$$\eta(rev) = \frac{q_1(rev) + q_2(rev)}{q_1(rev)} = 1 - \frac{|q_2(rev)|}{q_1(rev)}$$

$$\eta(irr) = \frac{q_1(irr) + q_2(irr)}{q_1(irr)} = 1 - \frac{|q_2(irr)|}{q_1(irr)}$$

Now since $q_1(rev) > q_1(irr)$ and $|q_2(rev)| < |q_2(irr)|$, therefore, it follows that

$$\frac{|q_2(rev)|}{q_1(rev)} < \frac{|q_2(irr)|}{q_1(irr)} \text{ or } \left\{1 - \frac{|q_2(rev)|}{q_1(rev)}\right\} > \left\{1 - \frac{|q_2(irr)|}{q_1(irr)}\right\}$$

i.e. $\eta(rev) > \eta(irr)$

Basic Conclusion from Efficiency of a Carnot Cycle :

For a reversible Carnot cycle operating between two temperatures T_H and T_C, the efficiency is given as

$$\eta = \frac{q_1 + q_2}{q_1} = \frac{T_H - T_C}{T_H}$$

where q_1 and q_2 are the heats exchanged with the thermal reservoirs at temperatures T_H and T_C, respectively. Rewriting the above expression, we have

Or, $1 + \frac{q_2}{q_1} = 1 - \frac{T_C}{T_H}$ or $\frac{q_2}{q_1} = -\frac{T_C}{T_H}$

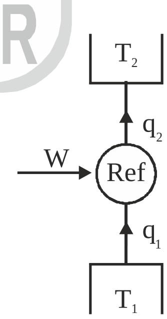
Or, $\frac{q_1}{T_H} + \frac{q_2}{T_C} = 0$

that is, the sum of the ratios of the heat involved and the corresponding temperature is zero for a Carnot cycle.

Carnot Refrigerator:

It is the reverse of Carnot engine i.e. the energy flow from low temperature body to a high temperature body by providing energy in the form of work to the system. It is the energy transfer device therefore, the ratio of its output to input is represented by coefficient of performance which can be greater than 1.

In case of Carnot refrigerator system absorbed heat from low temperature body and transfer it to the high temeprature body. In carnot engine heat is input work is output. In refrigerator heat is output and work is input.



Co-efficient of performance (β) of Carnot refrigerator:

It is defined as the ratio of heat transferred from a lower temperature to a higher temperature to the work done on the system, i.e. $\beta = \frac{|q_C|}{w}$

The lesser the work done the more efficient the operation and greater the coefficient of performance.

$$\beta = \frac{|q_C|}{|q_h| - |q_C|} = \frac{T_C}{T_H - T_C}$$

At $T_C \to 0K$, $\beta = 0$

$$\therefore \quad w = \frac{|q_C|}{\beta} = \frac{(q_C)}{0} = \infty$$

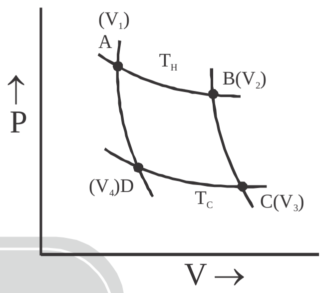
Thus as the temperature of a system is lowered the amount of work required to lower the temperature further increases rapidly and approaches infinity as the zero kelvin temperature is attained.

Efficiency of Carnot cycle $(\eta) = 1 - \frac{T_C}{T_H}$

For adiabatic curve $TV^{\gamma-1}$

$$\therefore \quad T_H V_2^{(\gamma-1)} = T_C V_3^{(\gamma-1)}$$

$$\frac{T_C}{T_H} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \Rightarrow \frac{T_C}{T_H} = \frac{1}{\left(\frac{V_2}{V_3}\right)^{\gamma-1}} = \frac{1}{(\rho)^{\gamma-1}}$$



$$\therefore \quad \eta = 1 - \left(\frac{1}{\rho}\right)^{\gamma-1} \quad \left\{\text{where, } \rho = \frac{V_3}{V_2} = \frac{V_4}{V_1}\right\}$$

Relation between η and β :

$$\because \quad \beta = \frac{T_C}{T_H - T_C} \qquad T_H > T_C$$

Again. $$\eta = \frac{T_H - T_C}{T_H} \Rightarrow \frac{1}{\eta} = \frac{T_H}{T_H - T_C} \Rightarrow \frac{1}{\eta} - 1 = \frac{T_H}{T_H - T_C} - 1 \Rightarrow \frac{1-\eta}{\eta} = \frac{T_H - T_H + 1}{T_H - T_C} = \frac{T_C}{T_H - T_C}$$

$$\beta = \frac{1-\eta}{\eta} \quad \text{or} \quad \eta = \frac{1}{\beta + 1}$$

Problem-1: A certain engine which operates in a Carnot cycle absorbs 4 kJ at 527ºC how much work is done on the engine per cycle and how much heat is evolved at 127ºC in each cycle?

Soln. The efficiency of the Carnot cycle is given by

$$\eta = \frac{T_H - T_C}{T_H} = \frac{q_1 + q_2}{q_1}$$

Thus, $$-\frac{T_C}{T_H} = \frac{q_2}{q_1} \text{ and hence } q_2 = -\left(\frac{T_C}{T_H}\right) q_1$$

Thus, the heat evolved in the present case is

$$q_2 = -\left(\frac{400K}{800K}\right)(4kJ) = -2kJ$$

and the work done on the engine is

$$w = -(q_1 + q_2) = -4 + 2 = -2kJ$$

The negative sign indicates that the work is actually done by the engine.