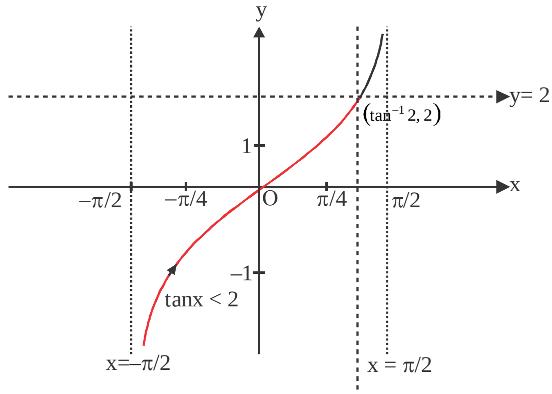
Solution: $\therefore$ solution of $\cos x \le -\frac{1}{2}$

$$\Rightarrow \quad x \in \left[2n\pi + \frac{2\pi}{3}, 2n\pi + \frac{4\pi}{3}\right]; n \in \mathbb{Z}$$



Example: Solve the inequality : tan x < 2

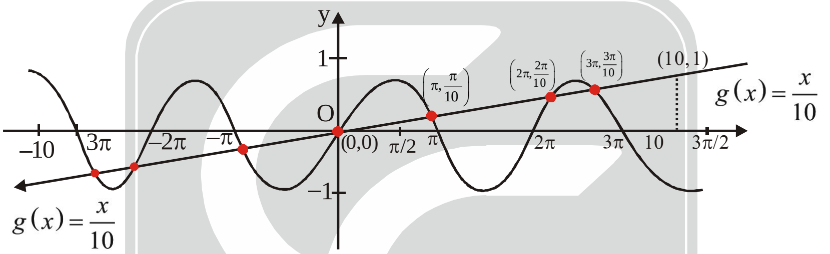
Solution: $x \in \left(2n\pi - \frac{\pi}{2}, 2n\pi + \text{arc}\tan 2\right), n \in \mathbb{Z}$



14. Solving Equations Graphically

Example : Find the number of solution of $\sin x = \dfrac{x}{10}$

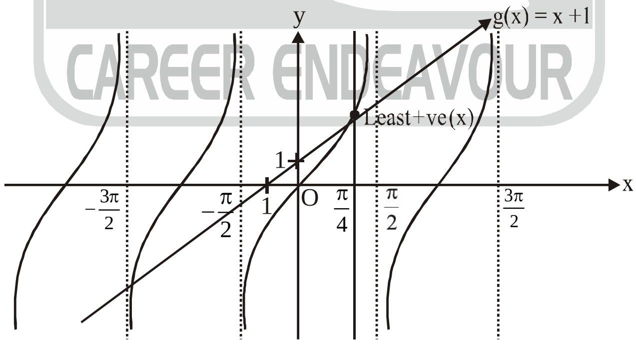
Solution: Here, let $f(x) = \sin x$ and $g(x) = \dfrac{x}{10}$



From above figure $f(x) = \sin x$ and $g(x) = \dfrac{x}{10}$ intersect at 7 points So, the number of solutions are 7.

Example: Find the least positive value of x, satisfying $\tan x = x + 1$ lies in the interval.

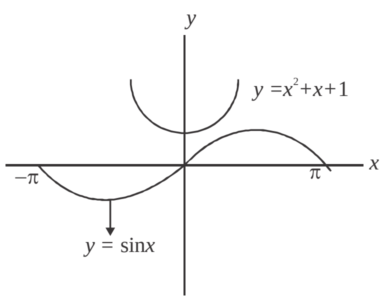
Solution: Let; $f(x) = \tan x$ and $g(x) = x + 1$;



From the above figure $\tan x = x + 1$ has infinity many solutions but the least positive value of $x \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

Example: Find the number of solutions of the equation,

$\sin x = x^2 + x + 1$

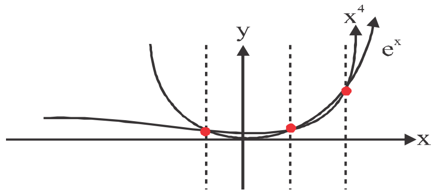


Solution: Let $f(x) = \sin x$ and $g(x) = x^2 + x + 1 = \left(x + \dfrac{1}{2}\right)^2 + \dfrac{3}{4}$

Which do not intersect at any point, therefore no solution.

Example: Find the number of solutions of $e^x = x^4$

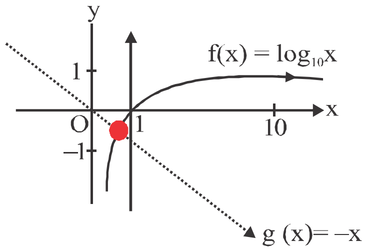
Solution: Let ; $f(x) = e^x$ and $g(x) = x^4$, which could be shown as



$\therefore$ We get 3 intersections.

Example: Find the number of solutions of ; $log_{10}x + x = 0$

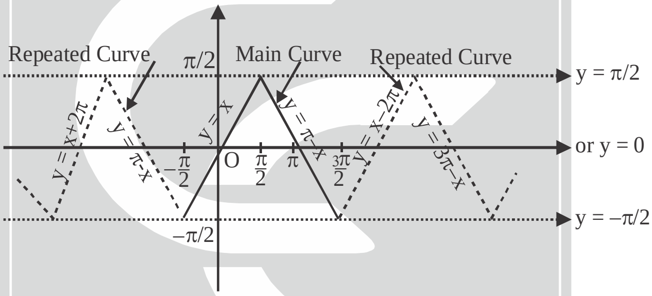
Solution: Let ; $f(x) = log_{10}\ x$ and $g(x) = -x$; which could be shown as;



From above figure, it is clear they intersect at one points, therefore 1 solution.

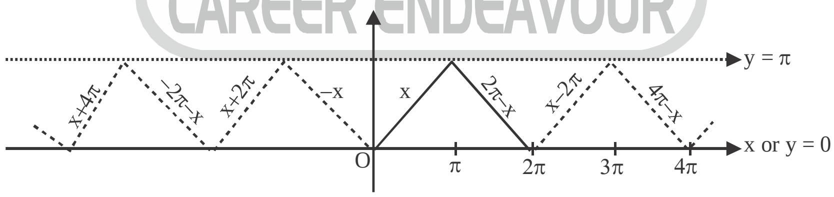
Example: Sketch the graph for $y = \sin^{-1}(\sin x)$.

Solution: As y = sin x is periodic with period 2π.



Example: Sketch the graph for $y = \cos^{-1}(\cos x)$

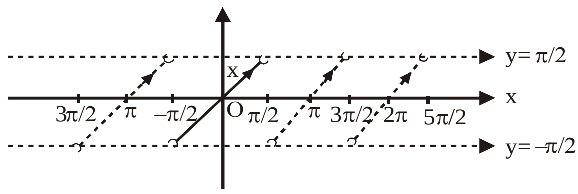
Solution: As $y = \cos^{-1}$ (cos x) is periodic with period 2π.



Thus, the curve $y = \cos^{-1}$ (cos x).

Example: Sketch the graph for $y = \tan^{-1}(\tan x)$

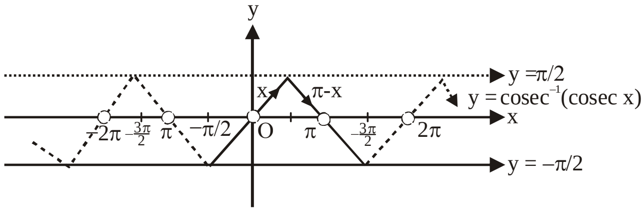
Solution: As $y = \tan^{-1}$ (tan x) is periodic with period π.



Thus, the curve for $y = \tan^{-1}(\tan x)$, where y is not defined for $x=(2n+1)\dfrac{\pi}{2}, n\in\mathbb{Z}$

Example: Sketch the graph for $y = \text{cosec}^{-1}(\text{cosec}\ x)$

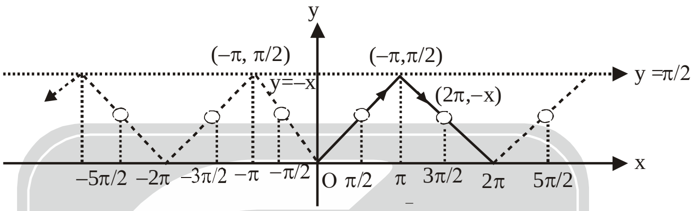
Solution: As y = cosec^{-1} (cosec x) is periodic with period 2π.



Thus, it has been defined for $\left[2n\pi-\frac{\pi}{2},2n\pi+\frac{3\pi}{2}\right]-\left\{2n\pi,(2n+1)\pi\right\}, n\in\mathbb{Z}$

Example: Sketch the graph for $y=\sec^{-1}(\sec x)$ has been defined for $x\neq(2n+1)\frac{\pi}{2}, n\in\mathbb{Z}$

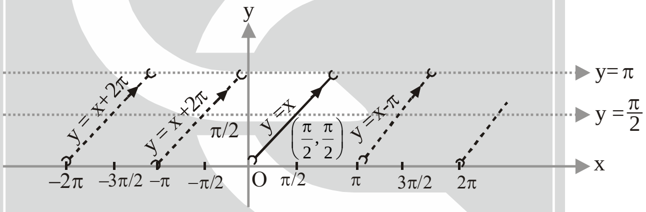
Solution: As y = sec^{-1} (sec x) is periodic with period 2π.



Thus, the curve for $y=\sec^{-1}(\sec x)$ has defined for $x\neq n\pi, n\in\mathbb{Z}$

Example: Sketch the graph for $y=\cot^{-1}(\cot x)$

Solution: As y = cot^{-1} (cot x) is periodic with period π.



Thus, the curve for $y=\cot^{-1}(\cot x)$.

Example: Sketch the graph for

(i) $\dfrac{|\sin x|}{\sin x}$ (ii) $\dfrac{|\cos x|}{\cos x}$ (iii) $\sin^{-1}\left(\dfrac{1+x^2}{2x}\right)$

(iv) $\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4\left(16x^2-8x+1\right)$

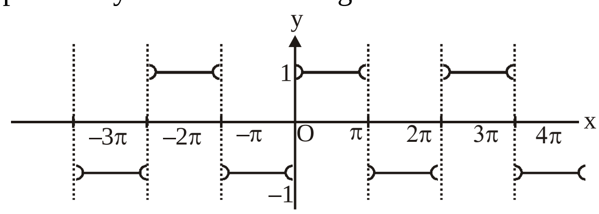
(v) $1+3\left(\log|\sin x|+\log|\text{cosec } x|\right)$ (vi) $1+3\left(\log\sin x+\log\text{cosec } x\right)$

Solution: As we known, to plot above curves we must check periodicity of domain and range:

(i) $=\dfrac{|\sin x|}{\sin x}$

Here, $y=\begin{cases}1; \sin x>0\\ -1; \sin x<0\end{cases}$

$y=\begin{cases}1; & 2n\pi<x<(2x+1)\pi; n\in z\\ -1; & (2n+1)\pi<x<(2n+2)\pi; n\in z\end{cases}$



$y=\dfrac{|\sin x|}{\sin x}$

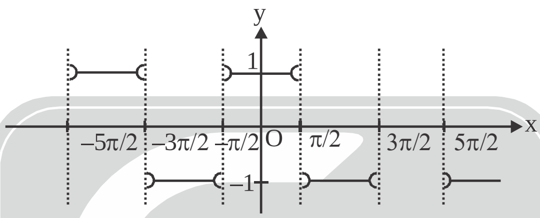
domain $= R - \{n\pi; n \in z\}$

$$\text{Range} = \begin{cases} 1; & 2n\pi < x < (2x+1)\pi \\ -1; & (2n+1)\pi < x < (2n+2)\pi \end{cases}$$

(ii) Sketch for $y = \dfrac{|\cos x|}{\cos x}$

$$y = \begin{cases} 1; & \cos x > 0 \\ -1; & \cos x < 0 \end{cases} \Rightarrow \quad y = \begin{cases} 1; & 2n\pi - \dfrac{\pi}{2} < x < 2n\pi \dfrac{\pi}{2} \\ -1; & 2n\pi + \dfrac{\pi}{2} < x < 2n\pi + \dfrac{3\pi}{2} \end{cases}$$

So, it could be plotted as:



$y = \dfrac{|\cos x|}{\cos x}$

(iii) sketch for $y = \sin^{-1}\left(\dfrac{1+x^2}{2x}\right)$

Here $y = \sin^{-1}\left(\dfrac{1+x^2}{2x}\right)$ is defined:

When $\left|\dfrac{1+x^2}{2x}\right| \le 1$ {as; $\sin^{-1} x$ is defined when $|x| \le 1$}

$\Rightarrow \quad 1 + x^2 \le 2|x|$ {as; $1 + x^2 > 0$}

$\Rightarrow \quad x^2 - 2|x| + 1 \le 0$ {as; $x^2 + |x|^2$}

$\Rightarrow \quad |x|^2 - 2|x| + 1 \le 0$

$\Rightarrow \quad (|x| - 1)^2 \le 0$

$\Rightarrow \quad (|x| - 1)^2 = 0$ {as $(|x| - 1)^2 < 0$ is not possible}

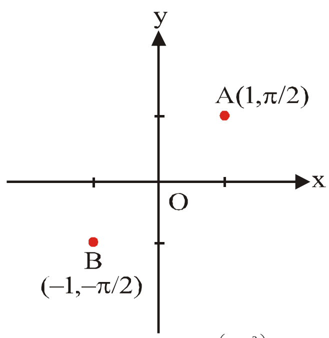
$\Rightarrow \quad x = \pm 1$

$\therefore$ Domain $= \{\pm 1\}$

$\therefore$ for range $y = \sin^{-1}\left(\dfrac{1+x^2}{2x}\right)$, where $x = +1, -1$

$y = \sin^{-1}(1)$ and $y = \sin^{-1}(-1)$

$\Rightarrow y = \pm\dfrac{\pi}{2}$



Graph for $\sin^{-1}\left(\frac{1+x^2}{2x}\right)$

Range $=\left\{\pm\frac{\pi}{2}\right\}$

Hence, the graph for $y=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is only two points , shown as:

Thus, the sketch for $y=\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is only two points A and B

(iv) **Skech for** $y=\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4\left(16x^2-8x+1\right)$

Here, $y=\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4(4x-1)^2$

$\Rightarrow y=\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4 16+\frac{1}{2}\log_4\left(x-\frac{1}{4}\right)^2$

or $y=-\log_4\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4 4^2+\frac{2}{2}\log_4\left(x-\frac{1}{4}\right)$ $\left\{as;\log_{b^n}a^m=\frac{m}{n}\log_b a\right\}$

$\Rightarrow y=-\log_4\left(x-\frac{1}{4}\right)+\log_4\left(x-\frac{1}{4}\right)+\frac{2}{2}\log_4 4$

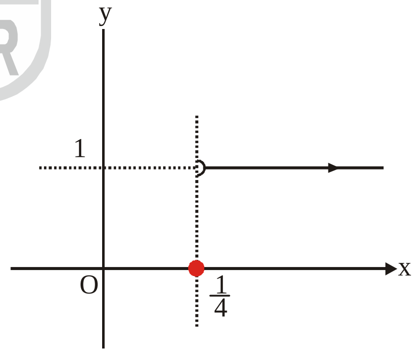
$\Rightarrow$ y = 1, wherever; $\left(x-\frac{1}{4}\right)>0$ {as; $\log_a x$ exists only when $a, x>0$ and $a\neq 1$}

Thus, $y=-\log_{1/4}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4+(4x-1)^2$

$\Rightarrow$ Domain $=\left(\frac{1}{4},\infty\right)$

Range $=\{1\}$

Thus, the graph is shown as:



(v) **Sketch for** $y = 1+3$ **(log |sin** x**| + log|cosec** x**|)**

Here $y=1+3\left[\log\left(|\sin x|.|\operatorname{cosec} x|\right)\right]$

When ever $|\sin x|\neq 0$ and $|\operatorname{cosec}x|\neq 0$

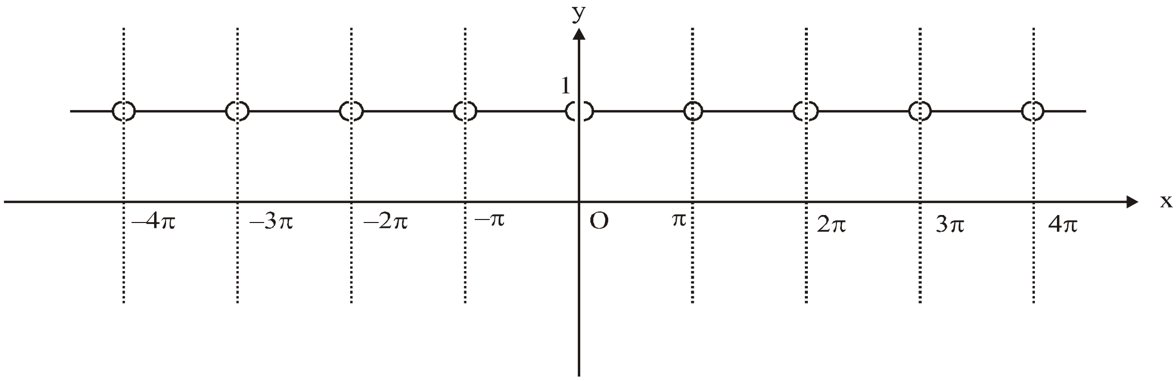
i.e., $y=1+3(\log 1)$; whenever $x\notin n\pi; n\in z$

$\Rightarrow$ y = 1 { as; log 1 = 0}

$\therefore$ **Domain = R – { nπ; n∈ℤ)**

Range = {1}

$\therefore$ it could plotted as:



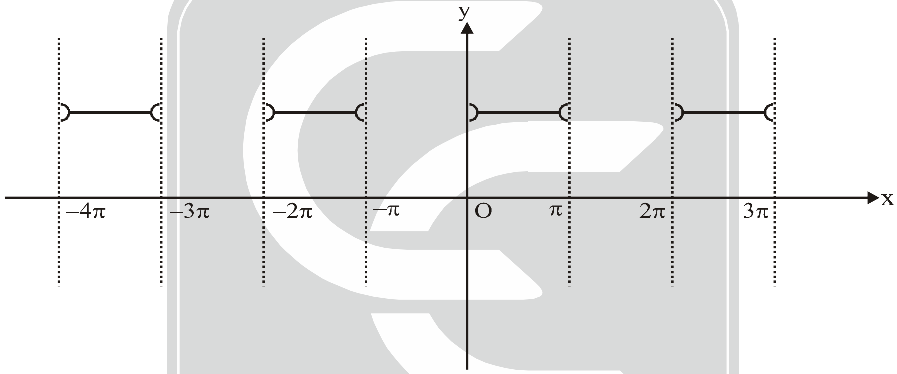
(vi) Sketch for $y = 1 + 3(\log \sin x + \log \text{cosec}\, x)$

Here $y = 1+3\,(\log \sin x.\ \text{cosec} x)$ whenever sin > 0 and cosec x > 0

$\Rightarrow$ $y = 1+ 3\log 1$; $\quad x \in (2n\pi, (2n+1)\pi)$, $n \in \mathbb{Z}$

or $y = 1$; whenever $x \in (2n\pi, (2n+1)\pi)$

$\therefore$ $y = 1+3(\log \sin x + \log \text{cos}\,ec x) = 1$ whenever $x \in (2n\pi, (2n+1)\pi), m \in \mathbb{Z}$



Example: Find the number of solution for ; $[x] = \{x\}$. where $[\bullet]$ $\{\bullet\}$ represents greatest integer and factional part of x.

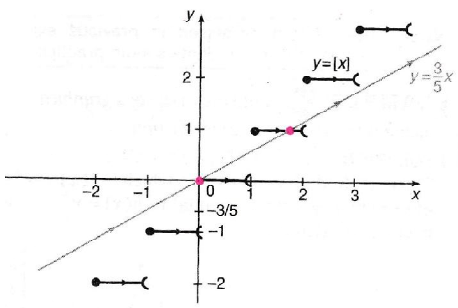
Solution: As $[x] \in \mathbb{Z}$ and $\{x\} \in \mathbb{Z}$ only if $\{x\} = 0$

$\therefore$ $[x] = \{x\} = 0 \Rightarrow x = [x] + \{x\} = 0$

$\therefore$ only one solution

Example: Find the number of solutions of $4\{x\} = x + [x]$
where {.}. [.] represents fractional part and greatest integer function.

Solution: As we know, to find number of solutions of two curves we should find the point of intersection of two curves.



$\therefore$ $4\{x\} = x + [x]$

$\Rightarrow$ $4(x-[x] = x + [x]$