



IIT-JAM-MATHEMATICS
FEBRUARY-2023

SECTION-A

1. Let $a_n = \left(1 + \frac{1}{n}\right)^n$ and $b_n = n \cos\left(\frac{n!\pi}{2^{10}}\right)$ for $n \in \mathbb{N}$. Then
- (a) (a_n) is not convergent and (b_n) is unbounded
 - (b) (a_n) is convergent and (b_n) is unbounded
 - (c) (a_n) is not convergent and (b_n) is bounded
 - (d) (a_n) is convergent and (b_n) is bounded
2. Let v_1, \dots, v_9 be the column vectors of a non-zero 9×9 real matrix A . Let $a_1, \dots, a_9 \in \mathbb{R}$, not all zero, be such that $\sum_{i=1}^9 a_i v_i = 0$. Then the system $Ax = \sum_{i=1}^9 v_i$ has
- (a) a unique solution
 - (b) more than one but only finitely many solutions
 - (c) infinitely many solutions
 - (d) no solution
3. Consider the initial value problem $\frac{dy}{dx} + \alpha y = 0$, $y(0) = 1$, where $\alpha \in \mathbb{R}$. Then
- (a) There is no α such that $y(2) = 1$
 - (b) There is a unique α such that $\lim_{x \rightarrow \infty} y(x) = 0$
 - (c) There is a unique α such that $y(1) = 2$
 - (d) There is an α such that $y(1) = 0$
4. Let G be a finite group. Then G is necessarily a cyclic group if the order of G is
- (a) 6
 - (b) 7
 - (c) 4
 - (d) 10
5. Let $p(x) = x^{57} + 3x^{10} - 21x^3 + x^2 + 21$ and $q(x) = p(x) + \sum_{j=1}^{57} p^{(j)}(x)$ for all $x \in \mathbb{R}$, where $p^{(j)}(x)$ denotes the j th derivative of $p(x)$. Then the function q admits
- (a) a global minimum but Not a global maximum on \mathbb{R}
 - (b) Neither a global maximum NOR a global minimum on \mathbb{R}
 - (c) a global minimum and a global maximum on \mathbb{R}
 - (d) a global maximum but not a global minimum on \mathbb{R}



6. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $f(x, y) = (e^x \cos(y), e^x \sin(y))$. Then the number of point in \mathbb{R}^2 that do not lies in the range of f is

- (a) 1 (b) 0 (c) 3 (d) infinite

7. Which of the following is a subspace of the real vector space \mathbb{R}^3 ?

- (a) $\{(x, y, z) \in \mathbb{R}^3 : y \in \mathbb{Q}\}$ (b) $\{(x, y, z) \in \mathbb{R}^3 : (y+z)^2 + (2x-3y)^2 = 0\}$
 (c) $\{(x, y, z) \in \mathbb{R}^3 : x+2y-3z+1=0\}$ (d) $\{(x, y, z) \in \mathbb{R}^3 : yz=0\}$

8. Let (a_n) be a sequence of real numbers defined by

$$a_n = \begin{cases} 1 & \text{if } n \text{ is prime} \\ -1 & \text{if } n \text{ is not prime} \end{cases}$$

Let $b_n = \frac{a_n}{n}$ for $n \in \mathbb{N}$. Then

- (a) (a_n) is convergent but (b_n) is not convergent
 (b) (a_n) is not convergent but (b_n) is convergent
 (c) both (a_n) and (b_n) are convergent
 (d) both (a_n) and (b_n) are not convergent

9. The value of $\int_0^1 \int_0^{1-x} \cos(x^3 + y^2) dy dx - \int_0^1 \int_0^{1-y} \cos(x^3 + y^2) dx dy$ is

- (a) $\cos\left(\frac{1}{2}\right) - \sin\left(\frac{1}{2}\right)$ (b) 0 (c) $\frac{\sin(1)}{2}$ (d) $\frac{\cos(1)}{2}$

10. The limit $\lim_{a \rightarrow 0} \frac{\int_0^a \sin(x^2) dx}{\int_0^a (\ln(x+1))^2 dx}$ is

- (a) non-existent (b) $\frac{\pi}{e}$ (c) 0 (d) 1

11. Let $f(x, y) = \iint_{(u-x)^2 + (v-y)^2 \leq 1} e^{-\sqrt{(u-x)^2 + (v-y)^2}} du dv$. Then $\lim_{n \rightarrow \infty} f(n, n^2)$ is

- (a) 0 (b) $2\pi(1-2e^{-1})$ (c) non-existent (d) $\pi(1-e^{-1})$

12. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be an infinitely differentiable function such that f' has exactly two distinct zeroes. Then
- (a) f' has a least 1 zero (b) f has a least 2 distinct zeroes
(c) f' has at most 3 distinct zeroes (d) f has at most 3 distinct zeroes
13. Consider the family of curves $x^2 + y^2 = 2x + 4y + k$ with a real parameter $k > -5$.
Then the orthogonal trajectory to this family of curves passing through $(2, 3)$ also passes through
- (a) $(-1, 1)$ (b) $(1, 0)$ (c) $(3, 5)$ (d) $(3, 4)$
14. From the additive group \mathbb{Q} to which one of the following groups does there exist a non-trivial group homomorphism?
- (a) \mathbb{Z} the additive group of integers
(b) \mathbb{Z}_2 the additive group of integers modulo 2
(c) \mathbb{R}^\times , the multiplicative group of non-zero real numbers
(d) \mathbb{Q}^\times , the multiplicative group of non-zero rational numbers
15. Consider the following statements:
- I. There exist a linear transformation from \mathbb{R}^3 to itself such that its range space and null space are the same
II. There exist a linear transformation from \mathbb{R}^2 to itself such that its range space and null space are the same
Then
- (a) Both I and II are false (b) II is True but I is false
(c) both I and II are True (d) I is True but II is false
16. Let $y : \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that y'' is continuous on $[0, 1]$ and $y(0) = y(1) = 0$.
Suppose $y''(x) + x^2 < 0$ for all $x \in [0, 1]$. Then
- (a) $y(x) > 0$ for all $x \in (0, 1)$
(b) $y(x) = 0$ has exactly one solution in $(0, 1)$
(c) $y(x) < 0$ for all $x \in (0, 1)$
(d) $y(x) = 0$ has more than one solution in $(0, 1)$
17. Let S and T be non-empty subsets of \mathbb{R}^3 , and W be a non-zero proper subspace of \mathbb{R}^2 . Consider the following statements:
- I. If $\text{span}(S) = \mathbb{R}^2$. Then $\text{span}(S \cap W) = W$
II. $\text{span}(S \cup T) = \text{span}(S) \cup \text{span}(T)$
Then
- (a) both I and II are True (b) both I and II are false
(c) I is true but II is false (d) II is True but I is false

21. Let $f(x) = \cos(x)$ and $g(x) = 1 - \frac{x^2}{2}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then

(a) $f(x) \leq g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(b) $f(x) - g(x)$ changes sign more than once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $f(x) \geq g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(d) $f(x) - g(x)$ changes sign exactly once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

22. Suppose $f : (-1, 1) \rightarrow \mathbb{R}$ is an infinitely differentiable function such that the series $\sum_{j=0}^{\infty} a_j \frac{x^j}{j!}$ converges to

$f(x)$ for each $x \in (-1, 1)$, where,

$$a_j = \int_0^{\pi/2} \theta^j \cos^j(\tan \theta) d\theta + \int_{\pi/2}^{\pi} (\theta - \pi)^j \cos^j(\tan \theta) d\theta$$

for $j \leq 0$. Then

(a) f is neither an odd function nor an even function on $(-1, 1)$

(b) f is a non-constant odd function on $(-1, 1)$

(c) f is a non-constant even function on $(-1, 1)$

(d) $f(x) = 0$ for all $x \in (-1, 1)$

23. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -2 & 2 & 2 \end{pmatrix}$ and $B = A^5 + A^4 + I_3$. Which of the following is not an eigenvalue of B?

(a) 49

(b) 3

(c) 2

(d) 1

24. How many group homomorphisms are there from \mathbb{Z}_2 to S_5 ?

(a) 41

(b) 26

(c) 25

(d) 40

25. The area of the curved surface $S = \{(x, y, z) \in \mathbb{R}^3 : z^2 = (x-1)^2 + (y-2)^2\}$ lying between the plane $z = 2$ and $z = 3$ is

(a) $4\pi\sqrt{2}$

(b) 9π

(c) $9\pi\sqrt{2}$

(d) $5\pi\sqrt{2}$



26. Let $a_n = \sin\left(\frac{1}{n^3}\right)$ and $b_n = \sin\left(\frac{1}{n}\right)$ for $n \in \mathbb{N}$. Then

(a) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are not convergent

(b) $\sum_{n=1}^{\infty} a_n$ not convergent but $\sum_{n=1}^{\infty} b_n$ is convergent

(c) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is not convergent

(d) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent

27. Let (a_n) and (b_n) be sequence of real numbers such that

$$|a_n - a_{n+1}| = \frac{1}{2^n} \text{ and } |b_n - b_{n+1}| = \frac{1}{\sqrt{n}} \text{ for } n \in \mathbb{N}. \text{ Then}$$

(a) (a_n) need not be a Cauchy sequence but (b_n) is a Cauchy sequence

(b) both (a_n) and (b_n) are Cauchy sequences

(c) (a_n) is Cauchy sequence but (b_n) need not be a Cauchy sequence

(d) Both (a_n) and (b_n) need not be Cauchy sequences

28. Let (a_n) be a sequence of real numbers such that the series $\sum_{n=0}^{\infty} a_n (x-2)^n$ converges at $x = -5$. Then This series also converges at

(a) $x = 12$

(b) $x = 5$

(c) $x = 9$

(d) $x = -6$

29. Consider the following statements:

I. Every infinite group has infinitely many subgroup.

II. There are only finitely many non-isomorphic groups of a given finite order.

Then

(a) I is true but II is false

(b) both I and II are True

(c) both I and II are false

(d) I is false but II is True

30. Let $f(x, y) = \ln(1 + x^2 + y^2)$ for $(x, y) \in \mathbb{R}^2$. Define

$$P = \frac{\partial^2 f}{\partial x^2} \Big|_{(0,0)} \quad Q = \frac{\partial^2 f}{\partial x \partial y} \Big|_{(0,0)}$$

$$R = \frac{\partial^2 f}{\partial y \partial x} \Big|_{(0,0)} \quad S = \frac{\partial^2 f}{\partial y^2} \Big|_{(0,0)}$$

Then

- (a) $PS - QR > 0$ and $P < 0$ (b) $PS - QR < 0$ and $P < 0$
 (c) $PS - QR < 0$ and $P > 0$ (d) $PS - QR > 0$ and $P > 0$

SECTION-B

31. Let R_1 and R_2 be the radii of convergence of the power series $\sum_{n=1}^{\infty} (-1)^n x^{n-1}$ and $\sum_{n=1}^{\infty} (-1)^n \frac{x^{n+1}}{n(n+1)}$, respectively.

Then

- (a) $\sum_{n=1}^{\infty} (-1)^n \frac{x^{n+1}}{n(n+1)}$ converges for all $x \in [-1, 1]$
 (b) $R_1 = R_2$
 (c) $R_2 > 1$
 (d) $\sum_{n=1}^{\infty} (-1)^n x^{n-1}$ converges for all $x \in [-1, 1]$

32. Which of the following is/are True ?

- (a) Every bijective linear transformation from \mathbb{R}^2 to \mathbb{R}^2 maps pairs of perpendicular lines to pairs of perpendicular lines.
 (b) Every bijective linear transformation from \mathbb{R}^2 to \mathbb{R}^2 maps pairs of parallel lines to pairs of parallel lines.
 (c) Every linear transformation from \mathbb{R}^2 to \mathbb{R}^2 maps lines onto points or lines
 (d) Every surjective linear transformation from \mathbb{R}^2 to \mathbb{R}^2 maps lines onto lines

33. Let $A \subseteq \mathbb{Z}$ with $0 \in A$. For $r, s \in \mathbb{Z}$, define

$$rA = \{ra : a \in A\}, rA + sA = \{ra + sb : a, b \in A\}$$

Which of the following imply that A is a subgroup of the additive group \mathbb{Z} ?

- (a) $A = -A, A + 2A = A$ (b) $2A \subseteq A, A + A = A$
 (c) $A = -A, A + A = A$ (d) $-2A \subseteq A, A + A = A$

34. Which of the following functions is/are Riemann integrable on $[0, 1]$?

- (a) $f(x) = \begin{cases} x \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (b) $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0, 1] \\ -1 & \text{otherwise} \end{cases}$
 (c) $f(x) = \int_0^x \left| \frac{1}{2} - t \right| dt$ (d) $f(x) = \begin{cases} x & \text{if } x \in [0, 1] \\ 0 & \text{if } x = 1 \end{cases}$

35. Let $y : (\sqrt{2/3}, \infty) \rightarrow \mathbb{R}$ be the solution of $(2x - y)y' + (2y - x) = 0$, $y(1) = 3$. Then

- (a) y' is bounded on $(1, \infty)$ (b) $y(2) = 4 + \sqrt{10}$
 (c) $y(3) = 1$ (d) y' is bounded on $(\sqrt{2/3}, 1)$



36. The subset $S \subseteq \mathbb{R}^2$ is said to be bounded if there is an $M > 0$ such that $|x| \leq M$ and $|y| \leq M$ for all $(x, y) \in S$. Which of the following subsets of \mathbb{R}^2 is/are bounded ?

- (a) $\{(x, y) \in \mathbb{R}^2 : e^{x^2} + y^2 \leq 4\}$ (b) $\{(x, y) \in \mathbb{R}^2 : e^{x^2} + y^2 \leq 4\}$
 (c) $\{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 4\}$ (d) $\{(x, y) \in \mathbb{R}^2 : x^4 + y^2 \leq 4\}$

37. Which of the following is/are linear transformations ?

- (a) $T : \mathbb{R} \rightarrow \mathbb{R}$ given by $T(x) = \sin(x)$ (b) $T : \mathbb{R} \rightarrow \mathbb{R}$ given by $T(x, y) = x + y + 1$
 (c) $T : P_2(\mathbb{R}) \rightarrow \mathbb{R}$ given by $T(p(x)) = p(1)$ (d) $T : M_2(\mathbb{R}) \rightarrow \mathbb{R}$ given by $T(A) = \text{trace}(A)$

38. Let $f : (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function satisfying $f(0) = 0$. Suppose there exists an $M > 0$ such that $|f'(x)| \leq M|x|$ for all $x \in (-1, 1)$. Then

- (a) f' is differentiable at $x = 0$ (b) $(f')^2$ is differentiable at $x = 0$
 (c) ff' is differentiable at $x = 0$ (d) f' is continuous at $x = 0$

39. $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined as follows:

$$f(x, y) = \begin{cases} \frac{x^4 y^3}{x^6 + y^6} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

then

- (a) $\frac{\partial f}{\partial x} \Big|_{(0,0)}$ exists and equal to 0 (b) $\lim_{t \rightarrow 0} \frac{f(t, t) - f(0, 0)}{t}$ exists and equal to $1/2$
 (c) $\lim_{t \rightarrow 0} \frac{f(t, 2t) - f(0, 0)}{t}$ exists and equal to $1/3$ (d) $\frac{\partial f}{\partial y} \Big|_{(0,0)}$ exists and equal to 0

40. For each $t \in (0, 1)$, the surface P_t in \mathbb{R}^3 is defined by

$$P_t = \{(x, y, z) : (x^2 + y^2)z = 1, t^2 \leq x^2 + y^2 \leq 1\}. \text{ Let } a_t \in \mathbb{R} \text{ be the surface area of } P_t. \text{ Then}$$

(a) the limit $\lim_{t \rightarrow 0^+} a_t$ does not exist

(b) $a_t = \iint_{t^2 \leq x^2 + y^2 \leq 1} \sqrt{1 + \frac{4x^2}{(x^2 + y^2)^2} + \frac{4y^2}{(x^2 + y^2)^2}} dx dy$

(c) $a_t = \iint_{t^2 \leq x^2 + y^2 \leq 1} \sqrt{1 + \frac{4x^2}{(x^2 + y^2)^4} + \frac{4y^2}{(x^2 + y^2)^4}} dx dy$



(d) the limit $\lim_{t \rightarrow 0^+} a_t$ exists

SECTION-C

41. Let $T : P_2(\mathbb{R}) \rightarrow P_4(\mathbb{R})$ be the linear transformation given by $T(p(x)) = p(x^2)$. Then the rank of T is equal to _____
42. Let $f(x) = \sqrt[3]{x}$ for $x \in (0, \infty)$ and $\theta(h)$ be a function such that $f(3+h) - f(3) = hf'(3 + \theta(h)h)$ for all $h \in (-1, 1)$. then $\lim_{h \rightarrow 0} \theta(h)$ is equal to _____ (round of to two decimal places)
43. For $\sigma \in S_8$ let $o(\sigma)$ denote the order of σ . Then $\max \{o(\sigma) : \sigma \in S_8\}$ is equal to _____
44. Let V be the volume of the region $S \subseteq \mathbb{R}^3$ defined by $S = \{(x, y, z) \in \mathbb{R}^3 : xy \leq z \leq 4, 0 \leq x^2 + y^2 \leq 1\}$. Then $\frac{V}{\pi}$ is equal to _____ (round of to two decimal places)
45. Let $g \in \mathbb{Z}$, let $\bar{g} \in \mathbb{Z}_8$ denote the residue class of g modulo 8. Consider the group $\mathbb{Z}_8^\times = \{\bar{x} \in \mathbb{Z}_8 : 1 \leq x \leq 7, \gcd(x, 8) = 1\}$ with respect to multiplication modulo 8. The number of group isomorphisms from \mathbb{Z}_8^\times onto itself is equal to _____
46. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be the function defined as follows:
- $$f(x, y) = \begin{cases} (x^2 - 1)^2 \cos^2\left(\frac{y^2}{(x^2 - 1)^2}\right) & \text{if } x \neq \pm 1 \\ 0 & \text{if } x = \pm 1 \end{cases}$$
- The number of points of discontinuity of $f(x, y)$ is equal to _____
47. The value of $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2^n} + \frac{1}{3^n} + \dots + \frac{1}{(2023)^n}\right)^{1/n}$ is equal to _____ (round of to two decimal places)
48. The sum of the series $\sum_{n=1}^{\infty} \frac{2n+1}{(n^2+1)(n^2+2n+2)}$ is equal to _____ (round of to two decimal places)
49. The value of $\lim_{n \rightarrow \infty} \left(n \int_0^1 \frac{x^n}{x+1} dx\right)$ is equal to _____ (round of to two decimal places)
50. If y is the solution of $y'' - 2y' + y = e^x$, $y(0) = 0$, $y'(0) = -1/2$ then $y(1)$ is equal to _____ (round of to two decimal places)
51. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a bijective function such that for all $x \in \mathbb{R}$, $f(x) = \sum_{n=1}^{\infty} a_n x^n$ and $f^{-1}(x) = \sum_{n=1}^{\infty} b_n x^n$, where f^{-1} is the inverse function of f . If $a_1 = 2$ and $a_2 = 4$ Then b_1 is equal to _____

52. The number of permutations in S_4 that have exactly two cycles in their cycle decompositions is equal to _____
53. Let S be the set of all real numbers α such that the solution y of the initial value problem $\frac{dy}{dx} = y(2-y)$, $y(0) = \alpha$, exists on $[0, \infty)$. Then the minimum of the set S is equal to _____ (round of to two decimal places)
54. Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ defined as $f(x, y, z) = x^3 + y^3 + z^3$, and let $L : \mathbb{R}^3 \rightarrow \mathbb{R}$ be the linear map satisfying
$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{f(1+x, 1+y, 1+z) - f(1,1,1) - L(x, y, z)}{\sqrt{x^2 + y^2 + z^2}} = 0$$
 Then $L(1, 2, 4)$ is equal to _____ (round of to two decimal places)
55. The maximum number of linearly independent eigenvectors of the matrix
$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$
 is equal to _____
56. Let $A = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 1 & 4 & 4 & 4 \end{pmatrix}$ and B be a 5×5 real matrix such that AB is the zero matrix. Then the maximum possible rank of B is equal to _____
57. Let W be the subspace of $M_3(\mathbb{R})$ consisting of all matrices with the property that the sum of the entries in each row is zero and the sum of the entries in each column is zero. Then the dimension of W is equal to _____
58. Let $y : (1, \infty) \rightarrow \mathbb{R}$ be the solution of the differentiable equation
$$y'' - \frac{2y}{(1-x)^2} = 0$$
 satisfying $y(2) = 1$ and $\lim_{x \rightarrow \infty} y(x) = 0$. Then $y(3)$ is equal to _____ (round of to two decimal places)
59. Let S be the triangular region whose vertices are $(0, 0)$, $(0, \frac{\pi}{2})$, and $(\frac{\pi}{2}, 0)$. The value of
$$\iint_S \sin(x) \cos(y) dx dy$$
 is equal to _____ (round of to two decimal places)
60. The global minimum value of $f(x) = |x-1| + |x-2|^2$ on \mathbb{R} is equal to _____ (round of to two decimal places)

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ANSWER KEY

SECTION-A

1. (2)	2. (3)	3. (3)	4. (2)	5. (2)
6. (1)	7. (2)	8. (2)	9. (2)	10. (4)
11. (2)	12. (3)	13. (4)	14. (3)	15. (2)
16. (1)	17. (2)	18. (4)	19. (2)	20. (4)
21. (3)	22. (3)	23. (3)	24. (2)	25. (3)
26. (3)	27. (3)	28. (2 & 3)	29. (2)	30. (4)

SECTION-B

31. (2, 1)	32. (2,3,4)	33. (3)	34. (1, 3, 4)	35. (1,2,3)
36. (2,3,4)	37. (3,4)	38. (1, 3, 4)	39. (1,2,4)	40. (2,4)

SECTION-C

41. (3)	42. (0)	43. (15)	44. (4)	45. (6)
46. (0)	47. (1)	48. (0.50)	49. (0.50)	50. (0)
51. (0.5)	52. (11)	53. (0)	54. (1)	55. (3)
56. (2)	57. (4)	58. (0.50)	59. (0.50)	60. (0.75)

