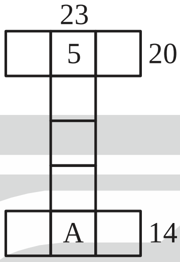
NTA-JOINT CSIR-UGC NET – June-2024

PHYSICAL SCIENCES

PART - A (General Aptitude)

1. The squares in the following grid are filled with numbers 1 to 9, without repetition, such that the numbers in the squares forming the top and bottom rows add to 20 and 14 respectively and those forming the column to 23. What is the value of A ?



(a) 4 (b) 6 (c) 7 (d) 8

2. The population of a town is increasing at a uniform rate. If its population was 90,000 and 96,000 in 2022 and 2023 respectively, what would be its population in 2024 ?

(a) 102,000 (b) 102,400 (c) 102,720 (d) 102,960

3. Canals A and B join to form canal C, all having semi-circular cross-sections of radii which are in the ratio 3:4:5, respectively. Assume smooth merger of A and B, and ignore the possibility of flooding. If the speed 's' of water is the same and uniform in both A and B, then the speed of water flowing in C is

(a) s (b) $7s/5$ (c) $2s$ (d) $5s/7$

4. How many three-digit numbers exist whose first and last digits add up to 9 ?

(a) 90 (b) 81 (c) 80 (d) 72

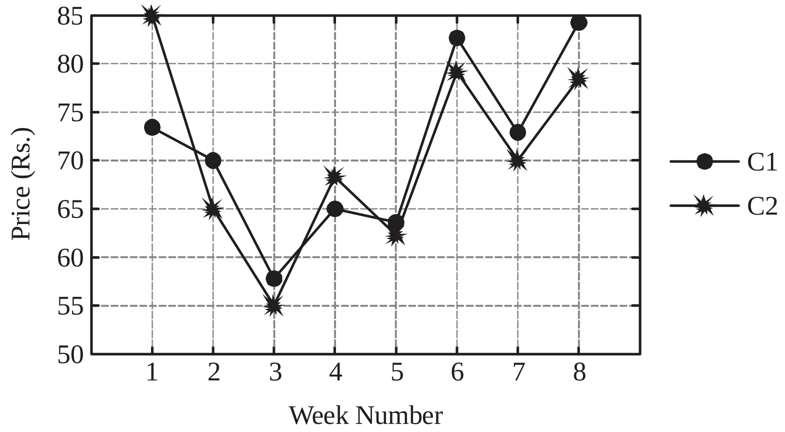
5. An egg tray has 30 cavities to hold eggs in 5 rows and 6 columns. Each cavity is surrounded by 4 raised corners shared by adjacent cavities. How many raised corners does the egg tray have ?

(a) 30 (b) 35 (c) 36 (d) 42

6. In how many distinct ways can 128 identical marbles be arranged in a complete rectangular grid (disregarding the orientation of the grid) ?

(a) 7 (b) 6 (c) 5 (d) 4

7. The two graphs show the change in price of two commodities C1 and C2 over 8 weeks.



Which of the statements is correct ?

(a) C1 has higher fluctuation than C2.

(b) Average price of C1 is lower than that of C2.

(c) The largest change in a week is shown by C2.

(d) C1 shows a tendency of reduction.

8. Suppose that the increase in a population can be modelled as:

$$\left(\frac{dN}{dt}\right) = rN\frac{(K-N)}{K},$$

where N is the size of the population, K is the carrying capacity, r is the per capita growth rate and t is time. Which of the following statements is correct ?

(a) When $N \approx 0$, the change in population N is nearly exponential.

(b) When $N = K$, the population goes extinct as dN/dt goes to zero.

(c) When $N \approx 0$, the population growth dN/dt is maximum.

(d) When $N \approx K/4$, the population growth dN/dt is maximum.

9. Among 1000 squirrel babies, 200 have three stripes on their back, 500 have two stripes on their back and the rest have four stripes on their back. While 90% of the three-striped babies survive to adulthood, only 80% of the two-striped and 70% of the four-striped babies survive to adulthood. The fraction of four-striped squirrels among the adults is nearest to

(a) 0.21 (b) 0.3 (c) 0.266 (d) 0.228

10. Among A, B, C, D, E and F, D is taller than B but shorter than F. E is taller than B, but shorter than C. B is not the shortest of all Then A is —

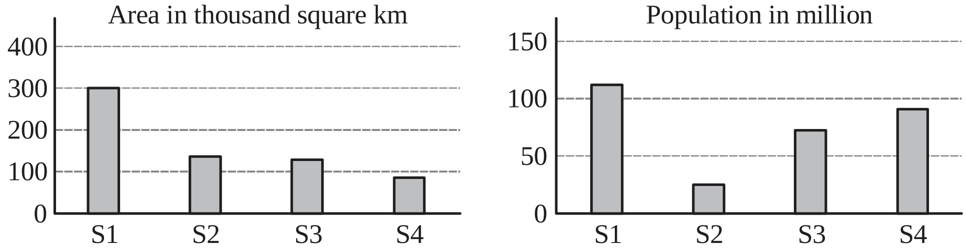
(a) the shortest of all. (b) the tallest of all.

(c) taller than E, but shorter than C. (d) taller than C, but shorter than F.

11. A referendum on a proposal involved 7000 participants. Among the participants 3600 were women and the rest were men. 2900 participants, of whom 1300 were women, voted against while 3000 participants voted in favour. 400 women abstained. The ratio of the number of men that did not vote to the total number of participants is

(a) 11 : 70 (b) 17 : 35 (c) 1 : 10 (d) 8 : 70

12. Areas and populations of four states S1, S2, S3 and S4 are shown.



Their arrangement **in decreasing order** of population density would be

(a) S4, S3, S1, S2 (b) S1, S2, S3, S4 (c) S4, S1, S3, S2 (d) S2, S1, S3, S4

13. If 32XY6 is divisible by 9, X and Y being even decimal digits, then X =

(a) 2 (b) 4 (c) 6 (d) 8

14. A rectangular tray of 30 cm × 60 cm size is used for baking circular biscuits. The diameter of each biscuit is 3 cm before baking, which increases by 10% on baking. What is the maximum number of biscuits that can be baked in the tray such that the base of each biscuit is in contact with the tray ?

(a) 171 (b) 162 (c) 180 (d) 200

15. On a one-way road, broken lines consisting of 2.5 m length segments separated by 2.5 m gaps are painted along the length of the road to demarcate 3 lanes, and continuous lines are painted along both the borders. What is the total length of painted lines (in m) over a 250 m stretched of the road ?

(a) 500 (b) 625 (c) 750 (d) 1000

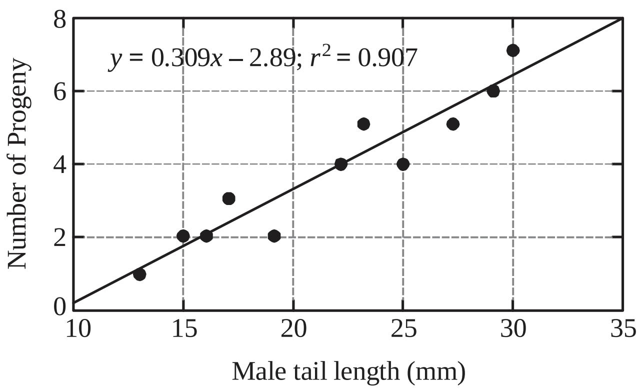
16. A patient requires administration of 500 ml of an intravenous fluid in 1 hour. What is the approximate drip rate (number of drops per minute) at which the fluid should be administered, if the volume of a drop is 0.05 ml?

(a) 76 (b) 152 (c) 167 (d) 332

17. A large number of birds, half of which belong to specie A and the other half to specie B, rest on a tree where they are distributed randomly across the branches. In a random sample of 5 birds from the tree, what is the probability that at least one is from specie A ?

(a) 0.03125 (b) 0.15625 (c) 0.84375 (d) 0.96875

18. The graph shows observations and a regression line of the number of progeny on the tail length of male birds.



Which of the following can be inferred from the graph ?

(a) Producing less progeny decreases the tail length of the males.

(b) Males cannot have a tail length lesser than 10 mm.

(c) Males with longer tails tend to father more progeny.

(d) For a male with a 25 mm tail, the expected number of progeny is 4.

19. In a class of 70 students, 20% of girls have spectacles and 40% of boys have spectacles. If the total number of students having spectacle is 23, the number of boys in the class is

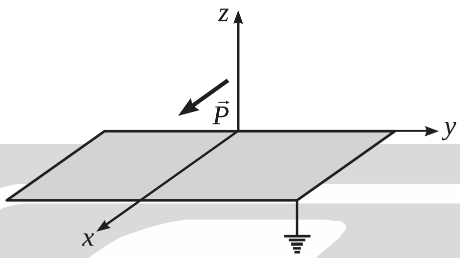
(a) 45 (b) 14 (c) 18 (d) 25

20. A record player stylus moves along a spiral groove cut on an annular portion of a disc with inner radius 4 cm and outer radius 10 cm. If the record turns 100 times when playing, the stylus travels approximately.

(a) 2.2 m (b) 4.4 m (c) 22 m (d) 44 m

PART - B (Physics)

1. A point electric dipole $\vec{P} = p_x \hat{i}$ is placed at a vertical distance *d* above a grounded infinite conducting *x*-*y* plane as shown in the figure.



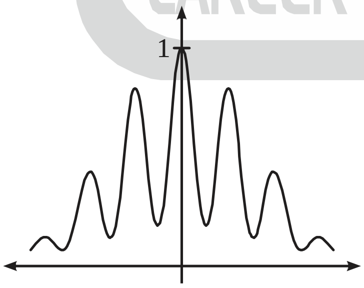
At a point $\vec{r}\,(r \gg d,\ z > 0)$ far away from the dipole, the electrostatic potential $V(r)$ varies approximately as:

(a) $\frac{1}{r^2}$ (b) $\frac{1}{r^6}$ (c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$

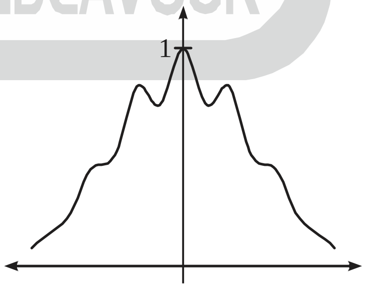
2. If $\vec{L}$ is the orbital angular angular momentum operator and $\vec{\sigma}$ are the Pauli matrices, which of the following operators commutes with $\vec{\sigma}\cdot\vec{L}$?

(a) $\vec{L} - \frac{\hbar}{2}\vec{\sigma}$ (b) $\vec{L} + \frac{\hbar}{2}\vec{\sigma}$ (c) $\vec{L} + \hbar\vec{\sigma}$ (d) $\vec{L} - \hbar\vec{\sigma}$

3. A finite sized light source is used in a double slit experiment. The observed intensity pattern changes from figure (A) to figure (B), as shown below:



(A)



(B)

The observed change can occur due to —

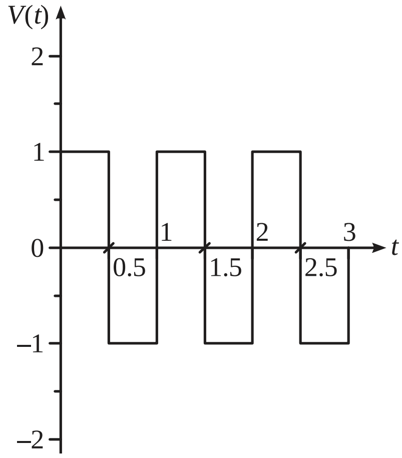
(a) narrowing of the slits.

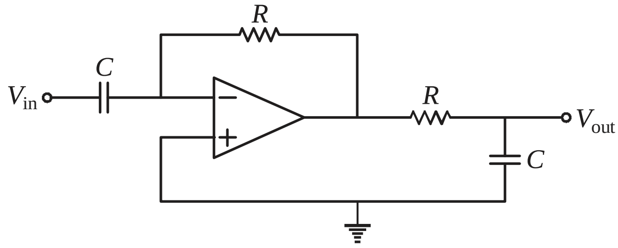
(b) a reduction in the distance between the slits.

(c) a decrease in the coherence length of the light source.

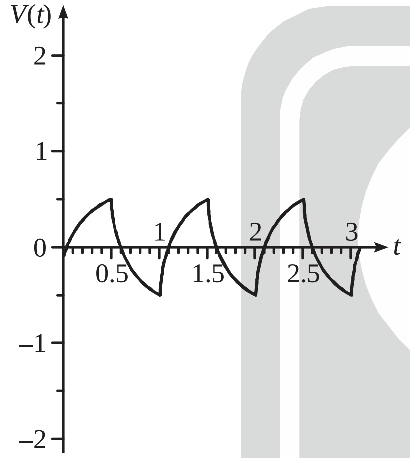
(d) a reduction in the size of the light source.

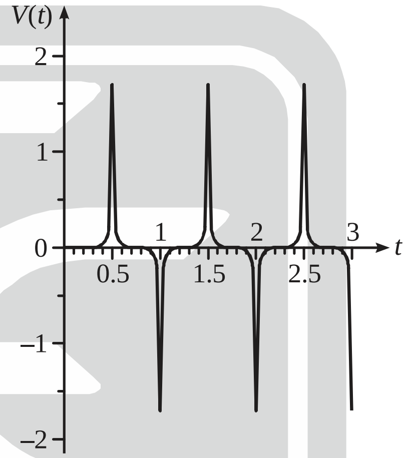
4. A train of square wave pulses is given to the input of an ideal op-amp circuit shown below:

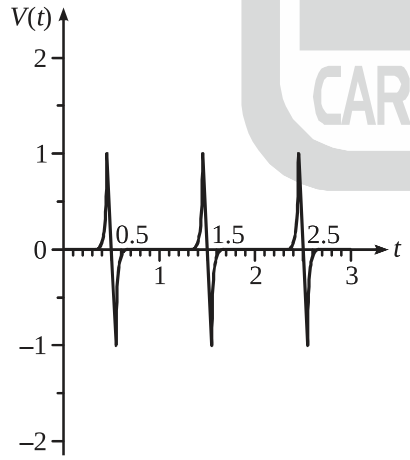


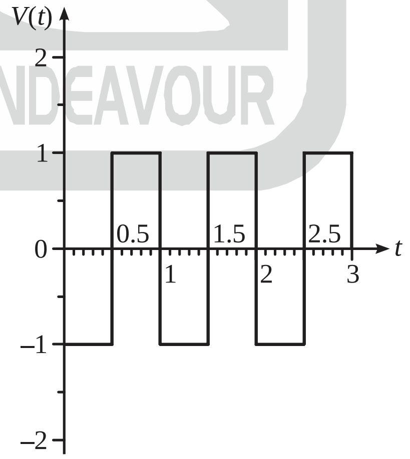


Given that the time period of the input pulses $T \ll RC$ and the op-amp does not get into saturation, which of the following best represents the output waveform ?

(a) 

(b) 

(c) 

(d) 

5. A body of mass m is acted upon by a central force $\vec{f}(\vec{r}) = -k\vec{r}$, where k is a positive constant. If the magnitude of the angular momentum is l, then the total energy for a circular orbit is

(a) $2\sqrt{\dfrac{kl^2}{m}}$ (b) $\dfrac{1}{2}\sqrt{\dfrac{kl^2}{m}}$ (c) $\dfrac{3}{2}\sqrt{\dfrac{kl^2}{m}}$ (d) $\sqrt{\dfrac{kl^2}{m}}$

6. If A and B are Hermitian operators and C is an antihermitian operators, then —

(a) $[[A, B], C]$ is hermitian and $[[A, C], B]$ is antihermitian.

(b) $[[A, B], C]$ and $[[A, C], B]$ are both antihermitian.

(c) $[[A, B], C]$ and $[[A, C], B]$ are both hermitian.

(d) $[[A, B], C]$ is antihermitian and $[[A, C], B]$ is hermitian.

7. The Hamiltonian for a one-dimensional simple harmonic oscillator is given by:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

The harmonic oscillator is in the state $|\psi\rangle = \frac{1}{\sqrt{1+\lambda^2}}\left(|1\rangle + \lambda e^{i\theta}|2\rangle\right)$, where $|1\rangle$ and $|2\rangle$ are the normalised first and second excited states of the oscillator and λ, θ are positive real constants. If the expectation value $\langle\psi|x|\psi\rangle = \beta\sqrt{\hbar/m\omega}$, the value of β is

(a) $\frac{1}{\sqrt{2}(1+\lambda^2)}$ (b) $\frac{\sqrt{2}\lambda\cos\theta}{1+\lambda^2}$ (c) $\frac{2\lambda\cos\theta}{1+\lambda^2}$ (d) $\frac{\lambda^2\cos\theta}{1+\lambda^2}$

8. Quantum particles of unit mass, in a potential

$$V(x) = \begin{cases} \frac{1}{2}\omega^2 x^2 & ; \quad x > 0 \\ \infty & ; \quad x \le 0 \end{cases}$$

are in equilibrium at a temperature T. Let n_2 and n_3 denote the numbers of the particles in the second and third excited states respectively. The ratio n_2/n_3 is given by:

(a) $\exp\left(\frac{2\hbar\omega}{k_B T}\right)$ (b) $\exp\left(\frac{\hbar\omega}{k_B T}\right)$ (c) $\exp\left(\frac{3\hbar\omega}{k_B T}\right)$ (d) $\exp\left(\frac{4\hbar\omega}{k_B T}\right)$

9. A battery with an open circuit voltage of 10 V is connected to a load resistor of $485\,\Omega$ and the voltage measured across the battery terminals using an ideal voltmeter is 9.7 V. The internal resistance of the battery is closest to

(a) $30\,\Omega$ (b) $15\,\Omega$ (c) $20\,\Omega$ (d) $40\,\Omega$

10. The matrix A is given by $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. The eigenvalues of $3A^3 + 5A^2 - 6A + 2I$, where I is the identity matrix, are

(a) 4, 9, 27 (b) 1, 9, 44 (c) 1, 110, 8 (d) 4, 110, 10

11. A quantum mechanical system is in the angular momentum state $|l = 4,\ l_z = 4\rangle$. The uncertainty in L_x is

(a) $\hbar\sqrt{2}$ (b) $2\hbar$ (c) 0 (d) $\hbar$

12. A hydrogen atom is in the state $|\psi\rangle = \sqrt{\frac{8}{21}}|\psi_{200}\rangle + \sqrt{\frac{3}{7}}|\psi_{210}\rangle + \sqrt{\frac{4}{21}}|\psi_{311}\rangle$, where $|\psi_{nlm}\rangle$ are normalised eigenstates. If $\hat{L}^2$ is measured in this state, the probability of obtaining the value $2\hbar^2$ is

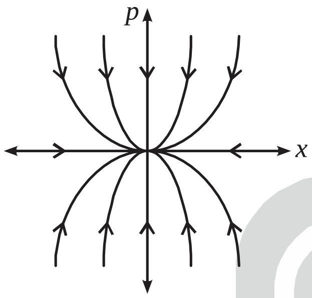
(a) $\frac{13}{21}$ (b) $\frac{4}{21}$ (c) $\frac{17}{21}$ (d) $\frac{3}{7}$

13. The evolution of the dynamical variables $x(t)$ and $p(t)$ is given by:

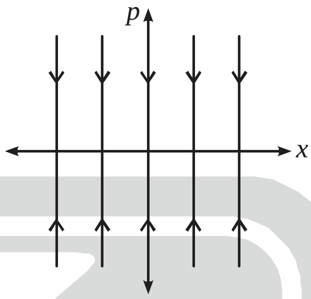
$$\dot{x} = ax \text{ and } \dot{p} = -p$$

where a is a constant. The trajectory in (x, p) space for $-1 < a < 0$ is best described by:

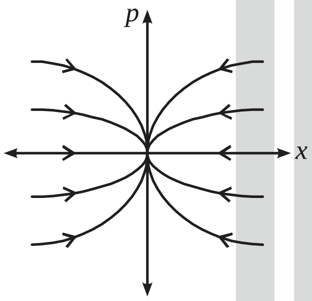
(a)



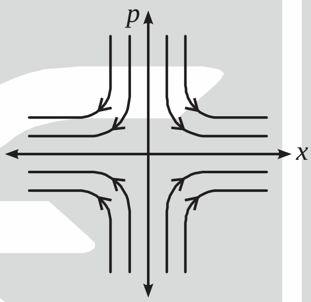
(b)



(c)



(d)



14. The electric field of an electromagnetic wave in free space is given by $\vec{E} = E_0 \sin(\omega t - k_z z)\hat{j}$. The magnetic field $\vec{B}$ vanishes for $t = \frac{k_z z}{\omega}$. The Poynting vector of the system is:

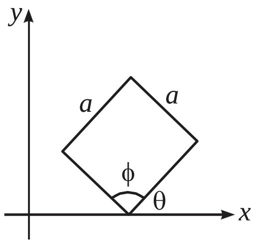
(a) $\frac{k_z}{2\mu_0\omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$ (b) $\frac{4k_z}{\mu_0\omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$

(c) $\frac{2k_z}{\mu_0\omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$ (d) $\frac{k_z}{\mu_0\omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$

15. Two non-interacting classical particles having masses m_1 and m_2 are moving in a one-dimensional box of length L. For total energy not exceeding a given value E, the phase space 'volume' is given by:

(a) $\pi L^2 E\left(\frac{m_1 m_2}{m_1 + m_2}\right)$ (b) $\pi L^2 E\sqrt{m_1 m_2}$ (c) $2\pi L^2 E\left(\frac{m_1 m_2}{m_1 + m_2}\right)$ (d) $2\pi L^2 E\sqrt{m_1 m_2}$

16. A square plate of dimension $a \times a$ makes an angle $\theta = (\pi/4)$ with the x-axis in its rest frame (S) as shown in the figure.



It is moving with a speed $v = \sqrt{\frac{2}{3}}c$ along the x-axis with respect to an observer S' (where c is the speed of light in vacuum). The value of the interior angle ϕ indicated in the figure (which is obviously $\pi/2$ in the frame S), as measured in S' is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{4\pi}{3}$

17. Probability density function of a variable x is given by:

$$P(x) = \frac{1}{2}\left[\delta(x-a) + \delta(x+a)\right]$$

The variance of x is

(a) a^2 (b) 0 (c) $2a^2$ (d) $a^2/2$

18. A single particle can exist in two states with energies 0 and E respectively. At high temperatures $(k_B T \gg E)$ the specific heat of the system (C_V) will be approximately.

(a) proportional to $1/T$ (b) proportional to $1/T^2$

(c) proportional to $e^{E/k_B T}$ (d) constant

19. A set of 100 data points yields an average $\bar{x} = 9$ and a standard deviation $\sigma_x = 4$. The error in the estimated mean is closest to

(a) 3.0 (b) 0.4 (c) 4.0 (d) 0.3

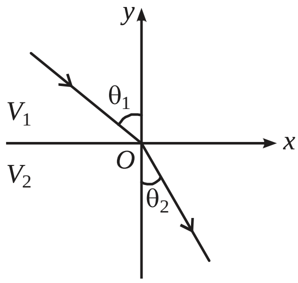
20. A uniform plane square sheet of mass m is centered at the origin of an inertial frame. The sheet is rotating about an axis passing through the origin. At an instant when all its vertices lie on x and y axes, the angular momentum is $\vec{L} = I_0\omega_0(2\hat{i} + \hat{j} + 2\hat{k})$, where I_0 is the moment of inertia about the x-axis. At this instant, the angular velocity of the sheet is

(a) $(2\hat{i} + \hat{j} + 2\hat{k})\omega_0$ (b) $(2\hat{i} + \hat{j} + \hat{k})\omega_0$ (c) $(2\hat{i} + \hat{j})\omega_0$ (d) $(\hat{i} + \hat{j})\omega_0$

21. An integral is given by: $\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \exp\left[-(x^2 + y^2 + 2axy)\right]$, where a is a real parameter. The full range of values of 'a' for which the integral is finite, is

(a) $-\infty < a < \infty$ (b) $-2 < a < 2$ (c) $-1 < a < 1$ (d) $-1 \le a \le 1$

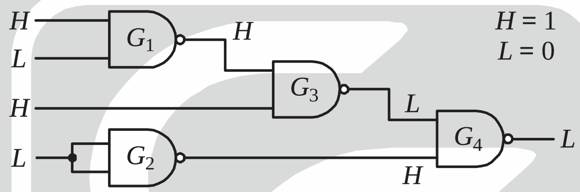
22. The region $y > 0$ has a constant electrostatic potential V_1 and $y < 0$ has a constant electrostatic potential $V_2 \neq V_1$. A charged particle with momentum $\vec{p}_1$ is incident at an angle θ_1 on the interface of the two regions (see figure below).



If the particle has momentum $\vec{p}_2$ in the region $y < 0$, then the angle θ_2 is given by:

(a) $\cos^{-1}\left(\frac{p_2}{p_1}\cos\theta_1\right)$ (b) $\cos^{-1}\left(\frac{p_1}{p_2}\cos\theta_1\right)$ (c) $\sin^{-1}\left(\frac{p_2}{p_1}\sin\theta_1\right)$ (d) $\sin^{-1}\left(\frac{p_1}{p_2}\sin\theta_1\right)$

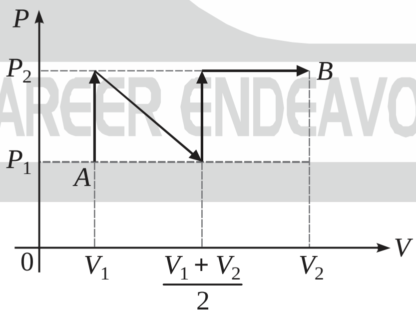
23. The logic levels H and L at different locations in a digital circuit are found to be as shown in the figure.



Based on these observations, which of the logic gates is not behaving as an ideal NAND gate ?

(a) G_2 (b) G_3 (c) G_4 (d) G_1

24. The following *P-V* diagram shows a process, where an ideal gas is taken quasi-statically from *A* to *B* along the path as shown in the figure.



The work done *W* in this process is

(a) $\frac{1}{4}(V_2 - V_1)(3P_2 + P_1)$ (b) $\frac{1}{4}(V_2 - V_1)(3P_2 - P_1)$

(c) $\frac{1}{2}(V_2 - V_1)(P_1 + P_2)$ (d) $\frac{1}{2}(V_2 + V_1)(P_2 - P_1)$

25. Vorticity of a vector field $\vec{B}$ is defined as $\vec{V} = \vec{\nabla} \times \vec{B}$. Given $\vec{B} = kxyz\hat{r}$, where *k* is a constant, which one of the following is ***correct***?

(a) Vorticity is a null vector for all finite x, y, z.
(b) Vorticity is parallel to the vector field everywhere.
(c) The angle between vorticity and vector field depends on x, y, z.
(d) Vorticity is perpendicular to the vector field everywhere.

PART - C (Physics)

1. π^- has spin 0 and negative intrinsic parity. In a reaction a deuteron in its ground state ($J=1$, parity is +1) captures a π^- in *p*-wave to produce a pair of neutrons (intrinsic parity is +1). The neutrons will be produced in a state with

(a) $l=1, S=0$ (b) $l=0, S=1$ (c) $l=1, S=1$ (d) $l=0, S=0$

2. Helium atom is excited to a state with the configuration $(2s2p)$ with an energy 58.3 eV. After some time, this atom spontaneously ejects a single electron. The value of the orbital angular momentum quantum number (l) of the ejected electron in the final state of the system is

[Ionization potential of He$(1s)^2$ is 24.6 eV]

(a) 1 (b) 0 (c) 2 (d) 3

3. The band dispersion of electrons in a two dimensional square lattice (lattice constant *a*) is given by:

$$E(k_x, k_y) = -2(t_x \cos k_x a + t_y \cos k_y a),$$

where $t_x, t_y > 0$. The effective mass tensor $m^* = \begin{pmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{pmatrix}$ of electrons at $\vec{k} = \left(\frac{\pi}{a}, \frac{\pi}{a}\right)$ is

(a) $\begin{pmatrix} 0 & \frac{\hbar^2}{2a^2\sqrt{t_x t_y}} \\ \frac{\hbar^2}{2a^2\sqrt{t_x t_y}} & 0 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{\hbar^2}{2a^2 t_x} & 0 \\ 0 & \frac{\hbar^2}{2a^2 t_y} \end{pmatrix}$

(c) $\begin{pmatrix} -\frac{\hbar^2}{2a^2 t_x} & 0 \\ 0 & -\frac{\hbar^2}{2a^2 t_y} \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -\frac{\hbar^2}{2a^2 (t_x + t_y)} \\ -\frac{\hbar^2}{2a^2 (t_x + t_y)} & 0 \end{pmatrix}$

4. In a scattering experiment, a beam of e^- with an energy of 420 MeV scatters off an atomic nucleus. If the first minimum of the differential cross-section is observed at a scattering angle of 45°, the radius of the nucleus (in Fermi) is closest to

(a) 0.4 (b) 8.0 (c) 2.5 (d) 0.8

5. The Hamiltonian of a particle of mass *m* is given by $H = \frac{p^2}{2m} + V(x)$, with

$$V(x) = \begin{cases} -\alpha x & \text{for} \quad x \le 0 \\ \beta x & \text{for} \quad x > 0 \end{cases}$$

where α, β are positive constants. The n^{th} energy eigenvalue E_n obtained using WKB approximation is

$$E_n^{3/2} = \frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2} \pi \left(n - \frac{1}{2}\right) f(\alpha, \beta); \quad (n = 1, 2, \ldots)$$

The function $f(\alpha, \beta)$ is

(a) $\sqrt{\frac{\alpha^2\beta^2}{2(\alpha^2+\beta^2)}}$ (b) $\frac{\alpha\beta}{\alpha+\beta}$ (c) $\frac{\alpha+\beta}{4}$ (d) $\frac{1}{2}\sqrt{\frac{\alpha^2+\beta^2}{2}}$

6. The general solution for the second order differential equation $\frac{d^2y}{dx^2} - y = x\sin x$ will be

(a) $C_1e^x + C_2e^{-x} - \frac{1}{2}(x\sin x + \cos x)$ (b) $C_1e^x + C_2e^{-x} - \frac{1}{2}(\sin x - x\cos x)$

(c) $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin x - \cos x)$ (d) $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin x + \cos x)$

7. A particle of unit mass and unit charge is moving in a magnetic field, which varies as $\vec{B}(\vec{r}) = \frac{b_0\vec{r}}{r^3}$, ($b_0$ is a constant) over a region far away from the origin. If $\vec{L}$ is the instantaneous angular momentum of the particle within that region, then $d\vec{L}/dt$ is

(a) $2b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ (b) $-b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ (d) $b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ (d) Zero

8. Rotational energy of a molecule in the angular momentum state j is given by $E_j = \frac{\hbar^2}{2I}j(j+1)$, where I is the moment of inertia of the molecule. The probability that the molecule will be in its ground state at temperature T $\left(\text{such that } k_BT \gg \frac{\hbar^2}{2I}\right)$, is

(a) $\frac{3}{2}\frac{\hbar^2}{Ik_BT}$ (b) $\frac{2}{3}\frac{\hbar^2}{Ik_BT}$ (c) $\frac{1}{2}\frac{\hbar^2}{Ik_BT}$ (d) $\frac{\hbar^2}{Ik_BT}$

9. A piezoresistive pressure sensor utilizes change in electrical resistance (ΔR) with change in pressure (ΔP) as $\Delta R = -R_0\log_{10}(\Delta P/P_0)$, where $R_0 = 500\,\Omega$ and $P_0 = 1000$ mbar. A current of 2μA is passed through the sensor and the resultant voltage drop is measured using an analog-to-digital (ADC) converter having a range 0 to 1V. If a pressure change of 1 mbar is to be measured, amongst the given options, the minimum number of bits needed for the ADC is

(a) 12 (b) 14 (c) 8 (d) 10

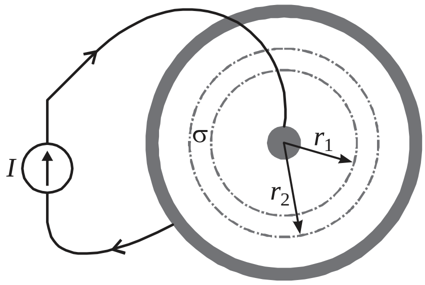
10. The bond dissociation energy of a molecule is defined as the energy required to dissociate it. For H_2 and H_2^+ molecules, the bond dissociation energies are 4.478 eV and 2.651 eV respectively. If the equilibrium bond lengths of both H_2 and H_2^+ are identical, the value of the ionization potential of hydrogen molecule will be closest to

(a) 15.427 eV (b) 11.773 eV (c) 20.729 eV (d) 6.471 eV

11. The integral $I = \int_0^1 \frac{2x}{1+x^2}dx$ is estimated using Simpson's 1/3rd rule with a grid value of $h = 0.5$. The difference $(I_{\text{estimated}} - I_{\text{exact}})$ is closest to

(a) 0.007 (b) 0.001 (c) 0.0007 (d) –0.005

12. A two dimensional sheet with a uniform sheet conductivity of σ has a central metallic point contact and a circular metal contact at the boundary as shown in the figure.



If a constant current I is injected through the central and collected at the boundary, then the voltage difference between two points on the sheet at radius r_1 and r_2 is proportional to

(a) $\frac{I}{\sigma}\left[\tan^{-1}\left(\frac{r_2}{r_1}\right)-\frac{\pi}{4}\right]$ (b) $\frac{I}{\sigma}\left[\ln\left(\frac{r_2}{r_1}\right)\right]$

(c) $\frac{I}{\sigma}\left(\frac{r_2-r_1}{r_2+r_1}\right)$ (d) $\frac{I}{\sigma}\left(\frac{r_2-r_1}{r_2+r_1}\right)^3$

13. In a non-magnetic material with no free charges and no free currents, the permittivity ε is a function of position. If $\vec{E}$ represents the electric field and μ_0, ε_0 are free space permeability and permittivity respectively, which one of the following expressions is ***correct***?

(a) $\nabla^2\vec{E}-\mu_0\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2}-\frac{1}{\varepsilon_0}\vec{\nabla}(\vec{E}\cdot\vec{\nabla}\varepsilon)=0$ (b) $\nabla^2\vec{E}-\mu_0\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2}+\frac{1}{\varepsilon_0}\vec{\nabla}(\vec{E}\cdot\vec{\nabla}\varepsilon)=0$

(c) $\nabla^2\vec{E}-\mu_0\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2}+\vec{\nabla}\left(\frac{1}{\varepsilon}\vec{E}\cdot\vec{\nabla}\varepsilon\right)=0$ (d) $\nabla^2\vec{E}-\mu_0\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2}-\vec{\nabla}\left(\frac{1}{\varepsilon}\vec{E}\cdot\vec{\nabla}\varepsilon\right)=0$

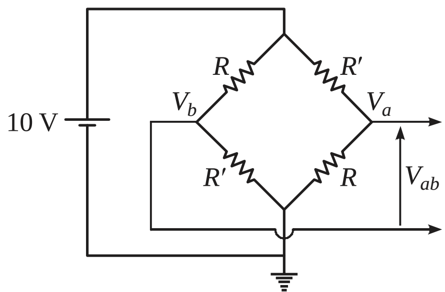
14. Five classical spins are placed at the vertices of a regular pentagon. The Hamiltonian of the system is $H=J\sum S_i S_j$, where $J>0,\ S_i=\pm1$ and the sum is over all possible nearest neighbour pairs. The degeneracy of the ground state is

(a) 8 (b) 5 (c) 4 (d) 10

15. An astronomer observes 500 objects and classifies them as either of type A to type B. She finds 148 objects to be of type B. Assuming a binomial distribution, the best estimate of the fraction of type A objects and its associated standard deviation respectively are

(a) 0.704, 0.002 (b) 0.70, 0.02 (c) 0.704, 0.031 (d) 0.72, 0.03

16. In the circuit shown in the figure, the resistance R and R' change due to strain. While R increases, R' decreases by the same amount ΔR due to the applied strain. The unstrained values of R and R' are 100 Ω each. If same strain is applied to all the resistors, and the output voltage (V_{ab}) changes to 0.3 V, then ΔR is closest to



(a) 3 Ω (b) 1.5 Ω (c) 4.5 Ω (d) 6 Ω

17. An atom of mass m, initially at rest, resonantly absorbs a photon. It makes a transition from the ground state to an excited state and also gets a momentum kick. If the difference between the energies of the ground state and the excited state is $\hbar\Delta$, the angular frequency of the absorbed photon is closest to

(a) $\Delta\left(1+\frac{3}{2}\frac{\hbar\Delta}{mc^2}\right)$ (b) $\Delta\left(1+\frac{1}{2}\frac{\hbar\Delta}{mc^2}\right)$ (c) $\Delta\left(1+\frac{\hbar\Delta}{mc^2}\right)$ (d) $\Delta\left(1+2\frac{\hbar\Delta}{mc^2}\right)$

18. An integral transform $\tilde{f}(x)$ of a function $f(x)$ can be regarded as a result of applying an operator F to the function such that

$$(Ff)(x) \equiv \tilde{f}(x) = \int_{-\infty}^{\infty} dy\, e^{-ixy} f(y)$$

If I is the identity operator, then the operator F^4 is given by:

(a) $(2\pi)^4 I$ (b) $(2\pi)I$ (c) I (d) $(2\pi)^2 I$

19. A random walker takes a step of unit length towards right or left at any discrete time step. Starting from $x=0$ at time $t=0$, it goes right to reach $x=1$ at $t=1$. Hereafter if it repeats the direction taken in the previous step with probability p, the probability that it is again at $x=1$ at $t=3$ is

(a) $1-p$ (b) $(1-p)^2$ (c) $2p(1-p)$ (d) $4p^2(1-p)$

20. The Δ^{++} can be produced by colliding a pion beam onto a H_2 target, in a reaction $\pi^+ + p \to \Delta^{++} \to \pi^+ + p$. In the rest frame of Δ^{++}, the energy and momentum of the pion in the final state (in MeV) are closest to

[Assume $c=1$ and $m_\pi \approx 140$ MeV, $m_p \approx 1$ GeV, $m_{\Delta^{++}} \approx 1.2$ GeV]

(a) 210, 156 (b) 230, 182 (c) 175, 105 (d) 190, 130

21. The Debye temperature of a two-dimensional insulator is 150 K. The ratio of the heat required to raise its temperature from 1K to 2K and from 2K to 3K is

(a) 7 : 19 (b) 3 : 13 (c) 1 : 1 (d) 3 : 5

22. A radio station antenna on the earth's surface radiates 50 kW power isotropically. Assume the electromagnetic waves to be sinusoidal and the ground to be a perfect absorber. Neglecting any transmission loss and effects of earth's curvature, the peak value of the magnetic field (in Tesla) detected at a distance of 100 km is closest to

(a) 1.5×10^{-11} (b) 5.5×10^{-11} (c) 8.5×10^{-11} (d) 3.5×10^{-11}

23. A particle of energy E is scattered off a one-dimensional potential $\lambda\delta(x)$, where λ is a real positive constant, with a transmission amplitude t_+. In a different experiment, the same particle is scattered off another one-dimensional potential $-\lambda\delta(x)$, with a transmission amplitude t_-. In the limit $E \to 0$, the phase difference between t_+ and t_- is

(a) $\pi/2$ (b) π (c) 0 (d) $3\pi/2$

24. For a simple harmonic oscillator, the Lagrangian is given by:

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$$

If $H(q, p)$ is the Hamiltonian of the system and $A(p,q) = \frac{1}{\sqrt{2}}(p+iq)$, the Poisson bracket $\{A, H\}$ is

(a) iA (b) A^* (c) $-iA^*$ (d) $-iA$

25. A linear molecule is modelled as two atoms of equal mass m placed at coordinates x_1 and x_2, connected by a spring of spring constant k. The molecule is moving in one dimension under an additional external potential $V(x_1, x_2) = \frac{1}{2}m\omega_0^2(x_1^2 + x_2^2)$. If one frequency of molecular vibration is ω_0, the other frequency is

(a) $\sqrt{\omega_0^2 - \frac{k}{m}}$ (b) $\sqrt{\omega_0^2 + \frac{k}{m}}$ (c) $\sqrt{\omega_0^2 + \frac{2k}{m}}$ (d) $\sqrt{\omega_0^2 - \frac{2k}{m}}$

26. Consider a body-centered tetragonal lattice with lattice constants $a = b = a_0$ and $c = \frac{a_0}{2}$. The number of nearest neighbours, the nearest neighbour distance, the number of next nearest neighbours and the next nearest neighbour distance, respectively, are

(a) $6, \frac{1}{2}a_0, 8, \frac{\sqrt{3}}{2}a_0$ (b) $8, \frac{\sqrt{3}}{2}a_0, 6, a_0$ (c) $2, \frac{1}{2}a_0, 8, \frac{3}{4}a_0$ (d) $8, a_0, 6, \frac{4}{3}a_0$

27. The following four matrices form a representation of a group

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Which of the following represents the multiplication table for the same group ?

(a)

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	A	I
C	C	B	I	A

(b)

	I	A	B	C
I	I	A	B	C
A	A	B	C	I
B	B	C	I	A
C	C	I	A	B

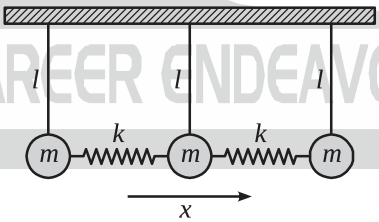
(c)

	I	A	B	C
I	I	A	B	C
A	A	C	I	B
B	B	I	C	A
C	C	B	A	I

(d)

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	I	A
C	C	B	A	I

28. Three identical simple pendula (of mass m and equilibrium string length l) are attached together by springs of spring constant k, as shown in the figure.



The frequencies of small oscillations are given by $\sqrt{\frac{g}{l}}, \sqrt{\frac{k}{m} + \frac{g}{l}}, \sqrt{\frac{3k}{m} + \frac{g}{l}}$. The normal modes (without normalisation) corresponding to these frequencies respectively are:

(a) (1, 1, 1), (1, 0, 1), (1, –2, 1) (b) (1, 1, 1), (1, 0, –1), (1, 2, 1)

(c) (1, 1, 1), (1, 0, –1), (1, –2, 1) (d) (1, 2, 1), (1, 0, –1), (1, 1, 1)

29. Using a normalized trial wavefunction $\psi(x) = \sqrt{\alpha}\, e^{-\alpha|x|}$, ($\alpha$ is a positive real constant) for a particle of mass m in the potential $V(x) = -\lambda\delta(x)$, $(\lambda > 0)$, the estimated ground state energy is

(a) $-\frac{m\lambda^2}{\hbar^2}$ (b) $\frac{m\lambda^2}{\hbar^2}$ (c) $\frac{m\lambda^2}{2\hbar^2}$ (d) $-\frac{m\lambda^2}{2\hbar^2}$

30. A particle of mass m is moving in a potential $V(r) = -\frac{k}{r}$, where k is a positive constant. If $\vec{L}$ and $\vec{p}$ denote the angular momentum and linear momentum respectively, the value of 'α' for which $\vec{A} = \vec{L} \times \vec{p} + \alpha mk\hat{r}$ is a constant of motion, is

(a) –2 (b) –1 (c) 2 (d) 1

****************** END OF QUESTION PAPER ******************

NTA-JOINT CSIR-UGC NET – June-2024
PHYSICAL SCIENCES

ANSWER KEY

PART–A

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (d)
6. (d)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (a)
11. (c)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (c)
16. (c)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (d)

PART–B

1. (c)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (d)
6. (b)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (d)
11. (a)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (d)
16. (a)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (b)
21. (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (d)

PART–C

1. (d)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (b)
6. (a)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (a)
11. (a)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (b)
16. (a)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (d)
21. (a)	**22.** (b)	**23.** (b)	**24.** (a)	**25.** (c)
26. (c)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (d)