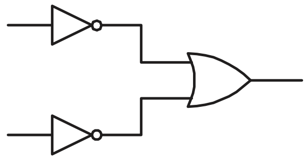
PHYSICS-PH

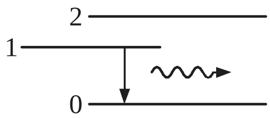
Q.1 – Q.25 : Carry ONE mark each.

1. The below combination of logic gates represents the operation



(a) AND (b) NOR (c) OR (d) NAND

2. To sustain lasing action in a three-level laser as shown in the figure, necessary condition(s) is(are):



(a) lifetime of the energy level 2 should be greater than that of energy level 0.
(b) lifetime of the energy level 1 should be greater than that of energy level 2.
(c) population of the particles in level 1 should be greater than that of level 0.
(d) population of the particles in level 2 should be greater than that of level 1.

3. Consider the potential $U(r)$ defined as:

$$U(r) = -U_0 \frac{e^{-\alpha r}}{r}$$

where α and U_0 are real constants of appropriate dimensions. According to the first Born approximation, the elastic scattering amplitude calculated with $U(r)$ for a (wave-vector) momentum transfer q and $\alpha \to 0$, is proportional to

$$\left(\text{Useful integral: } \int_0^\infty \sin(qr) e^{-\alpha r} dr = \frac{q}{\alpha^2 + q^2}\right)$$

(a) q^{-2} (b) q^2 (c) q^{-1} (d) q

4. Consider a system of three distinguishable particles, each having spin $S = 1/2$ such that $S_z = \pm 1/2$ with corresponding magnetic moments $\mu_z = \pm \mu$. When the system is placed in an external magnetic field H pointing along the z-axis, the total energy of the system is μH. Let x be the state where the first spin has $S_z = 1/2$. The probability of having the state x and the mean magnetic moment (in the +z direction) of the system in state x are

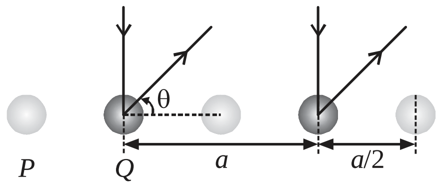
(a) $\frac{2}{3}, \frac{-2}{3}\mu$ (b) $\frac{2}{3}, \frac{1}{3}\mu$ (c) $\frac{1}{3}, \frac{2}{3}\mu$ (d) $\frac{1}{3}, \frac{-1}{3}\mu$

5. Consider a tiny current loop driven by a sinusoidal alternating current. If the surface integral of its time-averaged Poynting vector is constant, then the magnitude of the time-averaged magnetic field intensity, at any arbitrary position, $\vec{r}$, is proportional to

(a) $1/r^2$ (b) r (c) $1/r$ (d) $1/r^3$

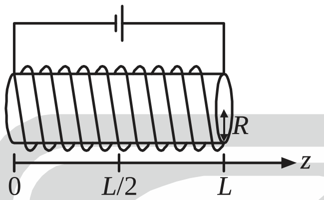
6. A light source having its intensity peak at the wavelength 289.8 nm is calibrated as 10,000 K which is the temperature of an equivalent black body radiation. Considering the same calibration, the temperature of light source (in K) having its intensity peak at the wavelength 579.6 nm **(rounded off to the nearest integer)** is _______.

7. As shown in the figure, X-ray diffraction pattern is obtained from a diatomic chain of atoms P and Q. The diffraction condition is given by $a\cos\theta = n\lambda$, where n is the order of the diffraction peak. Here, a is the lattice constant and λ is the wavelength of the X-rays. Assume that atomic form factors and resolution of the instrument do not depend on θ. Then, the intensity of the diffraction peaks is



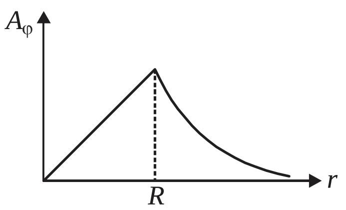
(a) lower for even values of n, when compared to odd values of n.
(b) zero for even values of n.
(c) lower for odd values of n, when compared to even values of n.
(d) zero for odd values of n.

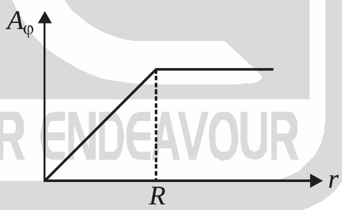
8. Consider a solenoid of length L and radius R, where $R \ll L$. A steady-current flows through the solenoid. The magnetic field is uniform inside the solenoid and zero outside.

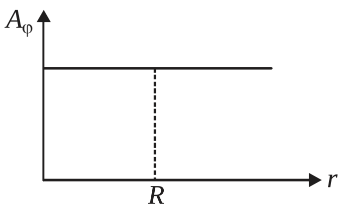


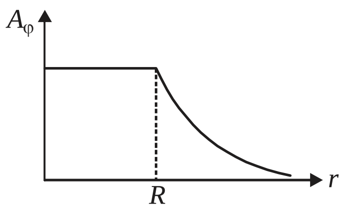
Among the given options, choose the one that best represents the variation in the magnitude of the vector potential $(0, A_\varphi, 0)$ at $z = L/2$, as a function of the radial distance (r) in cylindrical coordinates.
Useful information: The curl of a vector $\vec{F}$, in cylindrical coordinates is

$$\vec{\nabla}\times\vec{F}(r,\varphi,z) = \hat{r}\left[\frac{1}{r}\frac{\partial F_z}{\partial\varphi} - \frac{\partial F_\varphi}{\partial z}\right] + \hat{\varphi}\left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right] + \hat{z}\frac{1}{r}\left[\frac{\partial(rF_\varphi)}{\partial r} - \frac{\partial F_r}{\partial\varphi}\right]$$

(a)


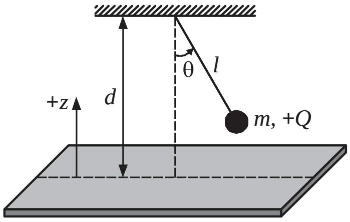
(b)


(c)


(d)


9. For a finite system of Fermions where the density of states increases with energy, the chemical potential
(a) increases with temperature.
(b) decreases with temperature.
(c) corresponds to the energy where the occupation probability is 0.5.
(d) does not vary with temperature.

10. Consider a point charge $+Q$ of mass m suspended by a massless, inextensible string of length l in free space (permittivity ε_0) as shown in the figure. It is placed at a height $d(d > 1)$ over an infinitely large, grounded conducting plane. The gravitational potential energy is assumed to be zero at the position of the conducting plane and is positive above the plane.



If θ represents the angular position and p_θ its corresponding canonical momentum, then the correct Hamiltonian of the system is

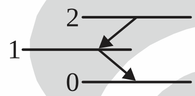
(a) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{8\pi\varepsilon_0(d - l\cos\theta)} + mg(d - l\cos\theta)$

(b) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{8\pi\varepsilon_0(d - l\cos\theta)} - mg(d - l\cos\theta)$

(c) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{16\pi\varepsilon_0(d - l\cos\theta)} + mg(d - l\cos\theta)$

(d) $\dfrac{p_\theta^2}{2ml^2} - \dfrac{Q^2}{16\pi\varepsilon_0(d - l\cos\theta)} - mg(d - l\cos\theta)$

11. Consider the atomic system as shown in the figure, where the Einstein A co-efficients for spontaneous emission for the levels are $A_{2\to1} = 2\times10^7\ \text{sec}^{-1}$ and $A_{1\to0} = 10^8\ \text{sec}^{-1}$. If 10^{14} atoms/cm^3 are excited from level 0 to level 2 and a steady state population in level 2 is achieved, then the steady state population at level 1 will be $x \times 10^{13}\ \text{cm}^{-3}$. The value of x (in integer) is __________.



12. If $\vec{a}$ and $\vec{b}$ are constant vectors, $\vec{r}$ and $\vec{p}$ are generalized positions and conjugate momenta, respectively, then for the transformation $Q = \vec{a}\cdot\vec{p}$ and $P = \vec{b}\cdot\vec{r}$ to be canonical, the value of $\vec{a}\cdot\vec{b}$ (in integer) is ________.

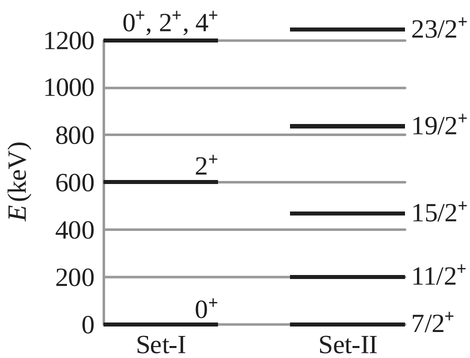
13. Consider an atomic gas with number density $n = 10^{20}\ \text{m}^{-3}$, in the ground state at 300 K. The valence electronic configuration of atoms is f^7. The paramagnetic susceptibility of the gas $\chi = m \times 10^{-11}$. The value of m (rounded off to two decimal places) is _________.

(Given: Magnetic permeability of free space $\mu_0 = 4\pi\times10^{-7}\ \text{H m}^{-1}$

Bohr magneton $\mu_B = 9.274\times10^{-24}\ \text{A m}^2$

Boltzmann constant $k_B = 1.3807\times10^{-23}\ \text{J K}^{-1}$)

14. For the given sets of energy levels of nuclei X and Y whose mass number are odd and even, respectively, choose the best suited interpretation.



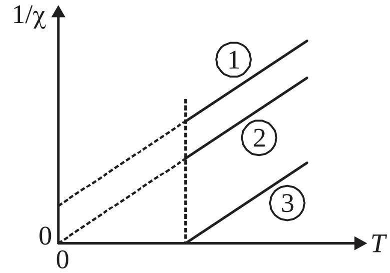
(a) Set-I: Vibrational band of X
Set-II: Rotational band of Y

(b) Set-I: Rotational band of Y
Set-II: Vibrational band of X

(c) Set-I: Rotational band of X
Set-II: Vibrational band of Y

(d) Set-I: Vibrational band of Y
Set-II: Rotational band of X

15. As shown in the figure, inverse magnetic susceptibility $(1/\chi)$ is plotted as a function of temperature (T) for three different materials in paramagnetic states.



(Curie temperature of ferromagnetic material $= T_C$)

Neel temperature of antiferromagnetic material $= T_N$)
Choose the correct statement from the following.

(a) Material 1 is ferromagnetic $(T < T_C)$, 2 is paramagnetic, and 3 is antiferromagnetic $(T < T_N)$.

(b) Material 1 is paramagnetic, 2 is antiferromagnetic $(T < T_N)$ and 3 is ferromagnetic $(T < T_C)$.

(c) Material 1 ferromagnetic $(T < T_C)$, 2 is antiferromagnetic $(T < T_N)$, and 3 is paramagnetic.

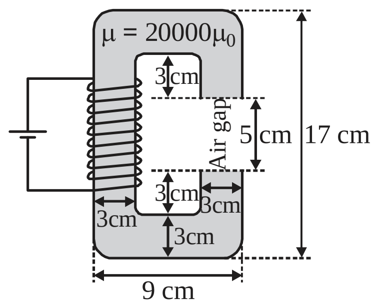
(d) Material 1 is antiferromagnetic $(T < T_N)$, 2 is paramagnetic, and 3 is ferromagnetic $(T < T_C)$.

16. The normalized radial wave function of the second excited state of Hydrogen atom is

$$R(r) = \frac{1}{\sqrt{24}} (a^{-3/2}) \frac{r}{a} (e^{-r/2a})$$

where a is the Bohr radius and r is the distance from the centre of the atom. The distance at which the electron is most likely to be found is $y \times a$. The value of y (in integer) is ________.

17. Consider a cross-section of an electromagnet having an air-gap of 5 cm as shown in the figure. It consists of a magnetic material $(\mu = 20000\,\mu_0)$ and is driven by a coil having $NI = 10^4$ A, where N is the number of turns and I is the current in Ampere.



Ignoring the fringe fields, the magnitude of the magnetic field $\vec{B}$ (in Tesla, rounded off to two decimal places) in the air-gap between the magnetic poles is ________.

18. A system of two atoms can be in three quantum states having energies 0, ε and 2ε. The system is in equilibrium at temperature $T = (k_B\beta)^{-1}$. Match the following **Statistics** with the **Partition function**.

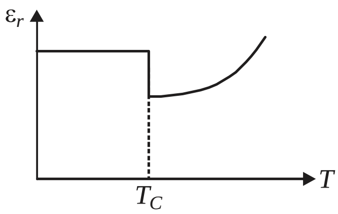
Statistics	Partition function
CD : Classical (distinguishable particles)	**Z1 :** $e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}$
CI : Classical (indistinguishable particles)	**Z2 :** $1+e^{-\beta\varepsilon}+2e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}+e^{-4\beta\varepsilon}$
FD : Fermi-Dirac	**Z3 :** $1+2e^{-\beta\varepsilon}+3e^{-2\beta\varepsilon}+2e^{-3\beta\varepsilon}+e^{-4\beta\varepsilon}$
BE : Bose-Einstein	**Z4 :** $\frac{1}{2}+e^{-\beta\varepsilon}+\frac{3}{2}e^{-2\beta\varepsilon}+e^{-3\beta\varepsilon}+\frac{1}{2}e^{-4\beta\varepsilon}$

(a) CD:Z1, CI:Z2, FD:Z3, BE:Z4 (b) CD:Z2, CI:Z3, FD:Z4, BE:Z1

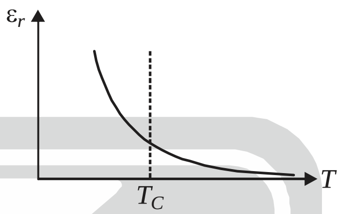
(c) CD:Z4, CI:Z1, FD:Z2, BE:Z3 (d) CD:Z3, CI:Z4, FD:Z1, BE:Z2

19. Choose the graph that best describes the variation of dielectric constant (ε_r) with temperature (T) in a ferro-electric material. (T_C is the Curie temperature)

(a)



(b)



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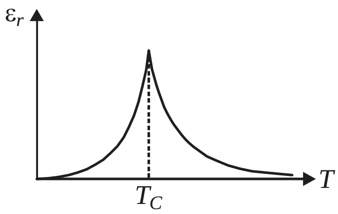
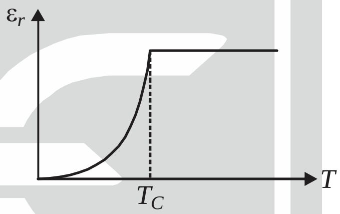


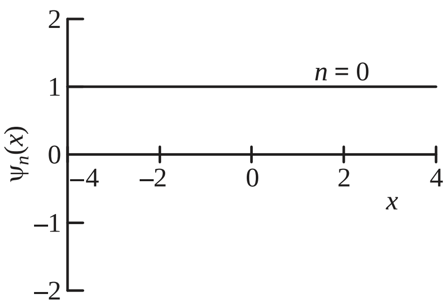
(d)



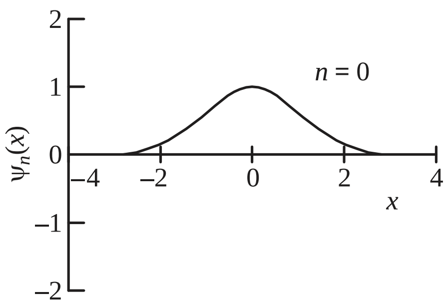
20. The spin-orbit effect splits the $^2\text{P} \rightarrow {}^2\text{S}$ transition (wavelength, $\lambda = 6521$ Å) in Lithium into two lines with separation of $\Delta\lambda = 0.14$ Å. The corresponding positive value of energy difference between the above two lines, in eV, is $m \times 10^{-5}$. The value of m (rounded off to the nearest integer) is ________.
[Given: Planck's constant, $h = 4.125 \times 10^{-15}$ eV-sec, Speed of light, $c = 3 \times 10^{8}$ m sec^{-1}]

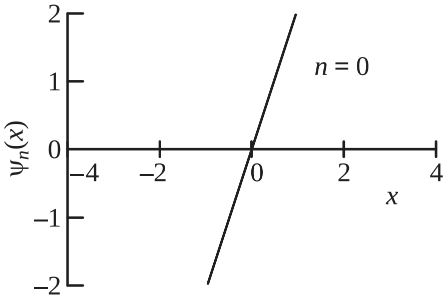
21. If $y_n(x)$ is a solution of the differential equation $y'' - 2xy' + 2ny = 0$, where n is an integer and the prime ($'$) denotes differentiation with respect to x, then acceptable plot(s) of $\psi_n(x) = e^{-x^2/2}\, y_n(x)$ is(are)

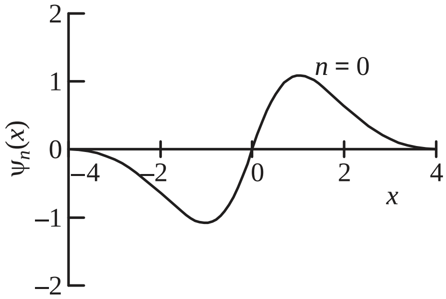
(a)



(b)



(c) 

(d) 

22. Consider a spin $S = \hbar/2$ particle in the state $|\phi\rangle = \frac{1}{3}\begin{bmatrix} 2+i \\ 2 \end{bmatrix}$. The probability that a measurement finds the state with $S_x = +\hbar/2$ is

(a) 17/18 (b) 5/18 (c) 15/18 (d) 11/18

23. Consider a state described by $\psi(x,t) = \psi_2(x,t) + \psi_4(x,t)$, where $\psi_2(x,t)$ and $\psi_4(x,t)$ are respectively the second and fourth normalized harmonic oscillator wave functions and ω is the angular frequency of the harmonic oscillator. The wave function $\psi(x, t=0)$ will be orthogonal to $\psi(x,t)$ at time t equal to

(a) $\frac{\pi}{2\omega}$ (b) $\frac{\pi}{4\omega}$ (c) $\frac{\pi}{6\omega}$ (d) $\frac{\pi}{\omega}$

24. Choose the correct statement from the following.
(a) Gallium Arsenide is an indirect band gap semiconductor.
(b) Conductivity of semiconductor decreases with increase in temperature.
(c) Silicon is a direct band gap semiconductor.
(d) Conductivity of metals decreases with increase in temperature.

25. A two-dimensional square lattice has lattice constant a.
k represents the wavevector in reciprocal space. The coordinates (k_x, k_y) of reciprocal space where band gap(s) can occur, are

(a) $\left(\pm\frac{\pi}{a}, \pm\frac{\pi}{1.3a}\right)$ (b) $\left(\pm\frac{\pi}{a}, \pm\frac{\pi}{a}\right)$ (c) $\left(\pm\frac{\pi}{3a}, \pm\frac{\pi}{a}\right)$ (d) (0, 0)

Q.26 – Q.55 : Carry TWO marks each.

26. A matter wave is represented by the wave function:

$$\psi(x, y, z, t) = Ae^{i(4x+3y+5z-10\pi t)}$$

where A is a constant. The unit vector representing the direction of propagation of this matter wave is

(a) $\frac{4}{5\sqrt{2}}\hat{x} + \frac{3}{5\sqrt{2}}\hat{y} + \frac{1}{\sqrt{2}}\hat{z}$ (b) $\frac{1}{5\sqrt{2}}\hat{x} + \frac{3}{5\sqrt{2}}\hat{y} + \frac{1}{\sqrt{2}}\hat{z}$

(c) $\frac{3}{5\sqrt{2}}\hat{x} + \frac{4}{5\sqrt{2}}\hat{y} + \frac{1}{5\sqrt{2}}\hat{z}$ (d) $\frac{1}{5\sqrt{2}}\hat{x} + \frac{4}{5\sqrt{2}}\hat{y} + \frac{3}{5\sqrt{2}}\hat{z}$

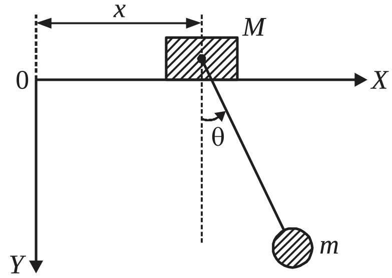
27. A material is placed in a magnetic field intensity $\vec{H}$. As a result, bound current density $\vec{J}_b$ is induced and magnetization of the material is $\vec{M}$. The magnetic flux density is $\vec{B}$. Choose the correct option(s) valid at the *surface* of the material.

(a) $\vec{\nabla}\cdot\vec{H} = 0$ (b) $\vec{\nabla}\cdot\vec{B} = 0$ (c) $\vec{\nabla}\cdot\vec{J}_b = 0$ (d) $\vec{\nabla}\cdot\vec{M} = 0$

28. Assume that $^{13}\text{N}\,(Z=7)$ undergoes first forbidden β^+ decay from its ground state with spin-parity J_i^π, to a final state J_f^π. The possible values for J_i^π and J_f^π, respectively, are

(a) $\frac{1^-}{2}, \frac{1^-}{2}$ (b) $\frac{1^-}{2}, \frac{5^+}{2}$ (c) $\frac{1^+}{2}, \frac{5^+}{2}$ (d) $\frac{1^+}{2}, \frac{1^-}{2}$

29. A uniform block of mass M slides on a smooth horizontal bar. Another mass m is connected to it by an inextensible string of length l of negligible mass, and is constrained to oscillate in the X-Y plane only. Neglect the sizes of the masses. The number of degrees of freedom of the system is two and the generalized coordinates are chosen as x and θ as shown in the figure.



If p_x and p_θ are the generalized momenta corresponding to x and θ, respectively, then the correct option(s) is(are)

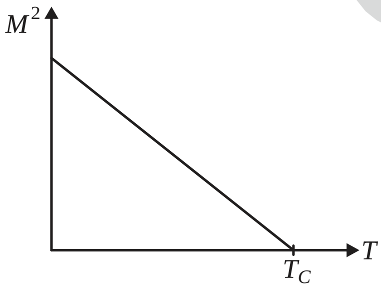
(a) p_θ is conserved (b) $p_\theta = ml^2\dot{\theta} - ml\cos\theta\,\dot{x}$

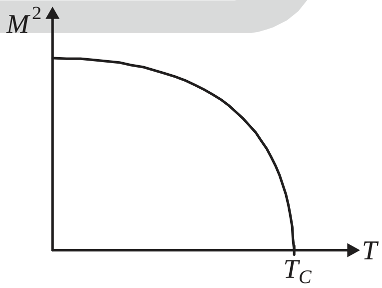
(c) p_x is conserved (d) $p_x = (m+M)\dot{x} + ml\cos\theta\,\dot{\theta}$

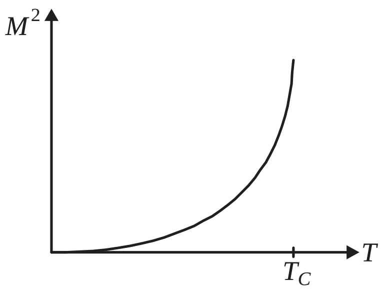
30. In a semiconductor, the ratio of the effective mass of hole to electron is 2:11 and the ratio of average relaxation time for hole to electron is 1 : 2. The ratio of the mobility of the hole to electron is

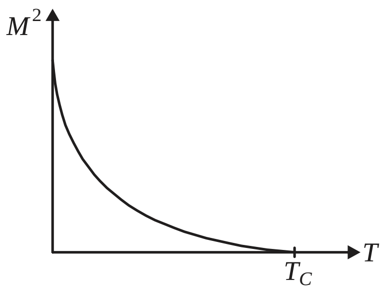
(a) 4 : 9 (b) 11 : 4 (c) 4 : 11 (d) 9 : 4

31. The free energy of a ferromagnet is given by $F = F_0 + a_0(T - T_C)M^2 + bM^4$, where F_0, a_0 and b are positive constants, M is the magnetization, T is the temperature, and T_C is the Curie temperature. The relation between M^2 and T is best depicted by

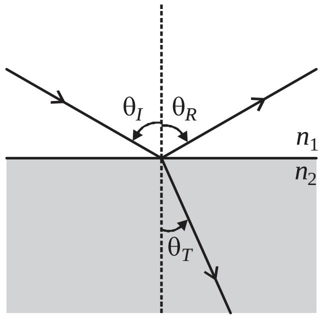
(a)


(b)


(c)


(d)


32. As shown in the figure, an electromagnetic wave with intensity I_I is incident at the interface of two media having refractive indices $n_1 = 1$ and $n_2 = \sqrt{3}$. The wave is reflected with intensity I_R and transmitted with intensity I_T. Permeability of each medium is the same. (Reflection coefficient $R = I_R / I_I$ and Transmission coefficient $T = I_T / I_I$).



Choose the correct statement(s).

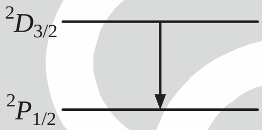
(a) $T = 1$ if $\theta_I = 60^\circ$ and polarization of incident light is perpendicular to the plane of incidence.

(b) $R = 0$ if $\theta_I = 60^\circ$ and polarization of incident light is perpendicular to the plane of incidence.

(c) $R = 0$ if $\theta_I = 0^\circ$ and polarization of incident light is parallel to the plane of incidence.

(d) $T = 1$ if $\theta_I = 60^\circ$ and polarization of incident light is parallel to the plane of incidence.

33. The transition line, as shown in the figure, arises between ${}^2D_{3/2}$ and ${}^2P_{1/2}$ states without any external magnetic field. The number of lines that will appear in the presence of a weak magnetic field (in integer) is________.



34. Consider a single one-dimensional harmonic oscillator of angular frequency ω, in equilibrium at temperature $T = (k_B \beta)^{-1}$. The states of the harmonic oscillator are all non-degenerate having energy $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ with equal probability, where n is the quantum number. The Helmholtz free energy of the oscillator is

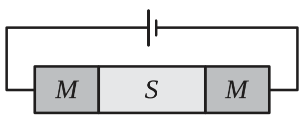
(a) $\frac{\hbar\omega}{2} + \beta^{-1} \ln\left[1 - \exp(-\beta\hbar\omega)\right]$

(b) $\frac{\hbar\omega}{2} + \beta^{-1} \ln\left[1 + \exp(-\beta\hbar\omega)\right]$

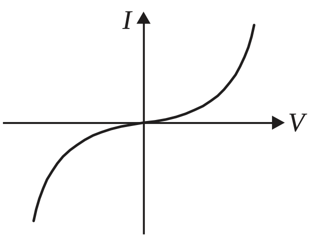
(c) $\frac{\hbar\omega}{2} + \beta^{-1} \ln\left[1 - \exp(\beta\hbar\omega)\right]$

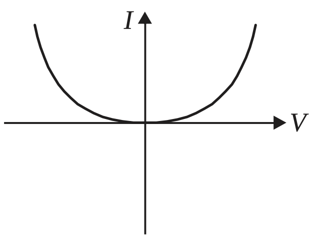
(d) $\beta^{-1} \ln\left[1 - \exp(-\beta\hbar\omega)\right]$

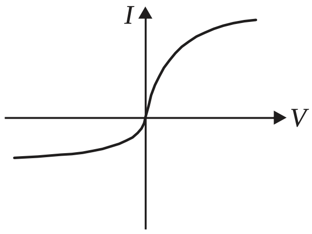
35. As shown in the figure, two metal-semiconductor junctions are formed between an n-type semiconductor S and metal M. The work functions of S and M are φ_S and φ_M, respectively with $\varphi_M > \varphi_S$.

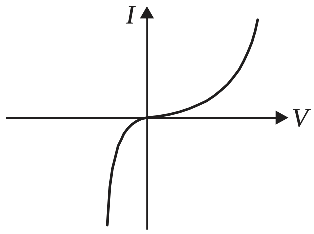


The I-V characteristics (on linear scale) of the junctions is best represented by

(a)


(b)


(c) 

(d) 

36. Two observers O and O' observe two events P and Q. The observers have a constant relative speed of $0.5c$. In the units, where the speed of light, c, is taken as unity, the observer O obtained the following coordinates:

Event P: $x = 5,\ y = 3,\ z = 5,\ t = 3$

Event Q: $x = 5,\ y = 1,\ z = 3,\ t = 5$

The length of the space-time interval between these two events, as measured by O', is L. The value of $|L|$ (in integer is ________.

37. P and Q are two Hermitian matrices and there exists a matrix R, which diagonalizes both of them, such that $RPR^{-1} = S_1$ and $RQR^{-1} = S_2$, where S_1 and S_2 are diagonal matrices. The correct statement(s) is/are

(a) The matrix QP can have complex eigenvalues.

(b) The matrices P and Q commute.

(c) All the elements of both matrices S_1 and S_2 are real.

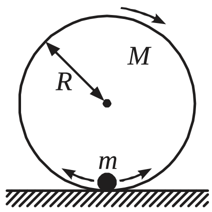
(d) The matrix PQ can have complex eigenvalues.

38. A linear charged particle accelerator is driven by an alternating voltage source operating at 10 MHz. Assume that it is used to accelerate electrons. After a few drift-tubes, the electrons attain a velocity $2.9 \times 10^8\ \text{ms}^{-1}$. The minimum length of each drift-tube, in m, to accelerate the electrons further (rounded off to one decimal place) is ________.

39. The Coulomb energy component in the binding energy of a nucleus is 18.432 MeV. If the radius of the uniform and spherical charge distribution in the nucleus is 3 fm, the corresponding atomic number (rounded off to the nearest integer) is ________.

$$\left[\text{Given}: \frac{e^2}{4\pi\varepsilon_0} = 1.44\ \text{MeV fm}\right]$$

40. A hoop of mass M and radius R rolls without slipping along a straight line on a horizontal surface as shown in the figure. A point mass m slides without friction along the inner surface of the hoop, performing small oscillations about the mean position. The number of degrees of freedom of the system (in integer) is ________.



41. The time derivative of a differentiable function $g(q_i, t)$ is added to a Lagrangian $L(q_i, \dot{q}_i, t)$ such that

$$L' = L(q_i, \dot{q}_i, t) + \frac{d}{dt} g(q_i, t)$$

where $q_i, \dot{q}_i, t$ are the generalized coordinates, generalized velocities and time, respectively. Let p_i be the generalized momentum and H the Hamiltonian associated with $L(q_i, \dot{q}_i, t)$. If p_i' and H' are those associated with L', then the correct option(s) is/are:

(a) Both L and L' satisfy the Euler-Lagrange's equations of motion.

(b) If p_i is conserved, then p_i' is necessarily conserved.

(c) $p_i' = p_i + \frac{\partial}{\partial q_i} g(q_i, t)$

(d) $H' = H + \frac{d}{dt} g(q_i, t)$

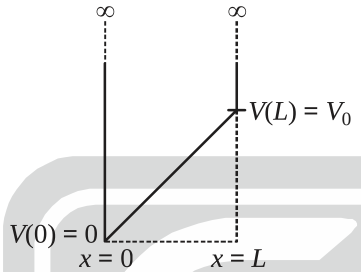
42. Among the term symbols

$^4S_1$, $^2D_{7/2}$, $^3S_1$ and $^2D_{5/2}$

Choose the option(s) possible in the LS coupling notation.

(a) $^2D_{7/2}$ (b) $^4S_1$ (c) $^3S_1$ (d) $^2D_{5/2}$

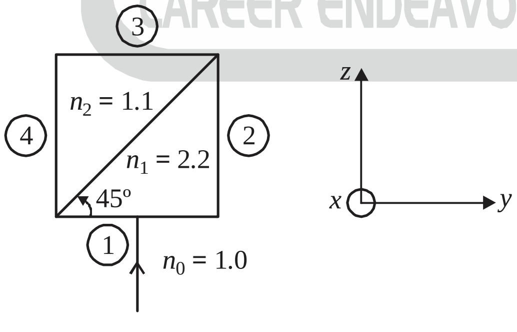
43. Consider a particle in a one-dimensional infinite potential well with its walls at $x = 0$ and $x = L$. The system is perturbed as shown in the figure.



The first order correction to the energy eigenvalue is

(a) $V_0/4$ (b) $V_0/2$ (c) $V_0/5$ (d) $V_0/3$

44. An electromagnetic wave having electric field $\vec{E} = 8\cos(kz - \omega t)\,\hat{y}$ V cm^{-1} is incident at 90º (normal incidence) on a square slab from vacuum (with refractive index $n_0 = 1.0$) as shown in the figure. The slab is composed of two different material with refractive indices n_1 and n_2. Assume that the permeability of each medium is the same. After passing through the slab for the first time, the electric field amplitude, in V cm^{-1}, of the electromagnetic wave, which emerges from the slab in region 2, is closest to



(a) $\frac{11}{3.2}$ (b) $\frac{11}{25.6}$ (c) $\frac{11}{13.8}$ (d) $\frac{11}{1.6}$

45. For a two-nucleon system in spin singlet state, the spin is represented through the Pauli matrices σ_1, σ_2 for particles 1 and 2, respectively. The value of $(\sigma_1 \cdot \sigma_2)$ (in integer) is _________.

46. In an experiment, it is seen that an electric-dipole $(E1)$ transition can connect an initial nuclear state of spin-parity $J_i^\pi = 2^+$ to final state J_f^π. All possible values of J_f^π are

(a) $1^+, 2^+, 3^+$ (b) $1^-, 2^-$ (c) $1^-, 2^-, 3^-$ (d) $1^+, 2^+$

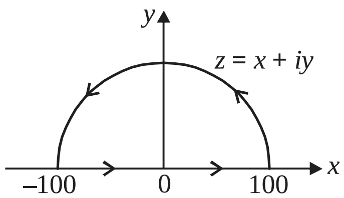
47. Three non-interacting bosonic particles of mass m each, are in a one-dimensional infinite potential well of width 'a'. The energy of the third excited state of the system is $x \times \frac{\hbar^2 \pi^2}{ma^2}$. The value of x (in integer) is ________.

48. The spacing between two consecutive S-branch lines of the rotational Raman spectra of hydrogen gas is 243.2 cm^{-1}. After excitation with a laser of wavelength 514.5 nm, the Stoke's line appeared at 17611.4 cm^{-1} for a particular energy level. The wavenumber (rounded off to the nearest integer), in cm^{-1}, at which Stoke's line will appear for the next higher energy level is _________.

49. A contour integral is defined as:

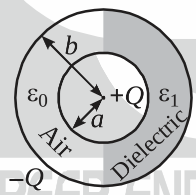
$$I_n = \oint_C \frac{dz}{(z-n)^2 + \pi^2}$$

where n is a positive integer and C is the closed contour, as shown in the figure, consisting of the line from –100 to 100 and the semicircle traversed in the counter-clockwise sense.



The value of $\sum_{n=1}^{5} I_n$ (in integer) is __________.

50. Consider two concentric conducting spherical shells as shown in the figure. The inner shell has a radius 'a' and carries a charge $+Q$. The outer shell has a radius 'b' and carries a charge $-Q$. The empty space between them is half-filled by a hemispherical shell of a dielectric having permittivity ε_1. The remaining space between the shells is filled with air having the permittivity ε_0.



The electric field at a radial distance r from the center and between the shells ($a < r < b$) is

(a) $\frac{Q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$ on the air side and $\frac{Q}{4\pi\varepsilon_1}\frac{\hat{r}}{r^2}$ on the dielectric side.

(b) $\frac{Q}{2\pi(\varepsilon_0+\varepsilon_1)}\frac{\hat{r}}{r^2}$ everywhere.

(c) $\frac{Q}{2\pi\varepsilon_0}\frac{\hat{r}}{r^2}$ on the air side and $\frac{Q}{2\pi\varepsilon_1}\frac{\hat{r}}{r^2}$ on the dielectric side.

(d) $\frac{Q}{4\pi(\varepsilon_0+\varepsilon_1)}\frac{\hat{r}}{r^2}$ everywhere.

51. A function $f(t)$ is defined only for $t \geq 0$. The Laplace transform of $f(t)$ is

$$L(f;s) = \int_0^\infty e^{-st} f(t)\,dt$$

whereas the Fourier transform of $f(t)$ is

$$\tilde{f}(\omega) = \int_0^\infty f(t)\,e^{-i\omega t}\,dt$$

The correct statement(s) is/are:

(a) $L(f;s)$ and $\tilde{f}(\omega)$ can be made connected.

(b) The variable s is always real.

(c) The variable s can be complex.

(d) $L(f;s)$ and $\tilde{f}(\omega)$ can never be made connected.

52. The spin $\vec{S}$ and orbital angular momentum $\vec{L}$ of an atom precess about $\vec{J}$, the total angular momentum. $\vec{J}$ precesses about an axis fixed by a magnetic field $\vec{B}_1 = 2B_0\hat{z}$, where B_0 is a constant. Now, the magnetic field is changed to $\vec{B}_2 = B_0\left(\hat{x} + \sqrt{2}\hat{y} + \hat{z}\right)$. Given the orbital angular momentum quantum number $l = 2$ and spin quantum number $s = 1/2$, θ is the angle between $\vec{B}_1$ and $\vec{J}$ for the largest possible values of total angular quantum number j and its z-component j_z. The value of θ (in degree, rounded off to the nearest integer) is ________.

53. The donor concentration in a sample of n-type silicon is increased by a factor of 100. Assuming the sample to be non-degenerate, the shift in the Fermi level (in meV) at 300 K (rounded off to the nearest integer) is ________.

$\left[\text{Given}: k_B T = 25 \text{ meV at } 300 \text{ K}\right]$

54. The Gell-Mann – Okuba mass formula defines the mass of baryons as $M = M_0 + aY + b\left[I(I+1) - \frac{1}{4}Y^2\right]$, where M_0, a and b are constants. I represents the isospin and Y represents the hypercharge. If the mass of Σ hyperons is same as that of Λ hyperons, then the correct option(s) is(are)

(a) $M \propto Y$

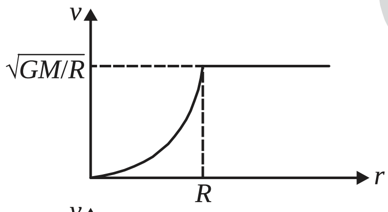
(b) $M \propto I(I+1)$

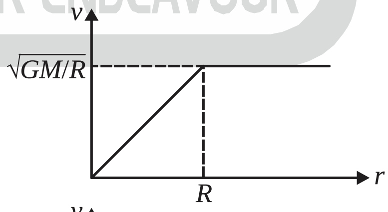
(c) M does not depend on I

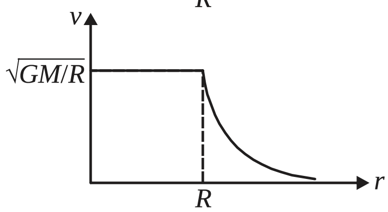
(d) M does not depend on Y

55. Consider a spherical galaxy of total mass M and radius R, having a uniform matter distribution. In this idealized situation, the orbital speed v of a star of mass $m\,(m \ll M)$ as a function of the distance r from the galactic centre is best described by

(*G is the universal gravitational constant*)

(a)


(b)


(c)


(d)