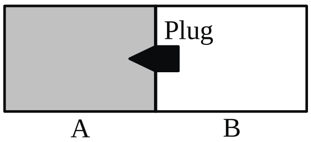
PHYSICS-PH

Q.1 – Q.25 : Carry ONE mark each.

1. As shown in the figure, an ideal gas is confined to chamber A of an insulated container, with vacuum in chamber B. When the plug in the wall separating the chambers A and B is removed, the gas fills both the chambers. Which one of the following statements is true ?



(a) Temperature of the gas decrease as it expands to fill the space in chamber B.
(b) Internal energy of the gas increases as its atoms have more space to move around.
(c) Internal energy of the gas decreases.
(d) The temperature of the gas remains unchanged.

2. Consider a diatomic molecule formed by identical atoms. If E_V and E_e represent the energy of the vibrational nuclear motion and electronic motion respectively, then in terms of the electronic mass m and nuclear mass M, E_V/E_e is proportional to

(a) $(m/M)^2$ (b) $(m/M)^{1/2}$ (c) m/M (d) $(m/M)^{3/2}$

3. Which one of the following is a solution of $\frac{d^2u(x)}{dx^2} = k^2u(x)$, for k real ?

(a) e^{-kx} (b) $\sin kx$ (c) $\sinh x$ (d) $\cos kx$

4. For a complex variable z and the contour $c: |z| = 1$ taken in the counter clockwise direction,

$\frac{1}{2\pi i}\oint_c \left(z - \frac{2}{z} + \frac{3}{z^2}\right)dz$ is equal to ________.

5. A particle is moving in a central force field given by $\vec{F} = -\frac{k}{r^3}\hat{r}$, where $\hat{r}$ is the unit vector pointing away from the center of the field. The potential energy of the particle is given by

(a) $-\frac{k}{r^2}$ (b) $-\frac{k}{2r^2}$ (c) $\frac{k}{2r^2}$ (d) $\frac{k}{r^2}$

6. The total angular momentum j of the ground state of the $^{17}_{8}\text{O}$ nucleus is

(a) 3/2 (b) 1/2 (c) 5/2 (d) 1

7. Let p be the momentum conjugate be the generalized coordinate q. If the transformation $Q = \sqrt{2}q^m \cos p$ and $P = \sqrt{2}q^m \sin p$ is canonical, then m is equal to ________.

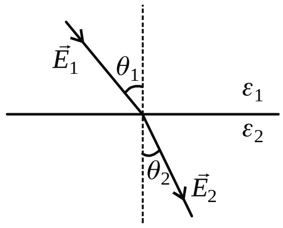
8. A particle Y undergoes strong decay $Y \to \pi^- + \pi^-$. The isospin of Y is _______.

9. Let $\hat{a}$ and $\hat{a}^\dagger$, respectively denote the lowering and raising operators of a one-dimensional simple harmonic oscillator. Let $|n\rangle$ be the energy eigenstate of the simple harmonic oscillator. Given that $|n\rangle$ is also an eigenstate of $\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}$, the corresponding eigenvalue is

(a) $n(n+1)$ (b) n^2 (b) $n(n-1)$ (d) $(n+1)^2$

10. Which one of the following is the correct binary equivalent of the hexadecimal F6C ?
(a) 0110 1100 0111 (b) 1111 0110 1100 (c) 1100 0110 1111 (d) 0110 1111 1100

11. Which one of the following relations determines the manner in which the electric field lines are refracted across the interface between two dielectric media having dielectric constants ε_1 and ε_2 (see figure) ?

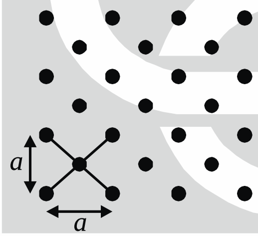


(a) $\varepsilon_1 \sin\theta_1 = \varepsilon_2 \sin\theta_2$ (b) $\varepsilon_1 \cot\theta_1 = \varepsilon_2 \cot\theta_2$
(c) $\varepsilon_1 \cos\theta_1 = \varepsilon_2 \cos\theta_2$ (d) $\varepsilon_1 \tan\theta_1 = \varepsilon_2 \tan\theta_2$

12. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields respectively, then $\vec{E}\cdot\vec{B}$ is
(a) odd under parity and even under time reversal.
(b) odd under parity and odd under time reversal.
(c) even under parity and odd under time reversal.
(d) even under parity and even under time reversal.

13. Which one of the following is a universal logic gate ?
(a) OR (b) NOT (c) AND (d) NAND

14. The number of distinct ways the primitive unit cell can be constructed for the two dimensional lattice as shown in the figure is _______.



15. A particle X is produced in the process $\pi^+ + p \to K^+ + X$ via the strong interaction. If the quark content of the K^+ is $u\bar{s}$, the quark content of X is
(a) $u\bar{d}$ (b) uud (c) $c\bar{s}$ (d) uus

16. A conducting sphere of radius 1 m is placed in air. The maximum number of electrons that can be put on the sphere to avoid electrical breakdown is about 7×10^n, where n is an integer. The value of n is _______.
Assume:
Breakdown electric field strength in air is $|\vec{E}| = 3\times10^6$ V/m
Permittivity of free space $\varepsilon_0 = 8.85\times10^{-12}$ F/m
Electron charge $e = 1.60\times10^{-19}$ C

17. Far from the Earth, the Earth's magnetic field can be approximated as due to a bar magnet of magnetic pole strength 4×10^{14} Am. Assume this magnetic field is generated by a current carrying loop encircling the magnetic equator. The current required to do so is about 4×10^n A, where n is an integer. The value of n is ______.
(Earth's circumference: 4×10^7 m)

18. Particle A with angular momentum j=3/2 decays into two particles B and C with angular momenta j_1 and j_2, respectively. If $\left|\frac{3}{2},\frac{3}{2}\right\rangle_A = \alpha\,|1,1\rangle_B \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle_C$, the value of α is _______.

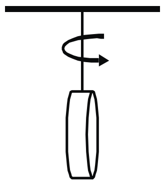
19. Choose the correct statement related to the Fermi energy (E_F) and the chemical potential (μ) of a metal.

(a) $\mu = E_F$ only at 0 K
(b) $\mu > E_F$ at finite temperature
(c) $\mu = E_F$ at finite temperature
(d) $\mu < E_F$ at 0 K

20. A small disc is suspended by a fiber such that it is free to rotate about the fiber axis (see figure). For small angular deflections, the Hamiltonian for the disc is given by

$$H = \frac{p_\theta^2}{2I} + \frac{1}{2}\alpha\theta^2,$$

where I is the moment of inertia and α is the restoring torque per unit deflection. The disc is subjected to angular deflections (θ) due to thermal collisions from the surrounding gas at temperature T and p_θ is the momentum conjugate to θ. The average and the root-mean-square angular deflection, θ_{avg} and $\theta_{\text{r.m.s}}$, respectively are



(a) $\theta_{\text{avg}} = 0$ and $\theta_{\text{r.m.s}} = \left(\frac{k_B T}{\alpha}\right)^{1/2}$
(b) $\theta_{\text{avg}} \neq 0$ and $\theta_{\text{r.m.s}} = \left(\frac{k_B T}{\alpha}\right)^{3/2}$
(c) $\theta_{\text{avg}} = 0$ and $\theta_{\text{r.m.s}} = \left(\frac{k_B T}{\alpha}\right)^{3/2}$
(d) $\theta_{\text{avg}} \neq 0$ and $\theta_{\text{r.m.s}} = \left(\frac{k_B T}{\alpha}\right)^{1/2}$

21. A quantum particle is subjected to the potential

$$V(x) = \begin{cases} \infty, & x \le -\frac{a}{2} \\ 0, & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, & x \ge \frac{a}{2} \end{cases}$$

The ground state wave function of the particle is proportional to

(a) $\cos(\pi x/2a)$
(b) $\cos(\pi x/a)$
(c) $\sin(\pi x/a)$
(d) $\sin(\pi x/2a)$

22. If a particle is moving along a sinusoidal curve, the number of degrees of freedom of the particle is ________.

23. A real, invertible 3 × 3 matrix M has eigenvalues $\lambda_i, (i = 1, 2, 3)$ and the corresponding eigenvectors are $|e_i\rangle, (i = 1, 2, 3)$ respectively. Which one of the following is correct ?

(a) The eigenvalues of M and M^{-1} are not related.
(b) $M^{-1}|e_i\rangle = \lambda_i|e_i\rangle$, for $i = 1, 2, 3$
(c) $M^{-1}|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1, 2, 3$
(d) $M|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1, 2, 3$

24. A medium $(\varepsilon_r > 1,\ \mu_r = 1,\ \sigma > 0)$ is semi-transparent to an electromagnetic wave when

(a) Conduction current $\ll$ Displacement current.
(b) Both conduction current and displacement current are zero.
(c) Conduction current $\gg$ Displacement current.
(d) Conduction current = Displacement current.

25. A hydrogenic atom is subjected to a strong magnetic field. In the absence of spin-orbit coupling, the number of doubly degenerate states created out of the d-level is ________.

Q.26 – Q.55 : Carry TWO marks each.

26. According to the Fermi gas model of the nucleus, the nucleons move in a spherical volume of radius $R(=R_0A^{1/3}$, where A is the mass number and R_0 is an empirical constant with the dimensions of length). The Fermi energy of the nucleus E_F is proportional to

(a) $\frac{1}{R_0^2}$ (b) $\frac{1}{R_0^3}$ (c) $\frac{1}{R_0}$ (d) R_0^2

27. Which one of the following matrices does NOT represent a proper rotation in a plane ?

(a) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{pmatrix}$ (b) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$ (d) $\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

28. $\hat{S}_x$ denotes the spin operator defined as $\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Which one of the following is correct ?

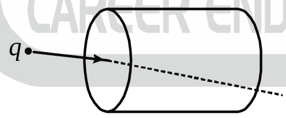
(a) The eigenstates of spin operator $\hat{S}_x$ are $\left|\uparrow\right\rangle_x \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\left|\downarrow\right\rangle_x \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

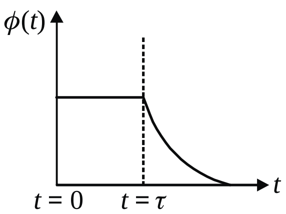
(b) The eigenstates of spin operator $\hat{S}_x$ are $\left|\uparrow\right\rangle_x \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\left|\uparrow\right\rangle_x \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

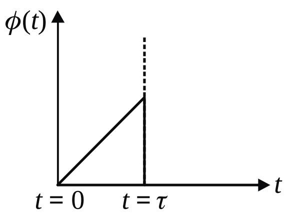
(c) In the spin state $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of $\hat{S}_x$, the probability for obtaining $\left|\uparrow\right\rangle_x$ is $\frac{1}{4}$.

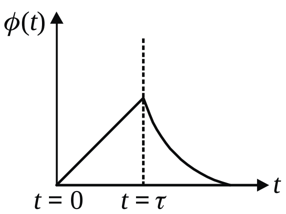
(d) In the spin state $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of $\hat{S}_x$, the probability for obtaining $\left|\uparrow\right\rangle_x$ is $\frac{2+\sqrt{3}}{4}$.

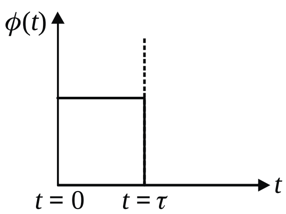
29. A charge q moving with uniform speed enters a cylindrical region in free space at $t=0$ and exits the region at $t=\tau$ (see figure). Which one of the following options best describes the time dependence of the total electric flux $\phi(t)$, through the entire surface of the cylinder ?



(a)


(b)


(c)


(d)


30. Let u^μ denote the 4-velocity of a relativistic particle whose square $u^\mu u_\mu = 1$. If $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor, then the value of $\varepsilon_{\mu\nu\rho\sigma}u^\mu u^\nu u^\rho u^\sigma$ is ______.

31. Consider the Lagrangian $L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$, where a, b and c are constants. If p_x and p_y are the momenta conjugate to the coordinates x and y respectively, then the Hamiltonian is

(a) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$ (b) $\frac{p_x^2}{a} + \frac{p_y^2}{b} + cxy$ (c) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} - cxy$ (d) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + cxy$

32. A free particle of mass M is located in a three-dimensional cubic potential well with impenetrable walls. The degeneracy of the 5^{th} excited state of the particle is ________.

33. For a gas of non-interacting particles, the probability that a particle has a speed v in the interval v to $v + dv$ is given by

$$f(v)\,dv = 4\pi v^2 dv\left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_BT}$$

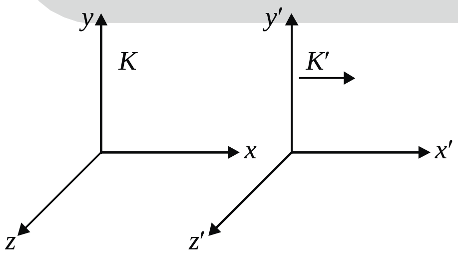
If E is the energy of a particle, then the maximum in the corresponding energy distribution in units of E/k_BT occurs at _______ (rounded off to one decimal place).

34. A plane electromagnetic wave of wavelength λ is incident on a circular loop of conducting wire. The loop radius is $a(a \ll \lambda)$. The angle (in degrees), made by the Poynting vector with the normal to the plane of the loop to generate a maximum induced electrical signal, is _______.

35. A uniform magnetic field $\vec{B} = B_0\hat{y}$ exists in an inertial frame K. A perfect conducting sphere moves with a constant velocity $\vec{v} = v_0\hat{x}$ with respect to this inertial frame. The rest frame of the sphere is K' (see figure). The electric and magnetic fields in K and K' are related as

$$\left.\begin{matrix} \vec{E}'_{\parallel} = \vec{E}_{\parallel} & \vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v}\times\vec{B}\right) \\ \vec{B}'_{\parallel} = \vec{B}_{\parallel} & \vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2}\times\vec{E}\right) \end{matrix}\right\}, \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

The induced surface charge density on the sphere (to the lowest order in v/c) in the frame K' is



(a) uniform over the sphere
(b) maximum along y'
(c) maximum along x'
(d) maximum along z'

36. Let $f_n(x) = \begin{cases} 0, & x < -\frac{1}{2n} \\ n, & -\frac{1}{2n} < x < \frac{1}{2n} \\ 0, & \frac{1}{2n} < x \end{cases}$

The value of $\lim_{n\to\infty}\int_{-\infty}^{\infty} f_n(x)\sin x\, dx$ is ________.

37. Consider a two dimensional crystal with 3 atoms in the basis. The number of allowed optical branches (n) and acoustic branches (m) due to the lattice vibrations are

(a) $(n, m) = (3, 3)$ (b) $(n, m) = (2, 4)$ (c) $(n, m) = (4, 2)$ (d) $(n, m) = (1, 5)$

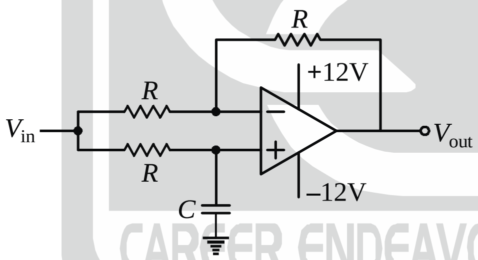
38. Consider a one-dimensional non-magnetic crystal with one atom per unit cell. Assume that the valence electrons (i) do not interact with each other and (ii) interact weakly with the ions. If n is the number of valence electrons per unit cell, then at 0 K.

(a) the crystal is metallic for even values of n (b) the crystal is metallic for any value of n
(c) the crystal is non-metallic for any value of n (d) the crystal is metallic for odd values of n

39. In the center of mass frame, two protons each having energy 7000 GeV, collide to produce protons and anti-protons. The maximum number of anti-protons produced is ________.
(Assume the proton mass to be 1 GeV/c^2)

40. The radial wave function of a particle in a central potential is given by $R(r) = A\frac{r}{a}\exp\left(-\frac{r}{2a}\right)$, where A is the normalization constant and a is positive constant of suitable dimensions. If γa is the most probable distance of the particle from the force center, the value of γ is ________.

41. The input voltage (V_{in}) to the circuit shown in the figure is 2 cos (100 t) V. The output voltage (V_{out}) is $2\cos\left(100t - \frac{\pi}{2}\right)$V. If $R = 1\text{ k}\Omega$, the value of C (in μF) is



(a) 100 (b) 0.1 (c) 10 (d) 1

42. The potential energy of a particle of mass m is given by

$$U(x) = a\sin\left(k^2x - \frac{\pi}{2}\right),\ a > 0,\ k^2 > 0$$

The angular frequency of small oscillations of the particle about $x = 0$ is

(a) $2k^2\sqrt{a/m}$ (b) $k^2\sqrt{a/2m}$ (c) $k^2\sqrt{a/m}$ (d) $k^2\sqrt{2a/m}$

43. Consider the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}'$, where

$\hat{H}_0 = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}$ and $\hat{H}'$ is the time independent perturbation given by $\hat{H}' = \begin{pmatrix} 0 & k & 0 \\ k & 0 & k \\ 0 & k & 0 \end{pmatrix}$, where $k > 0$.

If, the maximum energy eigenvalue of $\hat{H}$ is 3 eV corresponding to $E = 2$ eV, the value of k (rounded off to three dimensional places) in eV is ________.

44. If $x=\sum_{k=1}^{\infty} a_k \sin kx$, for $-\pi \le x \le \pi$, the value of a_2 is ________.

45. A sinusoidal voltage of the form $V(t)=V_0 \cos(\omega t)$ is applied across a parallel plate capacitor placed in vacuum. Ignoring the edge effects, the induced e.m.f. within the region between the capacitor plates can be expressed as a power series in ω. The lowest non-vanishing exponent in ω is ________.

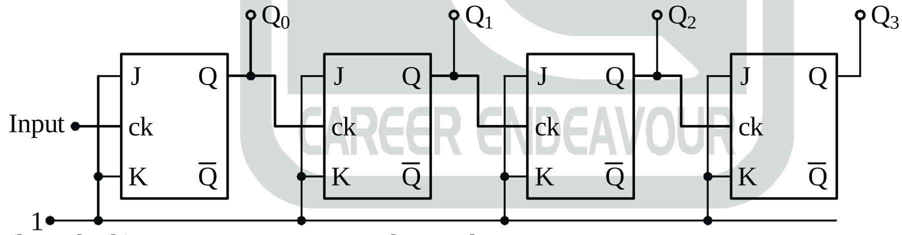
46. The product of eigenvalues of $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is

(a) 0 (b) –1 (c) 2 (d) 1

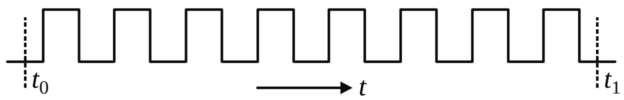
47. Let $|e_1\rangle \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |e_2\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $|e_3\rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Let $S=\{|e_1\rangle, |e_2\rangle, |e_3\rangle\}$. Let $\mathbb{R}^3$ denote the three-dimensional real vector space. Which one of the following is correct ?

(a) S is a linearly dependent set
(b) S is an orthonormal set
(c) S is a basis for $\mathbb{R}^3$
(d) $\sum_{i=1}^{3} |e_i\rangle\langle e_i| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

48. Consider a 4-bit counter constructed out of four flip-flops. It is formed by connecting the J and K inputs to logic high and feeding the Q output to the clock input of the following flip-flop (see the figure). The input signal to the counter is a series of square pulses and the change of state is triggered by the falling edge. At time $t=t_0$ the outputs are in logic low state $(Q_0=Q_1=Q_2=Q_3=0)$. Then at $t=t_1$, the logic state of the outputs is



4-bit ripple counter



Input Signal

(a) $Q_0=1, Q_1=0, Q_2=0$ and $Q_3=0$
(b) $Q_0=0, Q_1=0, Q_2=0$ and $Q_3=1$
(c) $Q_0=0, Q_1=1, Q_2=1$ and $Q_3=1$
(d) $Q_0=1, Q_1=0, Q_2=1$ and $Q_3=0$

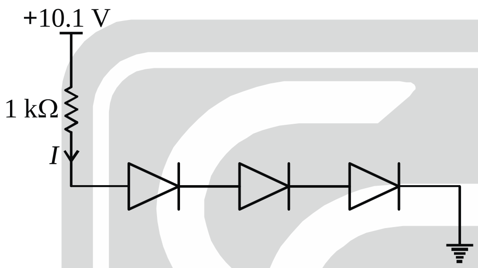
49. A Hydrogen atom is in an orbital angular momentum state $|l, m=l\rangle$. If $\vec{L}$ lies on a cone which makes a half angle 30° with respect to the z-axis, the value of l is ________.

50. The Planck's energy density distribution is given by $u(\omega)=\dfrac{\hbar\omega^3}{\pi^2c^3\left(e^{\hbar\omega/k_BT}-1\right)}$. At long wavelengths, the energy density of photons in thermal equilibrium with a cavity at temperature T varies T^{α}, where α is ______.

51. An electron in a hydrogen atom is in the state $n=3,\ l=2,\ m=-2$. Let $\hat{L}_y$ denote the y-component of the orbital angular momentum operator. If $(\Delta\hat{L}_y)^2=\alpha\hbar^2$, the value of α is _______.

52. Consider a simple cubic monoatomic Bravais lattice which has a basis with vectors $\vec{r}_1=0,\ \vec{r}_2=\frac{a}{4}(\hat{x}+\hat{y}+\hat{z})$, a is the lattice parameter. The Bragg reflection is observed due to the change in the wave vector between the incident and the scattered beam as given by $\vec{K}=n_1\vec{G}_1+n_2\vec{G}_2+n_3\vec{G}_3$, where $\vec{G}_1,\vec{G}_2$ and $\vec{G}_3$ are primitive reciprocal lattice vectors. For $n_1=3,\ n_2=3$ and $n_3=2$ the geometrical structure factor is _______.

53. Consider the circuit given in the figure. Let the forward voltage drop across each diode be 0.7 V. The current I (in mA) through the resistor is _______.



54. The internal energy U of a system is given by $U(S,V)=\lambda V^{-2/3}S^2$, where λ is a constant of appropriate dimensions; V and S denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system ?

(a) $\dfrac{P}{V^{1/3}T}=$ constant (b) $\dfrac{PV}{T^{1/3}}=$ constant (c) $\dfrac{PV^{2/3}}{T}=$ constant (d) $\dfrac{PV^{1/3}}{T^2}=$ constant

55. Consider a gas of hydrogen atoms in the atmosphere of the Sun where the temperature is 5800 K. If a sample from this atmosphere contains 6.023×10^{23} of hydrogen atoms in the ground state, the number of hydrogen atoms in the first excited state is approximately 8×10^{n}, where n is an integer. The value of n is _______.
(Boltzmann constant: 8.617×10^{-5} eV/K)

❍❍❍